5. Complete the circle with Centre 0 and radius ($\overline{OA} = \overline{OB} = \overline{OC} = \overline{OP} = \overline{OE} = \overline{OF}$). This will pass through all the points A, B, C, D, E and F on the given part of the circumference.

To complete the circle without finding the centre when a part of its circumference is given

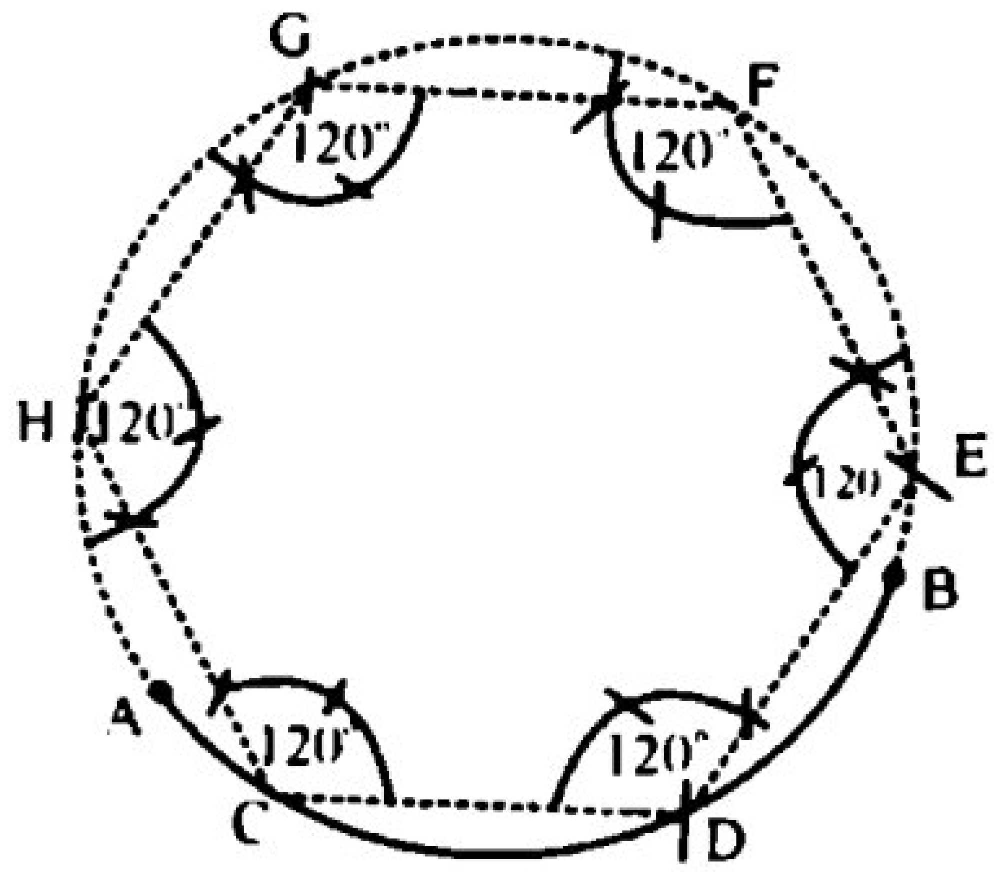
Given:

$\overset{\frown}{AB}$ is the part of circumference of a circle

Steps of Construction:

1. Take a chord $\overline{CP}$ of reasonable length on the arc AB.
2. Construct an internal angle of 120° at point D and draw a line segment $\overline{DE}$ equal to the length of $\overline{CD}$.
3. At point E again construct an internal angle of 120° and from point E draw line segment $\overline{EF}$ of length equal to $\overline{CD}$ etc.
4. Continue this practice until we reach at the starting point.
5. Now join the points D, E, F, G, H and C by arcs DE, EF and FG and HC all having length equal to the length of arc CD.

   As a result we get a circle including the given part of circumference.

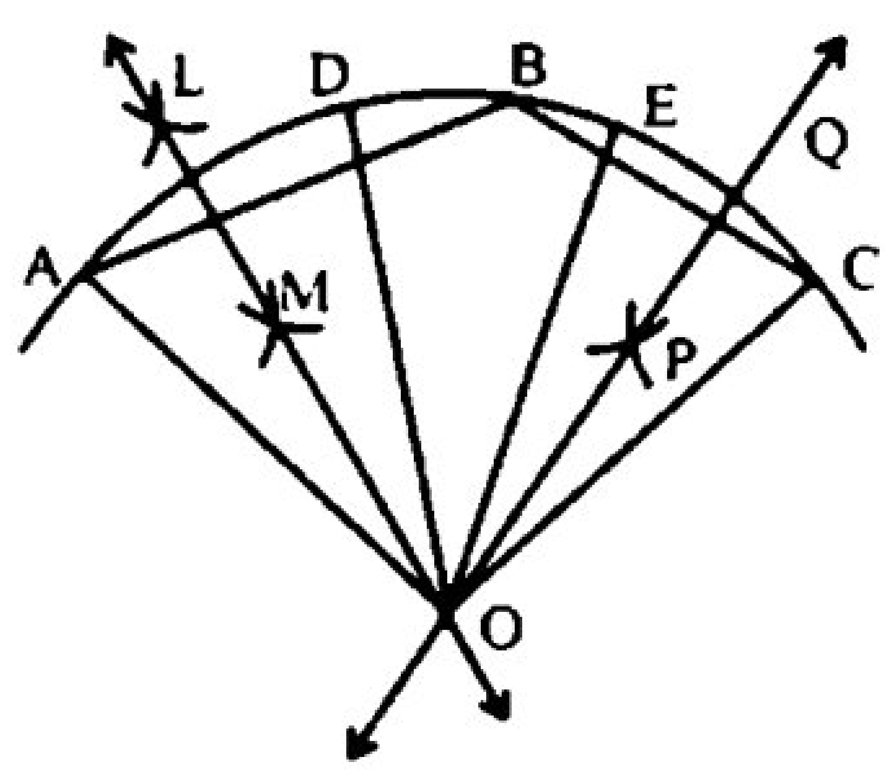


SOLVED EXERCISE 13.1

Q1. Divide an arc of any length.

(i) Into three equal parts.

Solution:



Steps of construction:

1. Draw an arc $\overset{\frown}{ABC}$.
2. Join A with B and B with C.
3. Draw $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ right

