	arcs.
So $m\overline{AQ} = m\overline{AP}$ or $m\overline{AP} = m\overline{AQ}$	Sides opposite to equal angles in ΔAPQ.

Example 2:

ΔBCD is a quadrilateral circumscribed about a circle Show that $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Given:

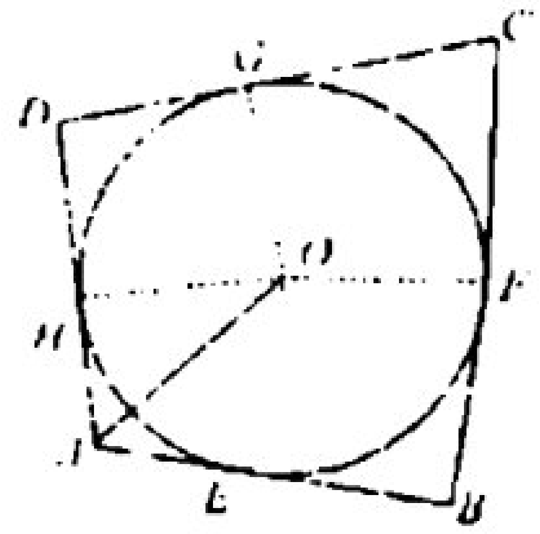
ΔBCD is a quadrilateral circumscribed about a circle with centre O.
So that each side becomes tangent to the circle.

To prove:

$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Construction:

Drawn $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{BC}$, $\overline{OG} \perp \overline{CD}$ and $\overline{OH} \perp \overline{DA}$



Proof:

Statements	Reasons
$\therefore m\overline{AE} = m\overline{HA};\ m\overline{EB} = m\overline{BF}$ ...(i) $m\overline{CG} = m\overline{FC}$ and $m\overline{GD} = m\overline{DH}$ ...(ii)	Since tangents drawn from a point to the circle are equal in length
$\left(m\overline{AE} + m\overline{EB}\right) + \left(m\overline{CG} + m\overline{GD}\right)$ $= \left(m\overline{BF} + m\overline{FC}\right) + \left(m\overline{DH} + m\overline{HA}\right)$ or $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$	Adding (i) & (ii)

SOLVED EXERCISE 12.1

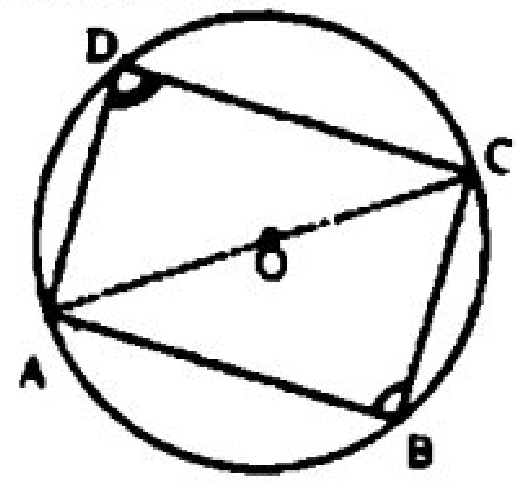
1. Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.

Solution:

Given:

A quadrilateral ABCD.

In scribed in a circle whose centre is O.



To Prove:

$\angle A + \angle C = 2rt \angle S$.

And $\angle B + \angle D = 2rt \angle 2$.

Construction:

Join AO and OC

Proof:

Arc ABC subtends $\angle AOC$ at the centre O and.

$\angle ADC$ at a point D on the remaining part of the circumference.

$$m\angle ADC = \frac{1}{2} \angle AOC \qquad \text{........(i)}$$

Similarly, arc ADC subtends reflex $\angle AOC$ at the centre and $\angle ABC$ on the circumferences.

$$m\angle ABC = \frac{1}{2} \angle AOC \qquad \text{........(ii)}$$

By Adding (i) & (ii)

$$m\angle ADC + m\angle ABC = \frac{1}{2} [m\angle AOC + m\angle AOC]$$

$$m\angle D + m\angle B \frac{1}{2} [4rt \angle 5]$$

$m\angle ADC + m\ 4B = 2rt \angle S$. Proved.

Similarly, by Join BO and OD it can be proved that.

