SOLVED EXERCISE 1.1

1. **Write the following quadratic equations in the standard form and point out pure quadratic equations.**

(i) $(x + 7)(x - 3) = -7$

Solution:

$$(x + 7)(x - 3) = -7$$
$$x(x - 3) + 7(x - 3) = -7$$
$$x^2 - 3x + 7x - 21 = -7$$
$$x^2 + 4x - 21 + 7 = 0$$
$$x^2 + 4x - 14 = 0$$

The above equation is a quadratic equation.

(ii) $\frac{x^2 + 4}{3} - \frac{x}{7} = 1$

Solution:

$$\frac{x^2 + 4}{3} - \frac{x}{7} = 1$$

Multiply both sides by 21, we get

$$21 \times \frac{x^2 + 4}{3} - 21 \times \frac{x}{7} = 1 \times 21$$
$$7\left(x^2 + 4\right) - 3x = 21$$
$$7x^2 + 28 - 3x = 21$$
$$7x^2 - 3x + 28 - 21 = 0$$
$$7x^2 - 3x + 7 = 0$$

(iii) $\frac{x}{x+1} + \frac{x+1}{x} = 6$

Solution:

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$
$$\frac{x^2 + \left(x^2 + 1\right)^2}{x\left(x + 1\right)} = 6$$
$$x^2 + x^2 + 2x + 1 = 6x\left(x + 1\right)$$
$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$2x^2 - 6x^2 + 2x - 6x + 1 = 0$$

$$-4x^2 - 4x + 1 = 0$$

$$-(4x^2 + 4x - 1) = 0$$

$$\Rightarrow \quad 4x^2 + 4x - 1 = 0$$

The above equation is a quadratic equation.

(iv) $\dfrac{x+4}{x-2} - \dfrac{x-2}{x} + 4 = 0$

Solution:

$$\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$$

$$\frac{x(x+4) - (x-2)^2 + 4x(x-2)}{x(x-2)}$$

$$\Rightarrow \quad (x^2 + 4x) - (x^2 - 4x + 4) + 4(x^2 - 8x) = 0$$

$$x^2 + 4x - x^2 + 4x - 4 + 4x^2 - 8x = 0$$

$$x^2 - x^2 + 4x^2 + 4x + 4x - 8x - 4 = 0$$

$$4x^2 + 8x - 8x - 4 = 0$$

$$4x^2 - 4 = 0$$

$$4(x^2 - 1) = 0 \quad \Rightarrow \quad x^2 - 1 = 0$$

The above equation is a pure quadratic equation.

(v) $\dfrac{x+3}{x+4} - \dfrac{x-5}{x} = 1$

Solution:

$$\frac{x+3}{x+4} - \frac{x-5}{x} = 1$$

$$\frac{x(x+3) - (x+4)(x-5)}{x(x+4)} = 1$$

$$\left(x^2+3x\right)-x(x-5)-4(x-5)=x(x+4)$$
$$x^2+3x-x^2+5x-4x+20=x^2+4x$$
$$x^2-x^2+3x+5x-4x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$-x^2+4x-4x+20=0$$
$$-x^2-20=0$$
$$-\left(x^2-20\right)=0 \Rightarrow x^2-20=0$$

The above equation is a pure quadratic equation.

(vi) $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$
$$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{x(x+3)+1(x+3)+\left(x^2+4x+4\right)}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6}=\frac{25}{12}$$
$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$
$$25\left(x^2+5x+6\right)=12\left(2x^2+8x+7\right)$$
$$25x^2+125x+150=24x^2+96x+84$$
$$x^2+29x+66=0$$

The above equation is a pure quadratic equation.

2. Solve by factorization:

(i) $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$
$$x^2-5x+4x-20=0$$
$$x(x-5)+4(x-5)=0$$
$$(x+4)(x-5)=0$$

(ii) $3y^2 = y(y-5)$

Solution:

$$3y^2 = y(y-5)$$
$$3y^2 = y^2 - 5y$$
$$3y^2 - y^2 + 5y = 0$$
$$2y^2 + 5y = 0$$
$$y(2y+5) = 0$$

Either $y = 0$ or $2y + 5 = 0$

$$2y = -5$$
$$y = -\frac{5}{2}$$

Thus, solution set $= \left\{0, -\frac{5}{2}\right\}$.

