Proof:

Statements	Reasons
Since ABCD and A'B'C'D' are two congruent circles with centres O and O' respectively.	Given
Place the circle ABCD on the circle A'B'C'D'	
So that point O falls on O' (i)	
Also $m\angle AOC = m\angle A'O'C'$	Given
$m\overline{OA} = m\overline{O'A'}$ (ii)	Radii for congruent circles
and $m\overline{OC} = m\overline{O'C'}$ (iii)	Radii for congruent circles
So point A will coincide with A' and point C will Coincide with C'	Using (i), (ii) and (iil)
Now every point on $\overparen{ADC}$ or on $\overparen{A'D'C'}$ is equidistant from the centres O and O' respectively.	
Hence $\overparen{ADC}$ coincides with $\overparen{A'D'C'}$.	Using theorem 1
or $m\overparen{AC} = m\overparen{A'C'}$	
i.e., $m(\overparen{ADC}) = m(\overparen{A'D'C'})$	

SOLVED EXERCISE 11.1

1. **In a circle two equal chords AB and CD intersect each other. Prove that mAD = mBC.**

Solution:

Given:

In a circle having centre at O.

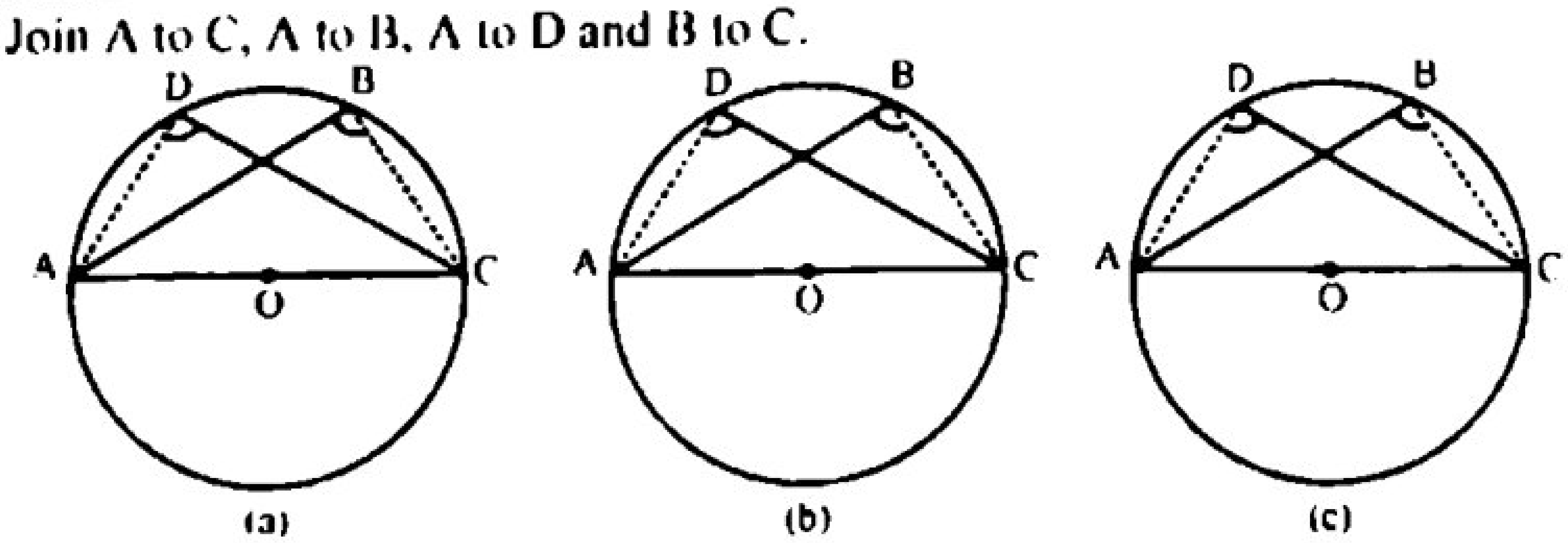
$m\overline{OB} \cong m\overline{CD}$

To prove:

$m\overline{AD} = m\overline{BC}$

Construction:

Join A to C, A to B, A to D and B to C.



SOLVED MISCELLANEOUS EXERCISE - 11

1. **Multiple Choice Questions**
Four possible answers are given for the following questions.
Tick (✓) the correct answer.