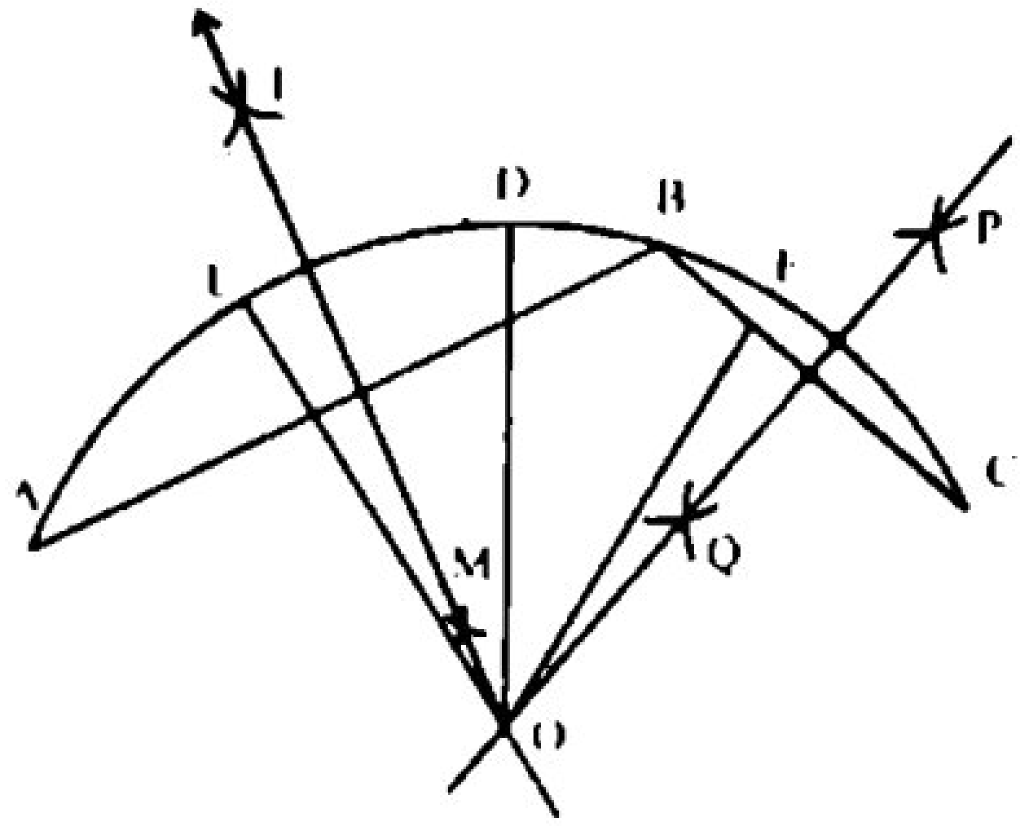
   bisectors of AB and BC respectively. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at pint O.
4. Divide the arc ABC in three equal parts, such that $\overset{\frown}{AD} = \overset{\frown}{DE} = \overset{\frown}{EC}$

(ii) Into four equal parts

Solution:

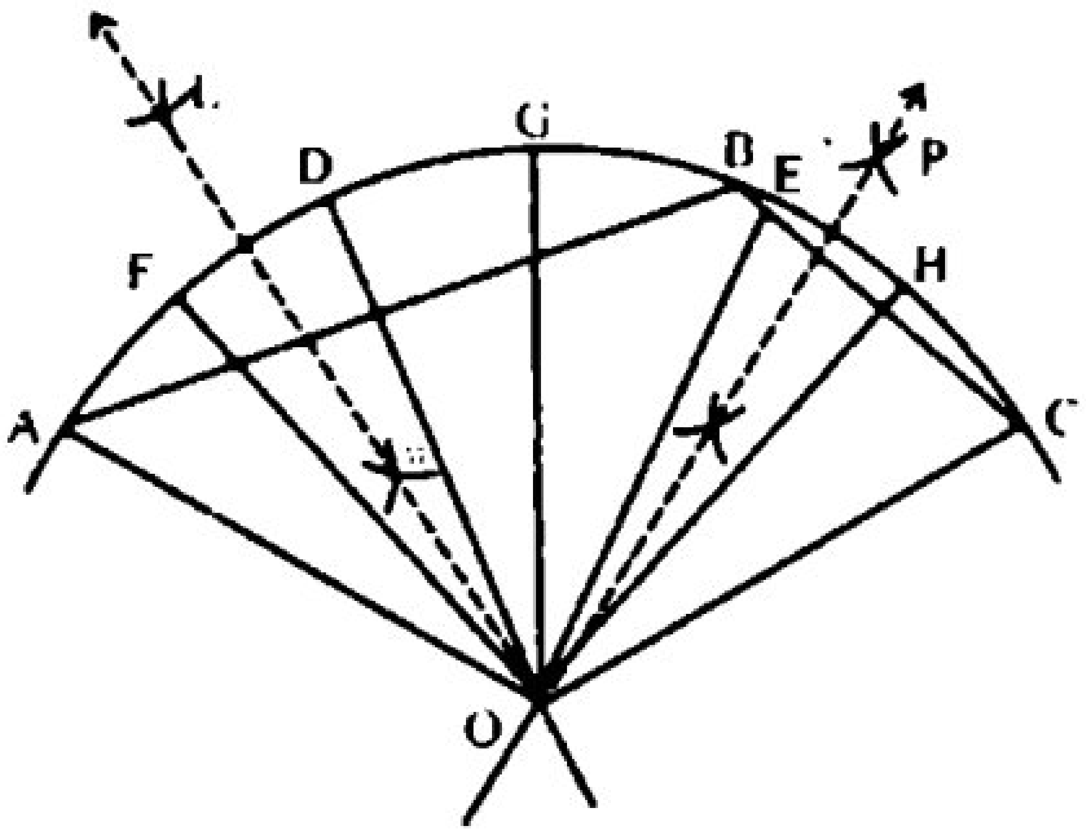
Steps of construction:

1. Draw an arc $\overset{\frown}{ABC}$.
2. Join A with B and B with C.
3. Divide the arc ABC in four equal parts, such that $\overset{\frown}{AE} = \overset{\frown}{ED} = \overset{\frown}{DF} = \overset{\frown}{FC}$.
4. Draw $\overleftrightarrow{LM}$ and $\overrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overleftrightarrow{LM}$ and $\overrightarrow{PQ}$ intersect at point O.



(iii) Into six equal parts

Solution:



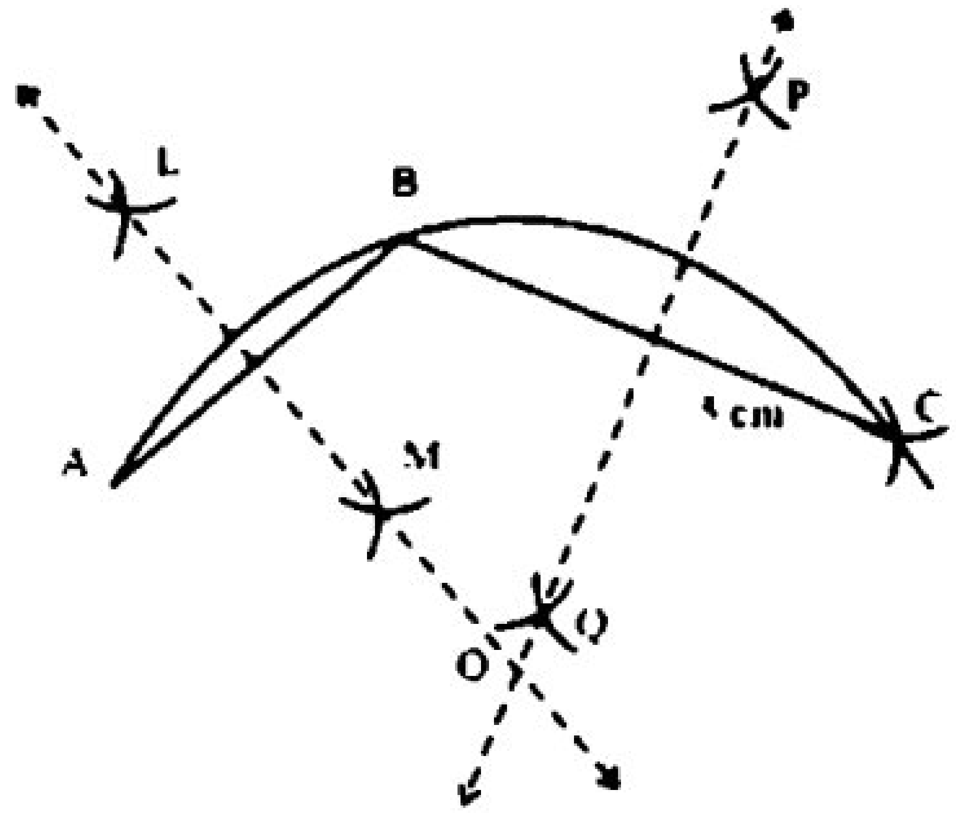
1. Draw an arc $\overset{\frown}{ABC}$.
2. Join A with B and B with C.
3. Divide the arc ABC in six equal parts, such that $\overset{\frown}{AF} = \overset{\frown}{FD} = \overset{\frown}{DG} = \overset{\frown}{GE} = \overset{\frown}{EH} = \overset{\frown}{HC}$
4. Draw $\overline{LM}$ and $\overrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overline{LM}$ and $\overrightarrow{PQ}$ intersects at point O.

