

## Quadratic Formula:

### Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Dividing each term of the equation by  $a$ , we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Shifting constant term  $\frac{c}{a}$  to the right, we have

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding  $\left(\frac{b}{2a}\right)^2$  on both sides, we obtain

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\text{or } \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking square root of both sides, we get

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\text{or } x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  is known as "quadratic formula".

## SOLVED EXERCISE 1.2

Q1. Solve the following equations using quadratic formula:

(i)  $2 - x^2 = 7x$

Solution:

$$2 - x^2 = 7x$$

$$-x^2 - 7x + 2 = 0$$

$$-(x^2 + 7x - 2) = 0$$

$$\Rightarrow x^2 + 7x - 2 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = 1, b = 7, c = -2$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

Thus, solution set =  $\left\{ \frac{-7 \pm \sqrt{57}}{2} \right\}$

(ii)  $5x^2 + 8x + 1 = 0$

Solution:

$$5x^2 + 8x + 1 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = 5, b = 8, c = 1$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{2(-4 \pm \sqrt{11})}{10}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Thus, solution set =  $\left\{ \frac{-4 \pm \sqrt{11}}{5} \right\}$

(iii)  $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution:

$$\sqrt{3}x^2 + x = 4\sqrt{3}$$

$$\sqrt{3}x^2 + x - 4\sqrt{3} = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = \sqrt{3}, b = 1, c = -4\sqrt{3}$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1 - 48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}} \quad \text{or} \quad x = \frac{-1 - 7}{2\sqrt{3}}$$

$$x = \frac{6}{2\sqrt{3}} \quad x = \frac{-8}{2\sqrt{3}}$$

$$x = \frac{3}{\sqrt{3}} \quad x = -\frac{4}{\sqrt{3}}$$

$$x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{3\sqrt{3}}{(\sqrt{3})^2}$$

$$x = \frac{3\sqrt{3}}{3}$$

$$x = \sqrt{3}$$

Thus, solution set =  $\left\{ \sqrt{3}, -\frac{4}{\sqrt{3}} \right\}$

(iv)  $4x^2 - 14 = 3x$

Solution:

$$4x^2 - 14 = 3x$$

$$4x^2 - 3x - 14 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

$$ax^2 + bx + c = 0$$

Here  $a = 4, b = -3, c = -14$

$$\text{Now } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

$$x = \frac{3 \pm \sqrt{233}}{8}$$

$$\text{Thus, solution set} = \left\{ \frac{3 \pm \sqrt{233}}{8} \right\}$$

$$(v) 6x^2 - 3 - 1x = 0$$

Solution:

$$6x^2 - 3 - 1x = 0$$

$$6x^2 - 7x - 3 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = 6, b = -7, c = -3$

$$\text{Now } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$$x = \frac{7 \pm 11}{12}, \quad x = \frac{7-11}{12}$$

$$= \frac{18}{12} \quad = \frac{4}{12}$$

$$= \frac{3}{2} \quad = -\frac{1}{3}$$

Thus, solution set =  $\left\{-\frac{1}{3}, \frac{3}{2}\right\}$

(vi)  $3x^2 + 8x + 2 = 0$

Solution:

$$3x^2 + 8x + 2 = 0$$

Compare it with

$$ax^2 + bx + c = 0$$

Here  $a = 3, b = 8, c = 2$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{10}}{6}$$

$$x = \frac{2(-4 \pm \sqrt{10})}{6}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

Thus, solution set =  $\left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii)  $\frac{3}{x-6} - \frac{4}{x-5} = 1$

Solution:

$$\frac{3}{x-6} - \frac{4}{x-5} = 1$$

$$\frac{3(x-5) - 4(x-6)}{(x-6)(x-5)} = 1$$

$$3x - 15 - 4x + 24 = (x-6)(x-5)$$

$$-x + 9 = x^2 - 11x + 30$$

$$x^2 - 11x + x + 30 - 9 = 0$$

$$x^2 - 10x + 21 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = 1, b = -10, c = 21$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$x = \frac{10 \pm \sqrt{16}}{2}$$

$$x = \frac{-8 \pm 4}{2}$$

$$x = \frac{10+4}{2}, \quad x = \frac{10-4}{2}$$

$$x = \frac{14}{2}, \quad x = \frac{6}{2}$$

$$x = 7, \quad x = 3$$

Thus, solution set =  $\{3, 7\}$

(viii)  $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

Solution:

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$$

$$\frac{2x(x+2) - (x-1)(4-x)}{2x(x-1)} = \frac{7}{3}$$

$$\frac{(2x^2 + 4x) - (4x - x^2 - 4 + x)}{2x^2 - 2x} = \frac{7}{3}$$

$$\frac{2x^2 + 4x + x^2 - 5x + 4}{2x^2 - 2x} = \frac{7}{3}$$

$$7(2x^2 - 2x) = 3(3x^2 - x + 4)$$

$$14x^2 - 14x = 9x^2 - 3x + 12$$

$$14x^2 - 9x^2 - 14x + 3x - 12 = 0$$

$$5x^2 - 11x - 12 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = 5, b = -11, c = -12$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(-12)}}{2(5)}$$

$$x = \frac{11 \pm \sqrt{121 - 240}}{10}$$

$$x = \frac{11 \pm \sqrt{361}}{10}$$

$$x = \frac{11 \pm 19}{10}$$

$$x = \frac{11+19}{10}, \quad x = \frac{11-19}{10}$$

$$x = \frac{30}{10}, \quad x = \frac{-8}{10}$$

$$x = 3, \quad x = -\frac{4}{5}$$

Thus, solution set =  $\left\{3, -\frac{4}{5}\right\}$

(ix)  $\frac{a}{x-b} + \frac{b}{x-a} = 2$

Solution:

$$\frac{a}{x-b} + \frac{b}{x-a} = 2$$

$$\frac{a(x-a) + b(x-b)}{(x-a)(x-b)} = 2$$

$$ax - a^2 + bx - b^2 = 2(x-a)(x-b)$$

$$ax + bx - a^2 - b^2 = 2(x^2 - ax - bx + ab)$$

$$ax + bx - a^2 - b^2 = 2x^2 - 2ax - 2bx + 2ab$$

$$2x^2 - 3ax - 3bx + 2ab + a^2 + b^2 = 0$$

$$2x^2 - 3(a+b)x + (2ab + a^2 + b^2) = 0$$

$$2x^2 - 3(a+b)x + (a+b)^2 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = 2$ ,  $b = -3(a+b)$ ;  $c = (a+b)^2$

Now  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-[-3(a+b)] \pm \sqrt{[-3(a+b)]^2 - 4(2)(a+b)^2}}{2(2)}$$

$$x = \frac{3(a+b) \pm \sqrt{9(a+b)^2 - 8(a+b)^2}}{4}$$

$$x = \frac{3(a+b) \pm \sqrt{(a+b)^2}}{4}$$

$$x = \frac{3(a+b) \pm (a+b)}{4}$$

$$x = \frac{3(a+b) + (a+b)}{4}, \quad x = \frac{3(a+b) - (a+b)}{4}$$

$$x = \frac{3a + 3b + a + b}{4}, \quad x = \frac{3a + 3b - a - b}{4}$$

$$x = \frac{4a + 4b}{4}, \quad x = \frac{2a + 2b}{4}$$

$$x = \frac{4(a+b)}{4}, \quad x = \frac{2(a+b)}{4}$$

$$x = a + b, \quad x = \frac{1}{2}(a+b)$$

Thus, solution set =  $\left\{ (a+b), \frac{1}{2}(a+b) \right\}$

(x)  $-(l+m) - lx^2 + (2l+m)x = 0$



Solution:

$$-(l+m) - lx^2 + (2l+m)x = 0, l \neq 0$$

$$-lx^2 + (2l+m)x - (l+m) = 0$$

$$-[lx^2 - (2l+m)x + (l+m)] = 0$$

$$\Rightarrow lx^2 - (2l+m)x + (l+m) = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here  $a = l$ ,  $b = -(2l+m)$ ,  $c = (l+m)$

$$\text{Now } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-(2l+m) \pm \sqrt{[-(2l+m)]^2 - 4(l)(l+m)}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{(2l+m)^2 - 4l(l+m)}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{4l^2 + 4lm + m^2 - 4l^2 - 4lm}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{m^2}}{2l}$$

$$x = \frac{(2l+m) \pm m}{2l}$$

$$x = \frac{2l+2m}{2l}, \quad x = \frac{2l+2m-m}{2l}$$

$$x = \frac{2l+2m}{2l} = \frac{2l}{2l}$$

$$= \frac{2(l+m)}{2l} = l$$

$$= \frac{l+m}{l}$$

Thus, solution set =  $\left\{ l, \frac{l+m}{l} \right\}$

### SOLVED EXERCISE 1.3

Q1. Solve the following equations.

(1)  $2x^4 - 11x^2 - 5 = 0$