(i) A 4 cm long chord sub stands a central angle of 60°. The radial segment this circle is:
(a) 1 (b) 1 (c) 3 (d) 4

(ii) The length of a chord and the radial segment of a circle are congruent, the central angle made by the chord will be:
(a) 30° (b) 45° (c) 60° (d) 75°

(iii) Out of two congruent arcs of a circle, if one are makes a central angle of 30° then the other arc will subtend the central angle of:
(a) 15° (b) 30° (c) 45° (d) 60°

(iv) An arc subtends a central angle of 40° then the corresponding chord will subtend a central angle of:
(a) 26° (b) 40° (c) 60° (d) 8.0°

(v) A pair of chords of a circle subtending two congruent central angles is:
(a) congruent (b) incongruent (c) overlapping (d) parallel

(vi) If an area of a circle subtends a central angle of 60°, then the corresponding chord of the area will make the central angle of:
(a) 20° (b) 40° (c) 60° (d) 80°

(vii) The semi circumference and the diameter of a circle both subtend a central angle of:
(a) 90° (b) 180° (c) 270° (d) 360°

(viii) The chord length of a circle subtending a central angle of 180° is always:
(a) less than radial segment (b) equal to the radial segment
(c) double of the radial segment (d) none of these

(ix) If a chord of a circle subtends, a central angle of 60°, then the length of the chord and the radial segment are:
(a) congruent (b) incongruent (c) parallel (d) perpendicular

(x) The arcs opposite to incongruent central angles of a circle are always:
(a) congruent (b) incongruent (c) parallel (d) perpendicular

Answers:

(i)	d	(ii)	c	(iii)	b	(iv)	b	(v)	a
(vi)	d	(vii)	d	(viii)	b	(ix)	a	(x)	b

SUMMARY

- ✔ The boundary traced by a moving point in a circle is called its circumference whereas any portion of the circumference will be known as an arc of the circle.
- ✔ The straight, line joining any two points of the circumference is called a chord of the circle.
- ✔ The portion of a circle bounded by an arc and a chord is known as the segment of a circle.
- ✔ The circular region bounded by an arc of a circle and its two corresponding radial segments is called a sector of the circle.
- ✔ A straight Sine, drawn from the centre of a circle bisecting a chord is perpendicular to the chord and conversely perpendicular drawn from the centre of a circle on a chord, bisects it.
- ✔ If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
- ✔ If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent.
- ✔ Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).
- ✔ If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, the chords are equal.

	arcs.
So $m\overline{AQ} = m\overline{AP}$ or $m\overline{AP} = m\overline{AQ}$	Sides opposite to equal angles in ΔAPQ.

Example 2:

ΔBCD is a quadrilateral circumscribed about a circle Show that
$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Given:

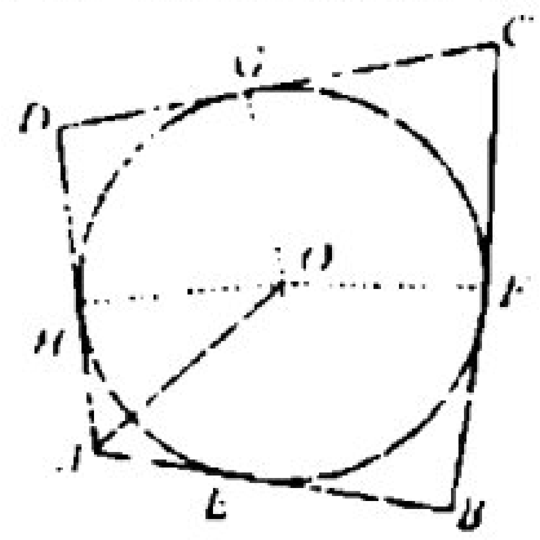
ΔBCD is a quadrilateral circumscribed about a circle with centre O.
So that each side becomes tangent to the circle.

To prove:

$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Construction:

Drawn $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{BC}$, $\overline{OG} \perp \overline{CD}$ and $\overline{OH} \perp \overline{DA}$



Proof:

Statements	Reasons
$\therefore m\overline{AE} = m\overline{HA};\ m\overline{EB} = m\overline{BF}$ ...(i) $m\overline{CG} = m\overline{FC}$ and $m\overline{GD} = m\overline{DH}$ ...(ii)	Since tangents drawn from a point to the circle are equal in length
$\left(m\overline{AE} + m\overline{EB}\right) + \left(m\overline{CG} + m\overline{GD}\right)$ $= \left(m\overline{BF} + m\overline{FC}\right) + \left(m\overline{DH} + m\overline{HA}\right)$ or $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$	Adding (i) & (ii)

SOLVED EXERCISE 12.1

1. Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.

Solution:

Given:

A quadrilateral ABCD.