2. Practically find the centre of an arc ABC.

Solution:

Steps of Construction:

1. Draw an arc $\overparen{ABC}$.
2. Join A with B and B with C.
3. Draw LM and PQ right Bisectors of AB and BC respectively. LM and PQ intersect at pint O.
4. O is the required centre of an arc ABC.

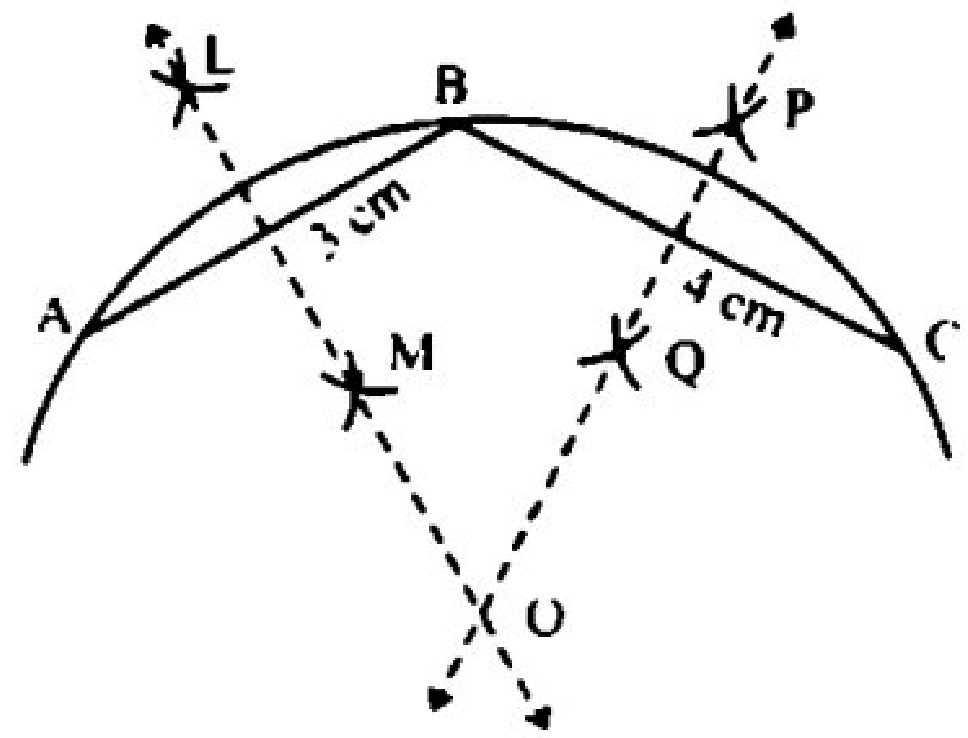


3. (i) If $|\overline{AB}| = 3$ cm and $|\overline{BC}| = 4$ cm are the lengths of two chords of an arc, then locate the centre of the arc.

Solution:

Steps of Construction:

1. Draw an arc $\overparen{ABC}$.
2. Draw $|\overline{AB}| = 3$cm and $|\overline{BC}| = 4$ cm
3. Draw $\overline{LM}$ and $\overleftrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overline{LM}$ and $\overleftrightarrow{PQ}$ intersect at pint O.
4. O is the required centre of an arc ABC.

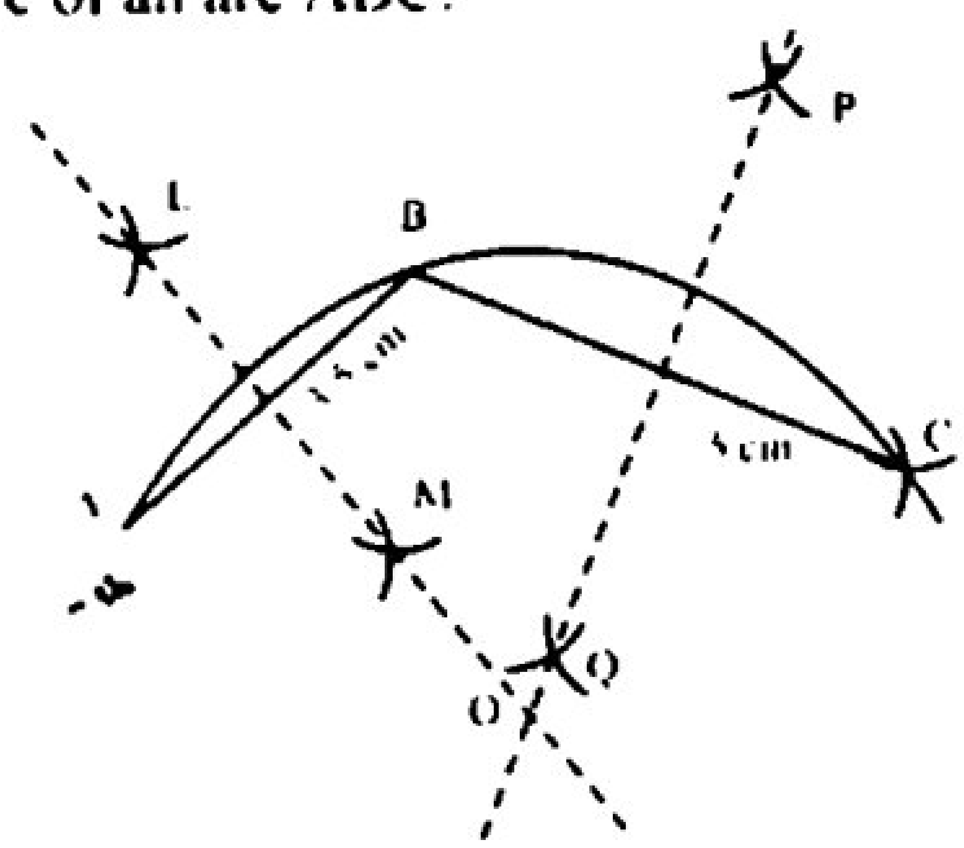


(ii) If $|\overline{AB}| = 3.5$ cm and $|\overline{BC}| = 5$ cm are the lengths of two chords of an arc, then locate the centre of the arc.