Converse:

If the opposite angles of a quadrilateral are supplementary, it vertigos are cycle.

Given:

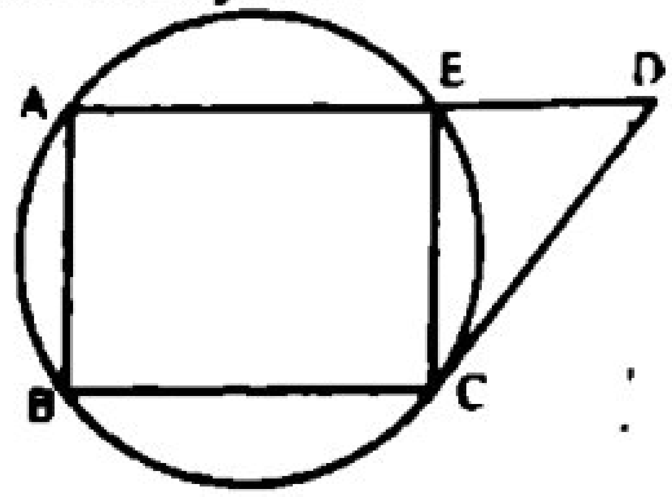
A quadrilateral ABCD such that

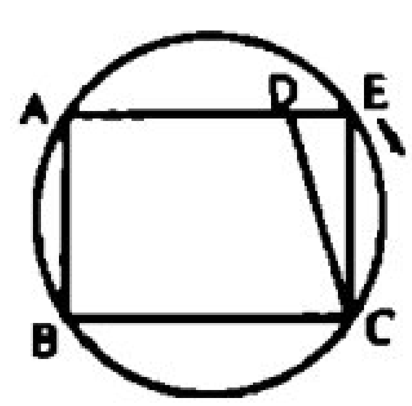
$\angle A + \angle C = 2rt. \angle S$

and $\angle B + \angle D = 2rt. \angle S$

To Prove:

Points A, B, C, D are cyclic.





Construction:

Draw a circle to pass through the points A, B, C. If it does not pass through D, Suppose it cuts AD or AD produced at E (as in fig i & ii).

Proof:

Now ABCE is a cyclic quadrilateral.

$\angle ABC + \angle CEA = 2rt. \angle S.$ _____ (i)

But $\angle ABC + \angle CDA = 2rt. \angle S.$ _____ (ii) (given)

$\therefore \angle ABC + \angle CEA = \angle ABC + \angle CDA.$ Add (i) & (ii).

$\Rightarrow$ $\angle CEA = \angle CDA.$

i.e. an ext. angle of Δ CDE is equal to its int. Opp. angle, which is impossible in less. E coincides with D.

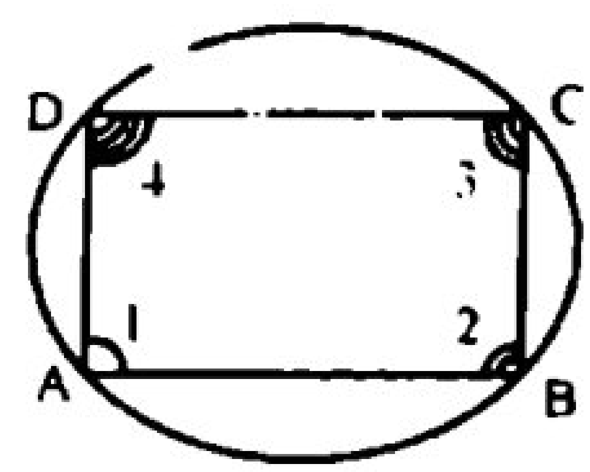
$\therefore$ the cycle which passes through A.B.D must pass through D,

2. Show that parallelogram inscribed in a circle will be a rectangle.

Solution:

Given:

ABCD is a parallelogram.



Top Prove:

ABCD is a rectangle.

Proof

$m\angle 1 + m\angle 3 = 180^\circ$ _______ (i)

[cyclic quadrilateral]

But $m\angle 1 + m\angle 3 = 180^\circ$ _______ (ii)

[opp. $\angle$sof a l lgrm)

From (i) & (ii) we get

$2m\angle 11 = 180^\circ$

$\Rightarrow$ $m\angle 1 = 90^\circ$

$\therefore m\angle = m\angle 3 = 90^\circ.$

By this

$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90^\circ.$

Hence, the l lgrm ABCD is a rectangle.