(iii) $4 - 32x = 17x^2$

Solution:

$$4 - 32x = 17x^2$$

or $17x^2 + 32x - 4 = 0$

$$17x^2 + 34x - 2x - 4 = 0$$
$$17x(x+2) - 2(x+2) = 0$$
$$(17x-2)(x+2) = 0$$

Either $17x - 2 = 0$ or $x = 2 = 0$

$17x = 2$ $\qquad$ $x = -2$

$$x = \frac{2}{17}$$

Thus, solution set $= \left\{\frac{2}{17}, -2\right\}$

(iv) $x^2 - 11x = 152$

Solution:

$$x^2 - 11x = 152$$
$$x^2 - 11x - 152 = 0$$
$$x^2 - 19x + 8x - 152 = 0$$
$$x(x-19) + 8(x-19) = 0$$
$$(x+8)(x-19) = 0$$

Either $x + 8 = 0$ or $x - 19 = 0$

$x = -8$ $\qquad$ $x = 19$

Thus, solution set $= \{-8, 19\}$

(v) $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

Solution:

$$\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$$

$$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$$

$$\frac{x^2+2x+1+x^2}{x^2+x}=\frac{25}{12}$$

$$\frac{2x^2+2x+1}{x^2+x}=\frac{25}{12}$$

$$25(x^2+x)=12(2x^2+2x+1)$$

$$25x^2+25x=24x^2+24x+12$$

$$25x^2-24x^2+25x-24x-12=0$$

$$x^2+x-12=0$$

$$x^2+4x-3x-12=0$$

$$x(x+4)-3(x+4)=0$$

$$(x-3)(x+4)=0$$

Either $x-3=0$ or $x+4=0$

$x=3$ $x=-4$

Thus, solution set = $\{3,-4\}$

(iv) $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

Solution:

$$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$

$$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$$

$$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-7x+12}$$

$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$

$$2(x^2-x+12)=-1(x-9)$$

$$2x^2-14x+24=-x+9$$

$$2x^2-14x+x+24-9=0$$

$$2x(x-5)-3(x-5)=0$$

$$(2x-3)(x-5)=0$$

Either $2x - 3 = 0$ or $x - 5 = 0$

$2x = 3$ $\quad$ $x = 5$

$x = \frac{3}{2}$

Thus, Solution set = $\left\{5, \frac{3}{2}\right\}$

Q3. Solve the following equations by completing square:

(i) $7x^2 + 2x - 1 = 0$

Solution:

$$7x^2 + 2x - 1 = 0$$

$$7x^2 + 2x = 1$$

$$\frac{7x^2}{7} + \frac{2x}{7} = \frac{1}{7}$$

$$x^2 + \frac{2x}{7} = \frac{1}{7}$$

$$(x)^2 + 2(x)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{1}{7} + \frac{1}{49}$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking square root on both sides, we get

$$x + \frac{1}{7} = \pm\sqrt{\frac{8}{49}}$$

$$x + \frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$

$$x = \frac{1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1 \pm 2\sqrt{2}}{7}$$

Thus, solution set = $\left\{\frac{-1 \pm 2\sqrt{2}}{7}\right\}$

(ii) $ax^2 + 4x - a = 0$

Solution:

$$ax^2 + 4x - a = 0$$

$$ax^2 + 4x = a$$

$$\frac{ax^2}{a} + \frac{4x}{a} = \frac{a}{a}$$

$$x^2 + \frac{4x}{a} = 1$$

$$(x)^2 + 2(x)\left(\frac{2}{a}\right) + \left(\frac{2}{a}\right)^2 = 1 + \left(\frac{2}{a}\right)^2$$