Solution:

Steps of Construction:

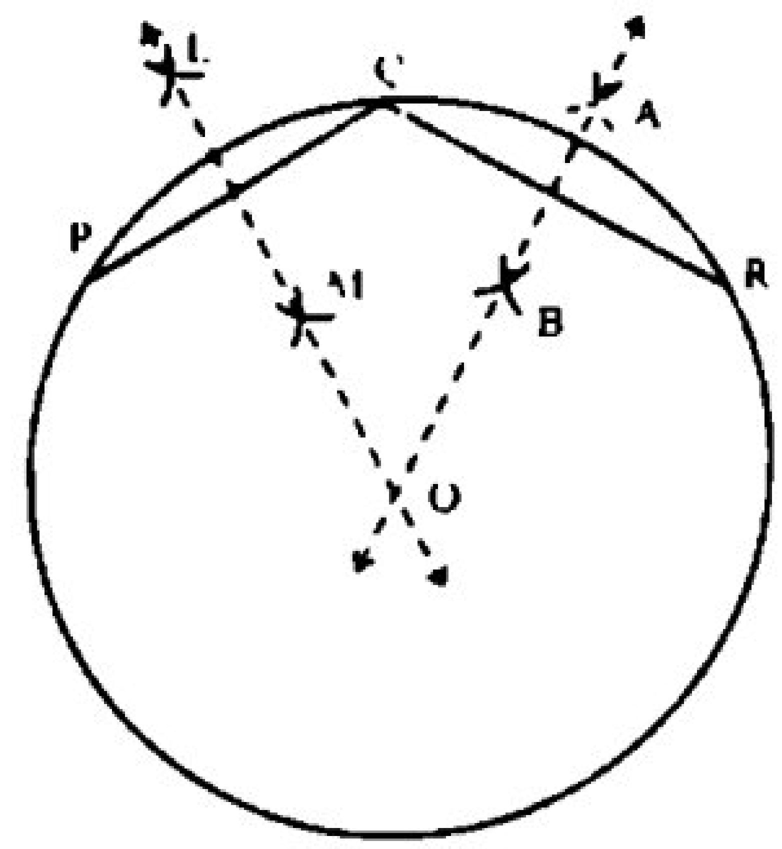
1. Draw an arc $\overparen{ABC}$
2. Draw $|AB| = 3.5$ cm and $|BC| = 5$ cm.
3. Draw $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at pint O.
4. O is the required centre of an arc ABC.



4. For an arc draw two perpendicular bisectors of the chords $\overline{PQ}$ and $\overline{QR}$ of this arc, construct a circle through P, Q and R.

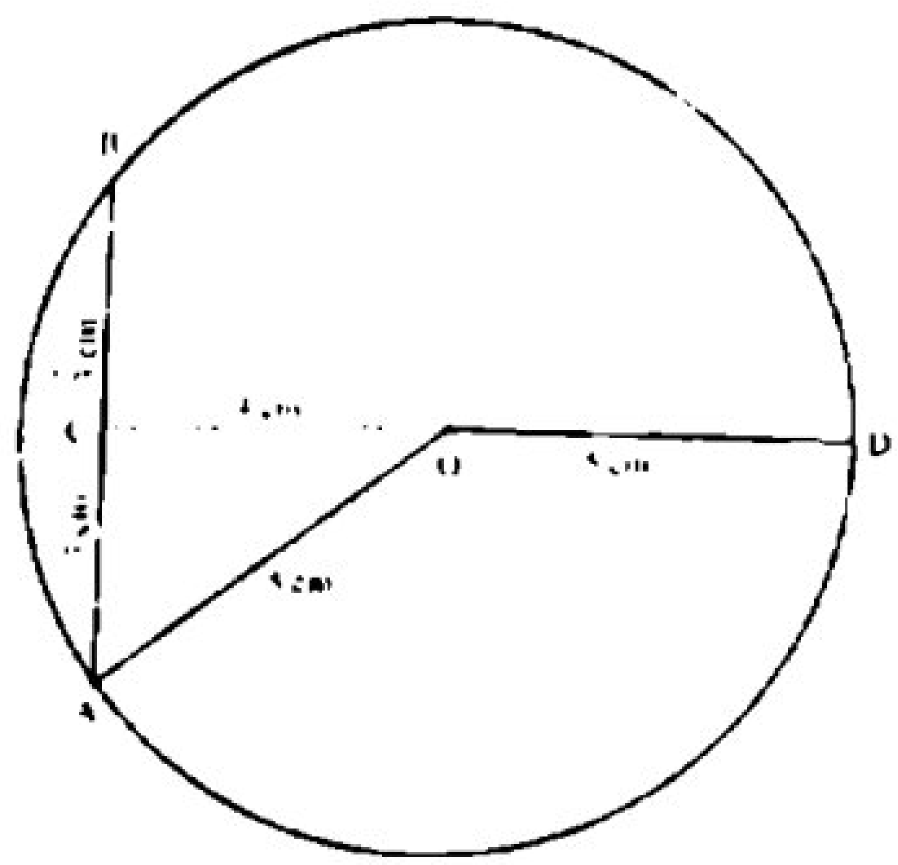
Solution:

Steps of Construction:



1. Draw an arc $\widehat{ABC}$.
2. Join P with Q and Q with R.
3. Draw $\overline{LM}$ and $\overline{PQ}$ right bisectors of $\overline{PQ}$ and $\overline{QR}$ respectively. $\overline{LM}$ and $\overline{AB}$ intersect at pint O.
4. O is the required centre of an arc ABC.
5. Draw a circle with radius $\overline{OP} = \overline{OQ} = \overline{OR}$ having centre at O, which is the required circle.

5. Describe circle of radius 5 cm passing through points A and B, 6 cm apart. Also find distance from the centre to the line segment AB.



Solution:

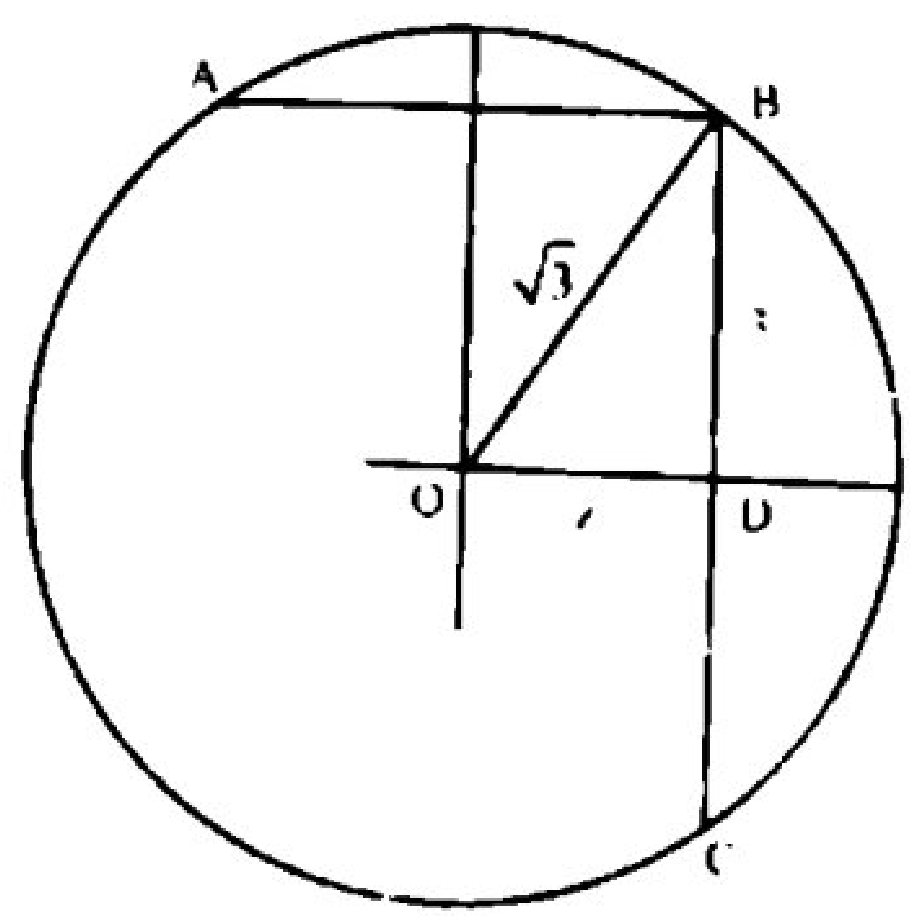
Steps of Construction:

1. Draw a circle of radius 5 cm passing through points A and B, 6 cm apat.
2. The distance from the centre O to the line segment AB is 4 cm.