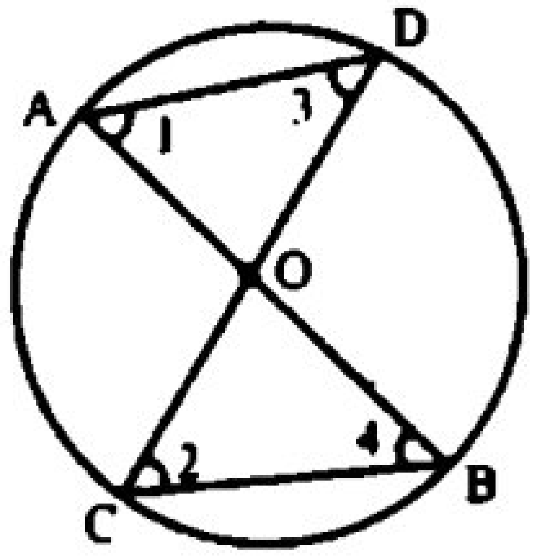
3. AOB and COD are two intersecting chords of a circle. Show that Δ^s AOD and BOC are equiangular.

Solution:

Given:

Two chords AOB & COD

Intersecting each other at O.



To Prove:

Δ' AOD and ΔCOD are equiangular.

Proof:

ΔAOB and ΔCOB

$\angle 1 = \angle 2.$

$\angle 3 = \angle 4$

[angles in the same segment of a circle].

$\therefore \angle AOB = \angle COB.$

(Vertical angles).

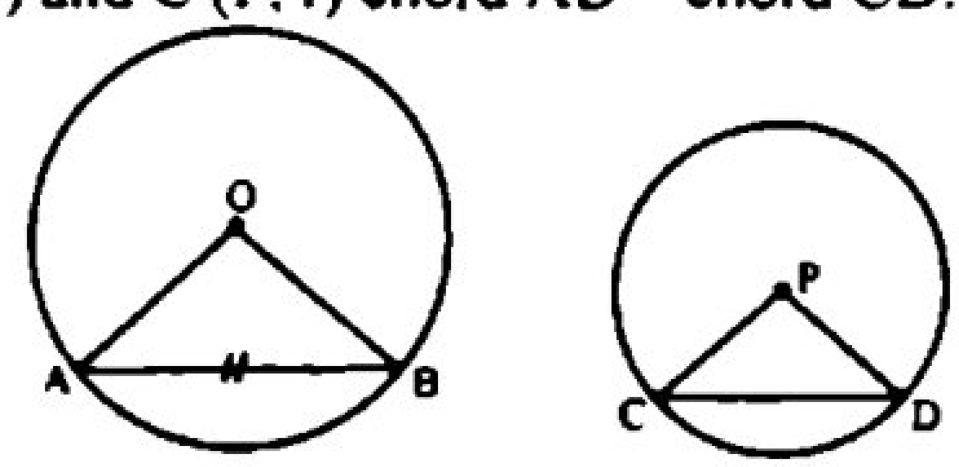
$\therefore$ ΔAOD and Δ COD are equiangular triangles.

4. $\overline{AD}$, and $\overline{BC}$ are two parallel chords of a circle prove that are $\overline{AB}$ ≅ arc $\overline{CD}$ and arc $\overline{AC}$ s arc $\overline{BD}$.

Solution:

Given:

Tow circles, C (O, r) and C (P, r) chord AB = chord CD.



To Prove:

$\overset{\frown}{AB} = \overset{\frown}{CD}$

Construction:

Join OA, OB and PC, PD.

Proof:

In Δ AOB & Δ CPD

OA ≅ PC

and OB ≅ PD [ralii of equal circles]

AB = CD [given].

$\therefore$ ΔAOB ≅ ΔCPD S.S.S.

$\therefore \angle AOB \cong \angle CPD.$

There are the angles subtended by the AB and CD at the centre of equal circle

$\overset{\frown}{AB} = \overset{\frown}{CD}$