$$\left(x + \frac{2}{a}\right)^2 = 1 + \frac{4}{a^2}$$

$$\left(x + \frac{2}{a}\right)^2 = \frac{a^2 + 4}{a^2}.$$

Taking square root on both sides, we get

$$x + \frac{2}{a} = \pm\sqrt{\frac{a^2 + 4}{a^2}}$$

$$x = -\frac{2}{a} \pm \frac{\sqrt{a^2 + 4}}{a}$$

$$x = \frac{-2 \pm \sqrt{a^2 + 4}}{a}.$$

Thus, solution set $= \left\{\frac{-2 \pm \sqrt{a^2 + 4}}{a}\right\}$

(iii) $11x^2 - 34x + 3 = 0$

Solution:

$$11x^2 - 34x + 3 = 0$$

$$11x^2 - 34x = -3$$

$$\frac{11x^2}{11} - \frac{34}{11}x = -\frac{3}{11}$$

$$x^2 - \frac{34}{11}x = -\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=-\frac{3}{11}+\left(\frac{34}{22}\right)^2$$

$$\left(x-\frac{34}{22}\right)^2=-\frac{3}{11}+\frac{1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{132+1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{1024}{484}$$

Taking square root on both sides we get

$$\left(x-\frac{34}{22}\right)^2=\pm\sqrt{\frac{1024}{484}}$$

$$x-\frac{34}{22}=\pm\frac{32}{22}$$

$$x=\frac{34}{22}\pm\frac{32}{22}$$

$$x=\frac{34\pm 32}{22}$$

$$x=\frac{34+32}{22},x=\frac{34-32}{22}$$

$$=\frac{66}{22} \qquad =\frac{2}{22}$$

$$=3 \qquad =\frac{1}{11}$$

Thus, solution set $\left\{3,\frac{1}{11}\right\}$

(iv) $\ell x^2-mx+n=0$

Solution:

$$\ell x^2-mx+n=0$$

$$\ell x^2+mx=-n$$

$$\frac{\ell x^2}{\ell}+\frac{mx}{\ell}=-\frac{n}{\ell}$$

$$x^2+\frac{mx}{\ell}=-\frac{n}{\ell}$$

$$(x)^2+2(x)\left(\frac{m}{2\ell}\right)+\left(\frac{m}{2\ell}\right)^2=-\frac{n}{\ell}+\left(\frac{m}{2\ell}\right)^2$$

$$\left(x+\frac{m}{2l}\right)^2=-\frac{n}{l}+\frac{m^2}{4l^2}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{-4ln+m^2}{4l^2}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{m^2-4ln}{4l^2}$$

Taking square root an both sides, we get

$$\sqrt{\left(x+\frac{m}{2l}\right)^2}=\pm\frac{\sqrt{m^2-4ln}}{4l^2}$$

$$x+\frac{m}{2l}=\pm\frac{\sqrt{m^2-4ln}}{2l}$$

$$x=\frac{m}{2l}\pm\frac{\sqrt{m^2-4ln}}{2l}$$

$$x=\frac{-m\pm\sqrt{m^2-4ln}}{2l}$$

Thus, solution set $=\left\{\frac{-m\pm\sqrt{m^2-4ln}}{2l}\right\}$

(v) $3x^2+7x=0$

Solution:

$$3x^2+7x=0$$

$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$

$$x^2+\frac{7}{3}x=0$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking square root on both sides, we get.

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\left(\frac{7}{6}\right)^2}$$

$$x+\frac{7}{6}=\pm\frac{7}{6}$$

$$x=-\frac{7}{6}\pm\frac{7}{6}$$

$$x=-\frac{7}{6}+\frac{7}{6} \quad \text{or} \quad x=-\frac{7}{6}-\frac{7}{6}$$

$$x=0 \qquad\qquad x=-\frac{14}{6}$$

$$x=-\frac{7}{3}$$

Thus, solution set = $\left\{0,-\frac{7}{3}\right\}$

(vi) $x^2-2x-195=0$

Solution:

$$x^2-2x-195=0$$

$$x^2-2x=195$$

$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root on both sides, we get

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$

$$x-1=\pm14$$

$$x=1\pm14$$

$$x=1\pm14 \quad \text{or} \quad x=1-14$$

$$=15 \qquad\qquad =-13$$

Thus, solution set = $\{-13.15\}$

(vii) $-x^2+\frac{15}{2}=\frac{7}{2}x$

Solution:

$$-x^2+\frac{15}{2}=\frac{7}{2}x$$

$$-x^2-\frac{7}{2}x=-\frac{15}{2}$$

$$-\left(x^2+\frac{7}{2}x\right)=-\frac{15}{2}$$

$$\Rightarrow \quad x^2+\frac{7}{2}x=\frac{15}{2}$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=-\frac{7}{4}\pm\frac{13}{4}$$

$$x=-\frac{7}{4}+\frac{13}{4} \quad \text{or} \quad x=-\frac{7}{4}-\frac{13}{4}$$

$$x=\frac{6}{4} \qquad x=-\frac{20}{4}$$

$$x=\frac{3}{2} \qquad x-5$$

(viii) $x^2+17x+\frac{33}{4}=0$

Solution:

$$x^2+17x+\frac{33}{4}=0$$

$$x^2 + 17x = -\frac{33}{4}$$

$$(x)^2 + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = -\frac{33}{4} + \left(\frac{17}{2}\right)^2$$

$$\left(x + \frac{17}{2}\right)^2 = -\frac{33}{4} + \frac{289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$$

Taking square root on both sides,

$$\sqrt{\left(x + \frac{17}{2}\right)^2} = \pm\sqrt{\frac{256}{4}}$$

$$x + \frac{17}{2} = \pm\frac{16}{2}$$

$$x = -\frac{17}{2} \pm \frac{16}{2}$$

$$x = -\frac{17}{2} + \frac{16}{2} \quad \text{or} \quad x = -\frac{17}{2} - \frac{16}{2}$$

Thus, solution set = $\left\{-\frac{1}{2}, -\frac{33}{2}\right\}$

(ix) $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

Solution:

$$4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$\frac{4(3x+1)-8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$\frac{12x+4-8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$\frac{12x-4}{3x+1} = \frac{3x^2+5}{3x+1}$$

Multiplying both sides by (3x + 1), we get

$$12x - 4 = 3x^2 + 5$$

or $$3x^2 + 5 - 12x + 4 = 0$$

$$3x^2 - 12x + 9 = 0$$

$$3(x^2 - 4x + 3) = 0$$

$\Rightarrow$ $$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x-3)-1(x-3) = 0$$
$$(x-1)(x-3) = 0$$

Either $x-1=0$ or $x-3=0$

$x=1$ $\quad x=3$

Thus, solution set = $\{1, 3\}$

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution:

$$7(x+2a)^2+3a^2 = 5a(7x+23a)$$
$$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$$
$$7x^2+28ax+28a^2+3a^2=35ax+115a^2$$
$$7x^2-7ax-84a^2=0$$
$$7(x^2-ax-12a^2)=0$$
$$x^2-ax-12a^2=0$$
$$x^2-ax=12a^2$$
$$\Rightarrow \quad (x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root on both sides, we get

$$\sqrt{\left(x-\frac{a}{2}\right)^2} = \pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a}{2}+\frac{7a}{2}, x=\frac{a}{2}-\frac{7a}{2}$$
$$=\frac{8a}{2}=-\frac{6a}{2}$$
$$=4a=-3a$$

Thus, solution set = $\{-3a, 4a\}$

Quadratic Formula:

Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$ax^2 + bx + c = 0, \ a \neq 0$

Dividing each term of the equation by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Shifting constant term $\frac{c}{a}$ to the right, we have

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding $\left(\frac{b}{a}\right)^2$ on both sides, we obtain

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

or $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Taking square root of both sides, we get

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

or $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Thus, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is known as "quadratic formula".

SOLVED EXERCISE 1.2

Q1. Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution:

$$2 - x^2 = 7x$$

$$-x^2 - 7x + 2 = 0$$

$$-(x^2 + 7x - 2) = 0$$

$\Rightarrow$ $x^2 + 7x - 2 = 0$

Compare it with, we have

$ax^2 + bx + c = 0$