6. If $|\overline{AB}|$ = A cm and $|\overline{BC}|$ = 6 m, such that $\overline{AB}$ is perpendicular to $\overline{BC}$, construct a circle through points A, B and C. Also measure its radius.

Solution:

Steps of Construction:



1. Draw $|\overline{AB}|$ = 4cm and $|\overline{BC}|$ = 6cm, such that $|\overline{AB}|$ is perpendicular to $\overline{BC}$.
2. Now in $\triangle OBD$, by Pythagoras theorem, we have

$$|\overline{OB}| = \sqrt{|\overline{OD}|^2 + |\overline{BD}|^2}$$

$$= \sqrt{(2)^2 + (3)^2}$$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13} \quad \text{(Radius)}$$

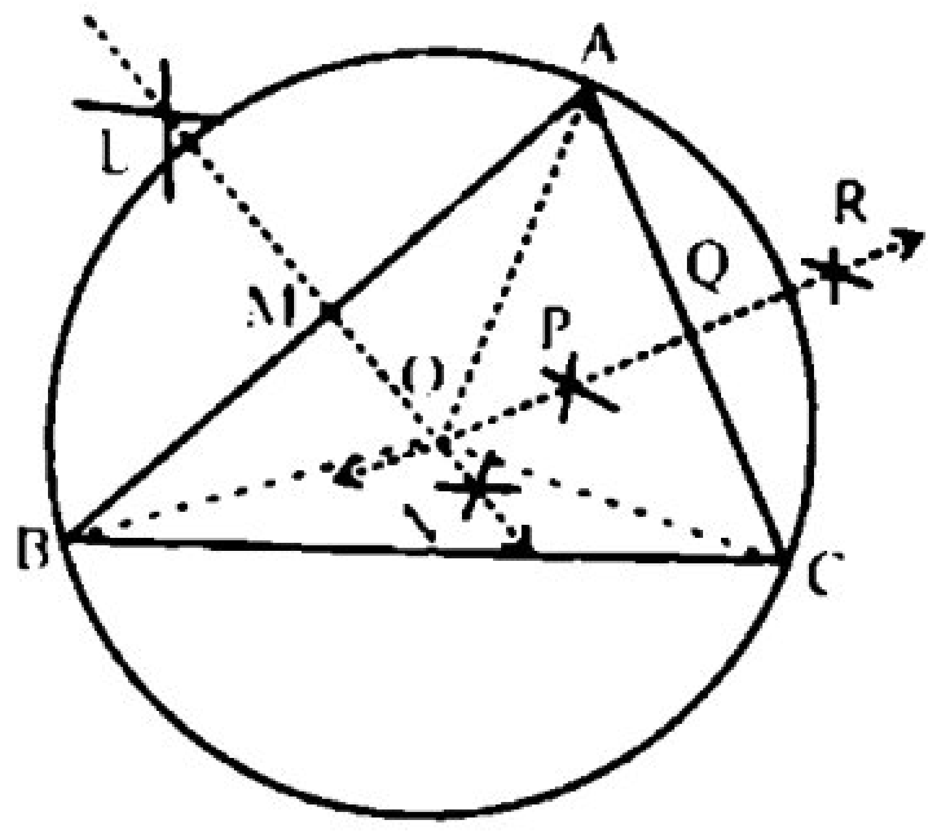
3. Draw a circle of radius $|\overline{OA}| = |\overline{OB}| = |\overline{OC}| = \sqrt{13}$ cm. which passes through points A, B and C.

Circumscribe a circle about a give a triangle:

Given:

Triangle ABC.

Steps of Construction:



1. Draw $\overleftrightarrow{LMN}$ as perpendicular bisector of side $\overline{AB}$.
2. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{AC}$.
3. $\overline{LN}$ and $\overline{PP}$. intersect at point O.
4. With centre O and radius

   $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

This circle will pass through A, B and C whereas O is the circumcentre of the circumscribed circle.

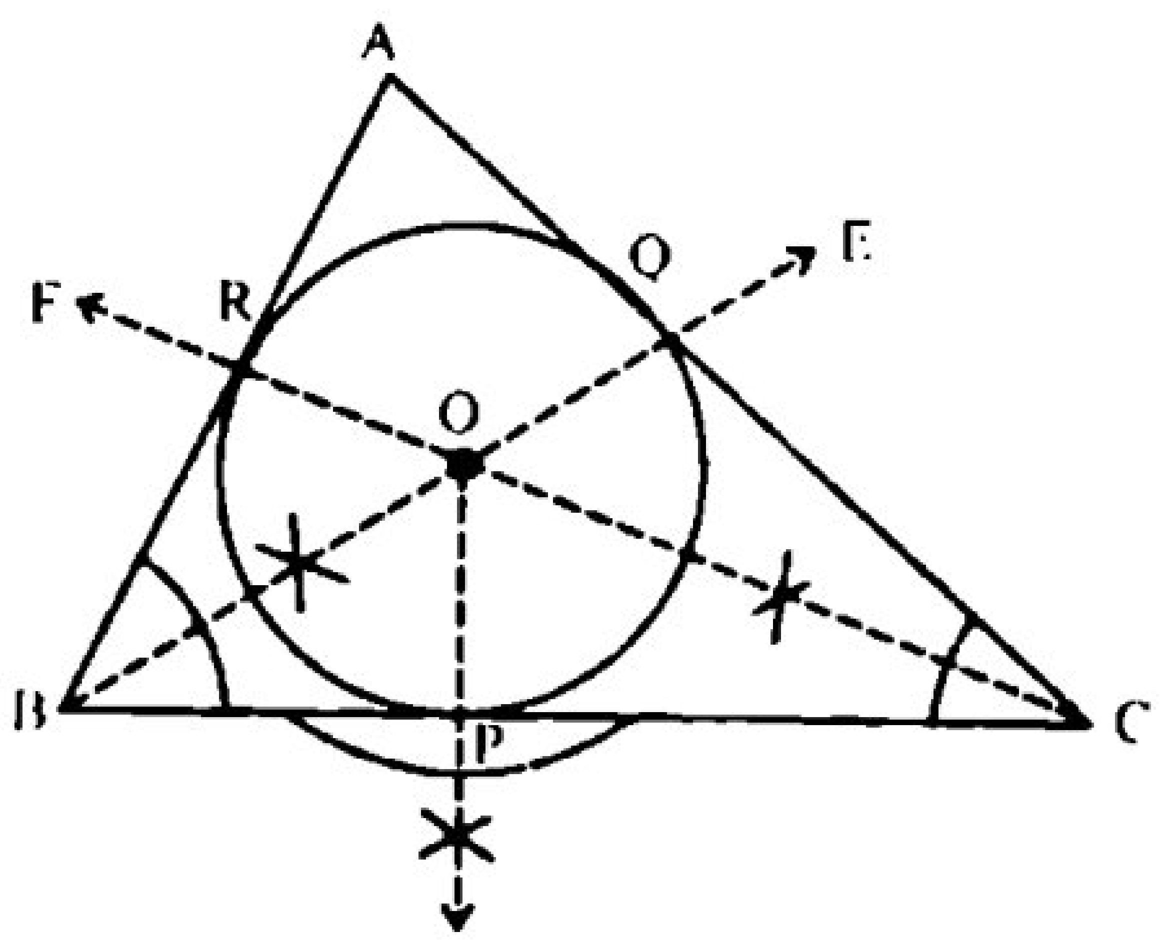
Remember: The circle passing through, the vertices of triangle ABC is known as **circumcircle**, its radius as **circumradius** and centre as **circumcentre**.

Inscribe a circle in a given triangle:

Given:

A triangle ABC.

Steps of Construction:



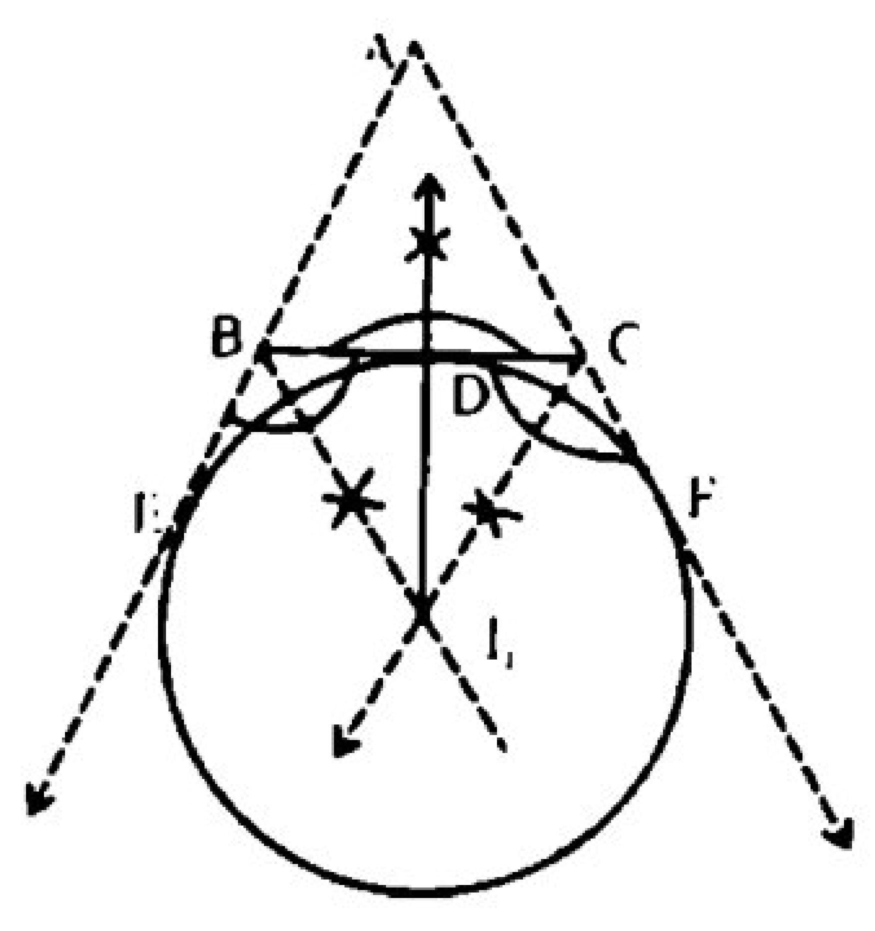
1. Draw $\overrightarrow{BE}$ and $\overrightarrow{CF}$ to bisect the angles ABC and ACB respectively. Rays $\overrightarrow{BE}$ and $\overrightarrow{CF}$ intersect each other at point O.
2. O is the centre of the inscribed circle.

   From O draw $\overrightarrow{OP}$ perpendicular to $\overline{BC}$.

   With centre O and radius $\overline{OP}$ draw a circle. This circle is the inscribed circle of triangle ABC.

Remember:

A circle which touches the three sides of a triangle internally is known as **incircle**, its radius as **in-radius** and centre as **in centre**:



Describe a circle to a given triangle:

Given:

A triangle ABC

Steps of Construction:

1. Produce the sides $\overline{AB}$ and $\overline{AC}$ of AABC.
2. Draw bisectors of exterior angles ABC and ACB.
   These bisectors of exterior angles meet at.
3. From I_1 draw perpendicular on Side $\overline{BC}$ of ΔABC.
   Which I_1D intersect BC at D. I_1D is the radius of the escribed circle with centre at I_1.
4. Draw the circle with radius I_1D and centre at I_1 that will touch the side BC of the ΔABC externally and the produced sides AB and AC.

Eescribed circle:
The circle touching one side of the triangle externally and two produced sides internally is called escribed circle (e-circle). The centre of e-circle is called **e-centre** and radius is called **e-radius**.

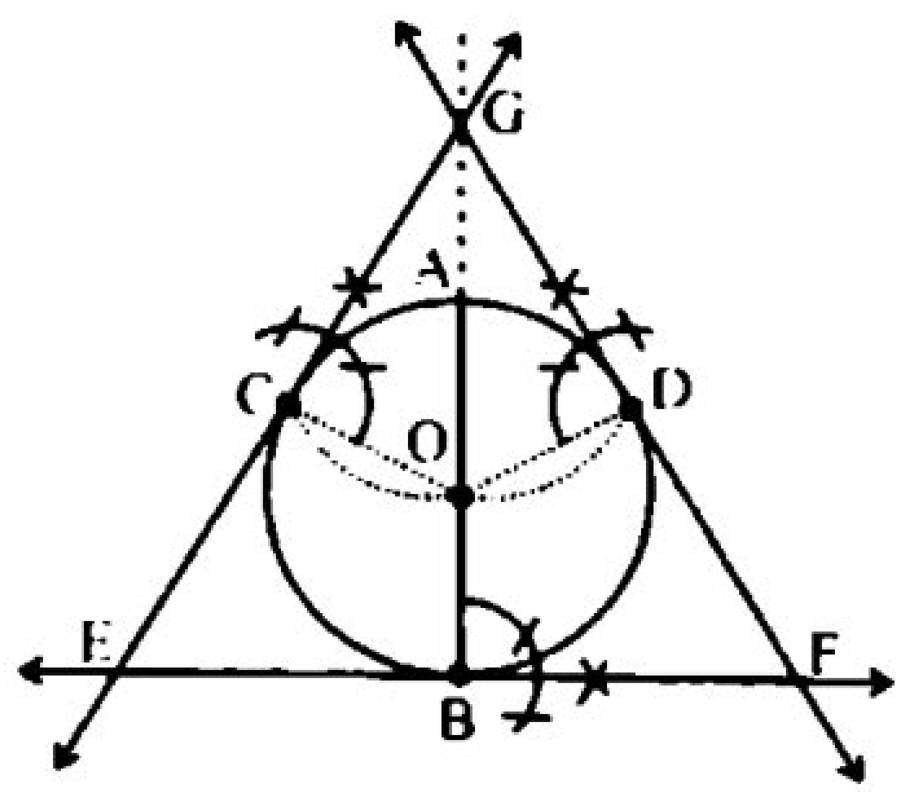
Circumscribe an equilateral triangle about a given circle

Given:

A circle with centre O of reasonable radius.

Steps of Construction:

1. Draw $\overline{AB}$, the diameter of the circle, for locating.
2. Draw an arc of radius m $\overline{OA}$ with centre at A for locating points C and D on the circle.
3. Join 0 to the points C and D.
4. Draw tangents to the circle at points B, C and D.
5. These tangents intersect at points E, F and G.



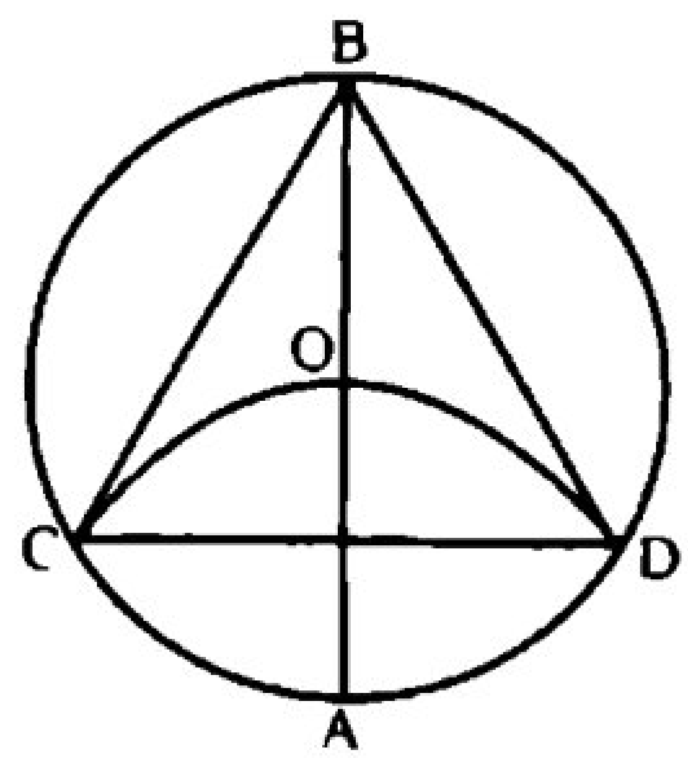
(v) Inscribe air equilateral triangle in a given circle.

Given:

A circle with centre at O.

Steps of Construction:

1. Draw any diameter $\overline{AB}$ of the circle.
2. Draw an arc of radius OA from point A. The arc cuts the circle at points C and D.
3. Join the prints B, C and D to form straight line segments $\overline{EC}$, $\overline{BC}$ and $\overline{BD}$.
   Triangle BCD is the required inscribed equilateral triangle.

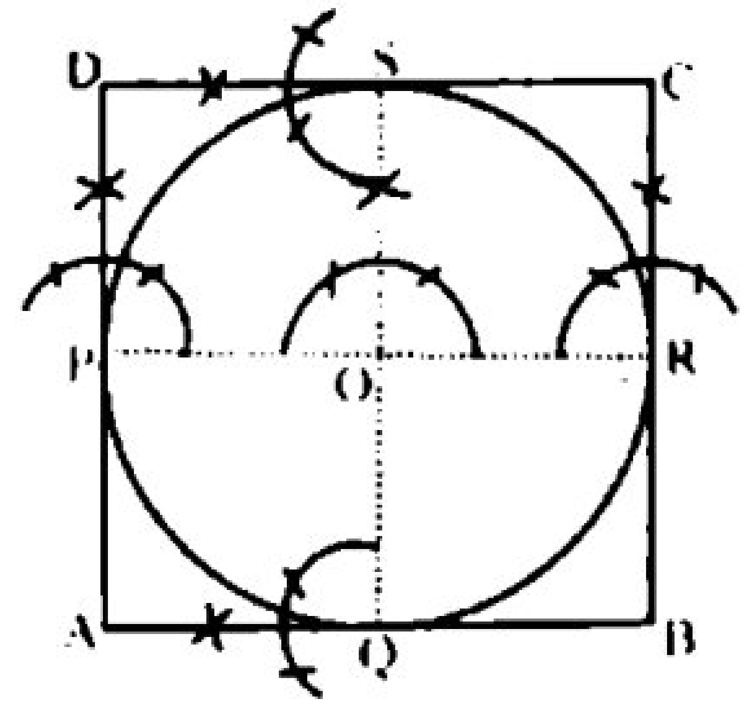


Circumscribe a square: about a given circle:

Given:

A circle with centre at O.

Steps of Construction:



1. Draw two diameters $\overline{PQ}$ and $\overline{OS}$ which bisect each other at right angle.
2. At points P, Q, R and S draw tangents to the circle.
3. Produce the tangents to meet each other at A, B, C and D. ABCD is the required circumscribed square.

Inscribe a square in a given circle:

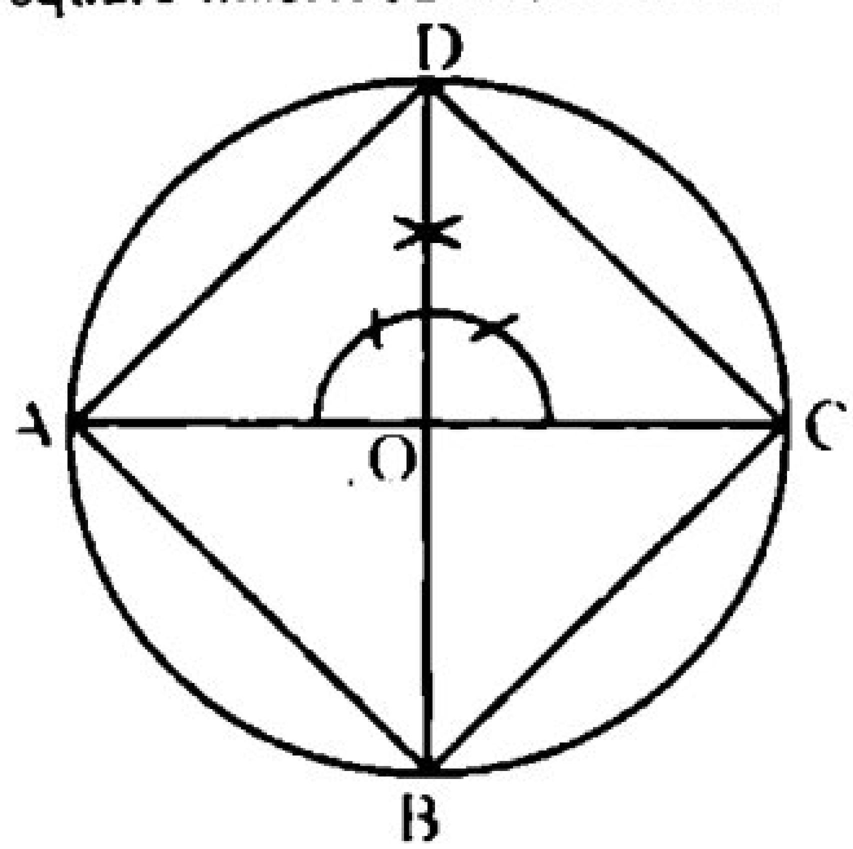
Given:

A circle, with centre at O.

Steps of Construction:

1. Through O draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.
2. Join A with B, B with C, C with D, and D with A.

ABCD is the required square inscribed in the circle.

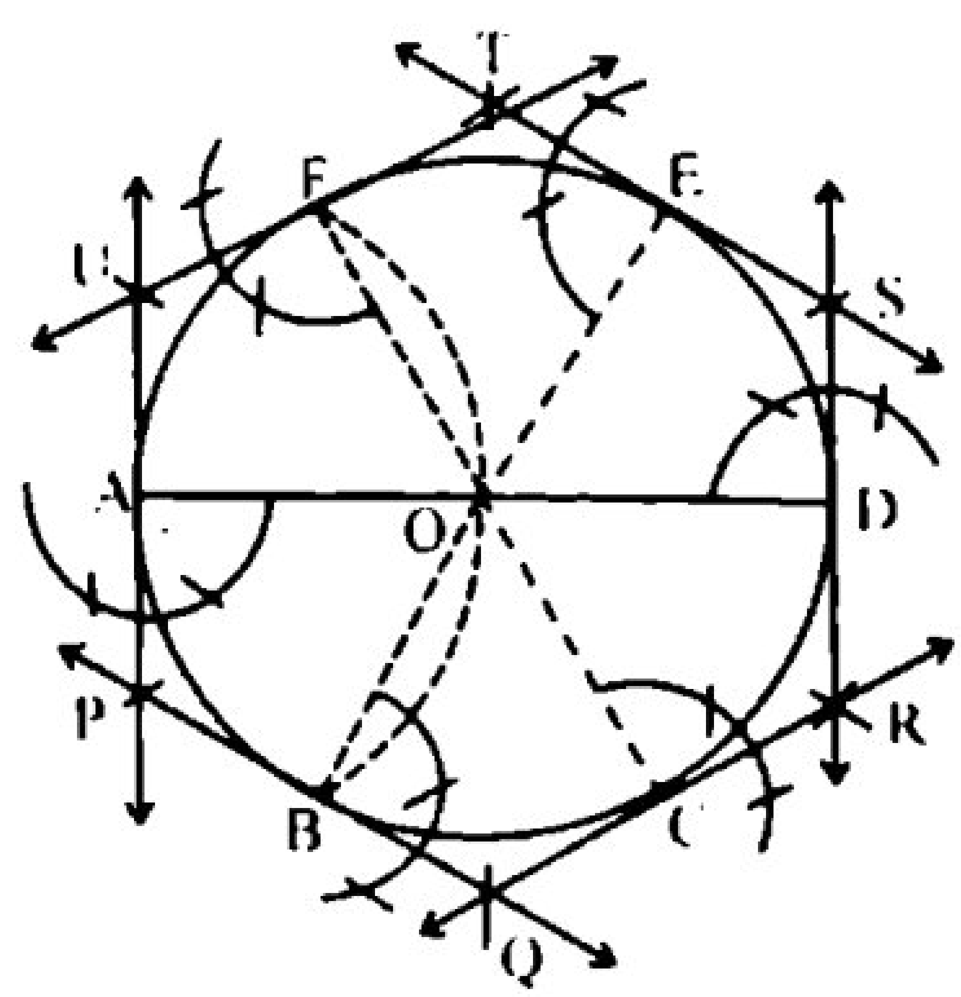


Circumscribe a regular hexagon about a given circle:

Given:

A circle with centre at O.

Steps of Construction:

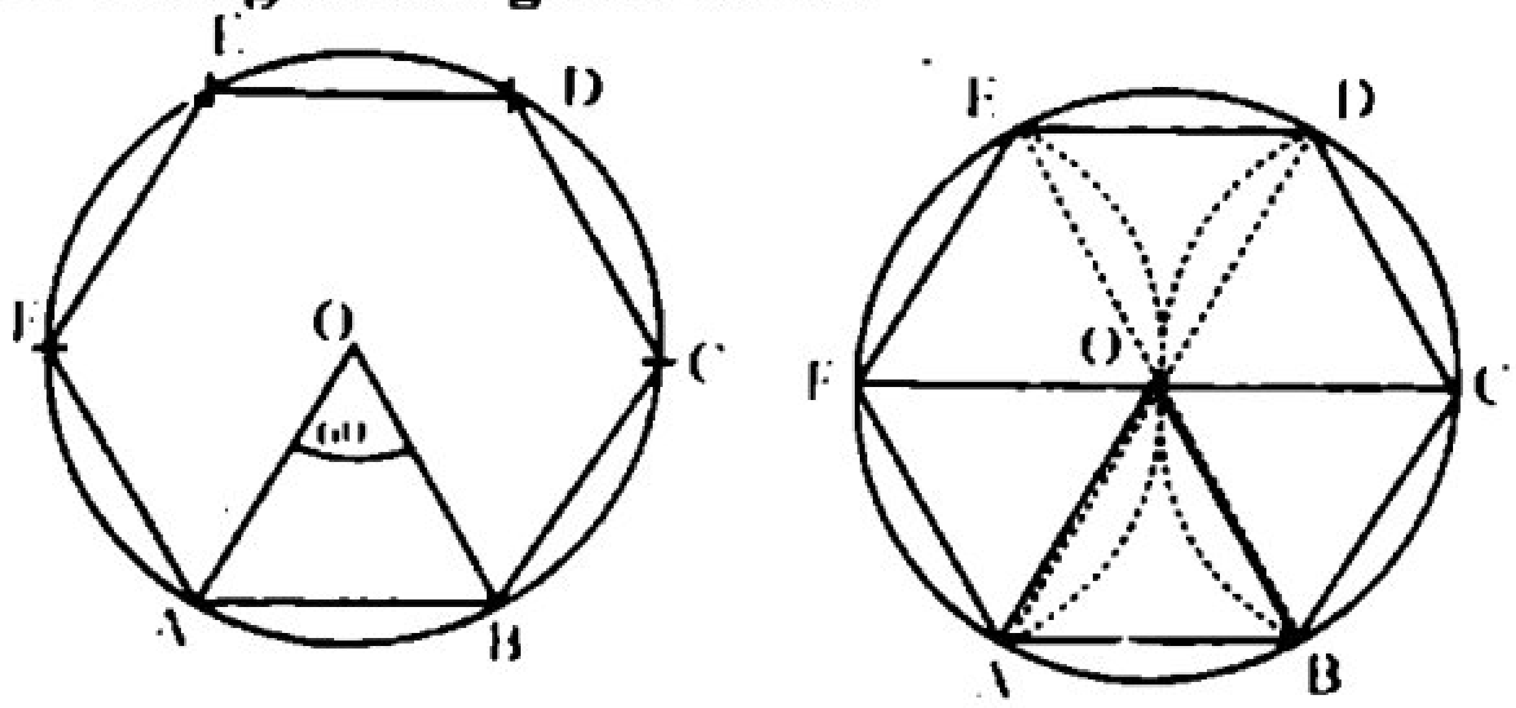


1. Draw any diameter $\overline{AD}$.
2. From point A draw an arc of radius $\overline{AO}$ (the radius of the circle), which cuts the circle at points B and F.
3. Join B with 0 and extend it to meet the circle at E.
4. Join F with 0 and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R.

S, T and U respectively.

6. Thus PQRSTU is the circumscribed regular hexagon.

Inscribe a regular hexagon in a given circle.



Given:

A circle, with centre at O.

Steps of Construction:

1. Take any point A on the circle and point with O.
2. From point A, draw an arc of radius $\overline{AO}$ which intersects the circle at points B and F.
3. Join (O and A with points B and F.
4. ΔOAB and ΔOAF are equilateral therefore $\angle AOB$ and $\angle AOF$ are of measure 60° i.e., $m\overline{AO} = m\overline{AB} = m\overline{AF}$.
5. Produce $\overline{FO}$ to meet the circle at C. Join B to C. Since in $\angle BOC = 60°$ therefore $m\overline{BC} = m\overline{OA}$.
6. From C and F, draw arcs of radius $\overline{OA}$, which intersect the circle at points D and E.
7. Join C to D, D to E and E' to F ultimately. We have
   $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$
   Thus the figure ABCDEF is a regular hexagon inscribed in the circle.

SOLVED EXERCISE 13.2

1. Circumscribe a circle about a triangle ABC with sides

$|\overline{AB}| = 6$ cm , $|\overline{BC}| = 3$cm , $|\overline{CA}| = 4$cm

Also measure its circum radius.

Solution:

Given:

Three sides

$|\overline{AB}| = 6$cm, $|\overline{BC}| = 3$cm, $|\overline{CA}| = 4$cm.

Required: