$$x^2 - (p^2 - 2q)\,x + q^2 = 0$$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

$$x^2 + Px + q = 0$$

Here $a = 1, b = P, c = q$

As $\propto, \beta$ be the roots of given equation.

Then $\propto + \beta = -\frac{b}{a}$ and $\propto\beta = \frac{c}{a}$

$= -\frac{p}{1}$ $\qquad$ $= \frac{q}{1}$

$= -p$ $\qquad$ $= q$

S = Sum of roots and P = Product of roots

$$= \frac{\propto}{\beta} + \frac{\beta}{\propto} \qquad = \left(\frac{\propto}{\beta}\right)\left(\frac{\beta}{\propto}\right)$$

$$= \frac{\propto^2 + \beta^2}{\propto\beta} \qquad = 1$$

$$= \frac{(\propto + \beta)^2 - 2\propto\beta}{\propto\beta}$$

$$= \frac{(-p)^2 - 2q}{q}$$

$$= \frac{p^2 - 2q}{q}$$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right) + 1 = 0$$

$$\Rightarrow \quad qx^2 - (p^2 - 2q) + q = 0$$

Synthetic Division:

Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In Fact synthetic division is simply a shortcut of long division method.

SOLVED EXERCISE 2.6

1. Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) + (x + 1)$

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Solution:

$(x^2 + 7x - 1) \div (x + 1)$

As $x + 1 = x - (-1)$, so $a = -1$

Now write the coefficients of dividend in a row and $a = -1$ on the left side.

	1	7	−1
−1	↓	−1	−6
	1	6	−7

Quotient $= Q(x) = x + 6$

Remainder $= R = -7$

(ii) $(4x^3 - 5x + 15) + (x + 3)$

Solution:

As $x + 3 = x - (-3)$, so $a = -3$

Now write the coefficients of dividend in a row and $a = -3$ on the left side.

	4	0	−5	15
−3	↓	−12	36	−93
	4	−12	31	−78

Quotient $= Q(x) = x^2 - 12x + 31$

Remainder $= R = -78$

(iii) $(x^3 + x^2 - 3x + 2) + (x - 2)$

Solution:

$(x^3 + x^2 - 3x + 2) \div (x - 2)$

As $x + 2 = x - 2$, so $a = 2$

Now write the coefficients of dividend in a row

and $a = 2$ on the left side.

	1	1	−3	2
2	↓	2	6	6
	1	3	3	8

Quotient $= Q(x) = x^2 + 3x + 3$

Remainder $= R = 8$

2. Find the value of A using synthetic division, if

(i) 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$

Solution:

$P(x) = 2x^3 - 3hx^2 + 9$ and its zero is 3

Then by synthetic division.

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$$\begin{array}{c|cccc}
 & 2 & -3h & 0 & 9 \\
3 & \downarrow & 6 & 18-9h & 54-27h \\
\hline
 & 2 & 6-3h & 18-9h & \big| \; 63-27h
\end{array}$$

Remainder = 63 − 27 h since 1 is the zero of the polynomial, therefore
Remainder = 0, that is

$$63 - 27\,h = 0$$
$$27\,h = 63$$
$$h = \frac{63}{27} = \frac{7}{3}$$

(ii) 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$

Solution:

$P(x) = x^3 - 2hx^2 + 11$ and its zero is 1
Then by synthetic division.

$$\begin{array}{c|cccc}
 & 1 & -2h & 0 & 11 \\
1 & \downarrow & 1 & 1-2h & 1-2h \\
\hline
 & 1 & 1-2h & 1-2h & \big| \; 12-2h
\end{array}$$

Remainder = 12 − 2h
Since 1 is the zero of the polynomial, therefore
Remainder = 0, that is

$$12 - 2h = 0$$
$$-2h = -12$$
$$h = \frac{12}{2} = 6$$

(iii) −1 is the zero of the polynomial $2x^3 + 5hx - 23$

Solution:

$P(x) = 2x^3 + 5hx - 23$ and its zero is −1
Then by synthetic division.

$$\begin{array}{c|cccc}
 & 2 & 0 & 5h & -23 \\
-1 & \downarrow & -2 & 2 & -5h-2 \\
\hline
 & 2 & -2 & 5h+2 & \big| \; -5h-25
\end{array}$$

Remainder = − 5h − 25
Since −1 is the zero of the polynomial, therefore,
Remainder = 0, that is

$$-5h - 25 = 0$$
$$-5h = 25$$
$$h = \frac{25}{-5} = -5$$

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3. Use synthetic division to find the values of l and m, if

(i) $(x + 3)$ and $(x - 2)$ are the factors of the polynomial $x^3 - 4x^2 + 2lx + m$

Solution:

Since $(x + 3)$ and $(x - 2)$ are the factors of $p(x) = x^3 + 4x^2 + 2lx + m$.
Therefore -3 and 2 are zeros of polynomial $p(x)$.
Now by synthetic division.

$$\begin{array}{r|cccc} & 1 & 4 & 2l & m \\ -3 & \downarrow & -3 & -3 & -6l+9 \\ \hline & 1 & 1 & 2l-3 & \multicolumn{1}{|c}{m-6l+9} \end{array}$$

Remainder $= m - 6l + 9$
Since -3 is the zero of polynomial, therefore,
Remainder $= 0$, that is,
$m - 6l + 9 = 0$ _______ (i)
and

$$\begin{array}{r|cccc} & 1 & 4 & 2l & m \\ 2 & \downarrow & 2 & 12 & 4l+24 \\ \hline & 1 & 6 & 2l+12 & \multicolumn{1}{|c}{m+4l+24} \end{array}$$

Remainder $= m + 4l + 24$
Since 2 is the zero of polynomial, therefore,
Remainder $= 0$, that is,
$m + 4l + 24 = 0$ _______ (ii)
Now, subtract eq. (ii) from eq. (i), we get

$$\begin{array}{r} m - 6l + 9 = 0 \\ \pm m \pm 4l \pm 24 = 0 \\ \hline -10l - 15 = 0 \end{array}$$

$$-10l = 15$$

$$l = -\frac{15}{10}$$

$$l = -\frac{3}{2}$$

Put $l = -\frac{3}{2}$ in eq. (i), we get

$$m - 6\left(-\frac{3}{2}\right) + 9 = 0$$

$$m + 9 + 9 = 0$$

$$m + 18 = 0$$

$$m = -18$$

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Thus, $l = -\frac{3}{2}, m = -18$

(ii) (x – 1) and (x + 1) are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

Solution:

Since $(x - 1)$ and $(x + 1)$ are the factors of $p(x) = x^3 - 3lx^2 + 2mx + 6$.
Therefore 1 and – 1 are zeros of polynomial p (x).
Now by synthetic division.

$$\begin{array}{c|cccc} & 1 & -3l & 2m & 6 \\ 1 & \downarrow & 1 & 1-3l & 2m-3l+1 \\ \hline & 1 & 1-3l & 2m-3l+1 & \big|\ 2m-3l+7 \end{array}$$

Remainder = $2m - 3l + 7$
Since 1 is the zero of polynomial, therefore,
Remainder = 0, that is,
$2m - 3l + 7 = 0$ ________ (i)
and

$$\begin{array}{c|cccc} & 1 & -3l & 2m & 6 \\ -1 & \downarrow & -1 & 3l+1 & -2m-3l-1 \\ \hline & 1 & -3l-1 & 2m+3l+1 & \big|\ -2m-3l+5 \end{array}$$

Remainder = $-2m - 3l + 5$
Since –1 is the zero of polynomial, therefore,
Remainder = 0, that is,
$-2m - 3l + 5 = 0$ _______ (ii)
Add eq. (ii) from eq. (i), we get

$$\begin{array}{c} 2m - 3l + 7 = 0 \\ \underline{-2m - 3l + 5 = 0} \\ -6l + 12 = 0 \\ -6l = -12 \\ l = 2 \end{array}$$

Put l = 2 in eq. (i), we get

$$2m - 3(2) + 7 = 0$$
$$2m - 6 + 7 = 0$$
$$2m + 1 = 0$$
$$2m = -1$$
$$m = -\frac{1}{2}$$
$$m = -18$$

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Thus, $l = 2, m = -\frac{1}{2}$

4. Solve by using synthetic division, if

(i) 2 is the root of the equation $x^3 - 28x + 48 = 0$

Solution:

Since 2 is the root of the equation $x^3 - 28x + 48 = 0$

Then by synthetic division, we get

$$\begin{array}{c|cccc} & 1 & 0 & -28 & 48 \\ 2 & \downarrow & 2 & 4 & -48 \\ \hline & 1 & 2 & -24 & |\,0 \end{array}$$

The depressed equation is

$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x + 6) - 4(x + 6) = 0$$
$$(x - 4)(x + 6) = 0$$

Either $x - 4 = 0$ or $x + 6 = 0$

$x = 4$ $\quad$ $x = -6$

Hence, -6, 2 and 4 are the roots of the given equation.

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

Solution:

Since 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

Then by synthetic division, we get

$$\begin{array}{c|cccc} & 2 & -3 & -11 & 6 \\ 3 & \downarrow & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & |\,0 \end{array}$$

The depressed equation is

$$2x^2 + 3x - 2 = 0$$
$$2x^2 + 4x - x - 2 = 0$$
$$2x(x + 2) - 1(x + 2) = 0$$
$$(2x - 1)(x + 2) = 0$$

Either $2x - 1 = 0$ or $x + 2 = 0$

$2x = 1$ $\quad$ $x = -2$

$x = \frac{1}{2}$

Hence, $-2, \frac{1}{2}$ and 3 are the roots of the given equation.

(iii) -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

Solution:

Since -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$
Then by synthetic division, we get

	4	−1	−11	−6
−1	↓	−4	5	6
	4	−5	−6	0

The depressed equation is

$$4x^2 - 5x - 6 = 0$$
$$4x^2 - 8x + 3x - 6 = 0$$
$$4x(x + 2) + 3(x - 2) = 0$$
$$(4x + 3)(x - 2) = 0$$

Either $4x + 3 = 0$ or $x - 2 = 0$

$4x = -3$ $\quad$ $x = 2$

$x = -\frac{3}{4}$

Hence, $-\frac{3}{4}, -1$ and 2 are the roots of the given equation.

5. Solve by using synthetic division, if

(i) 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

Solution:

Since 1 and 3 are the root of the equation
$x^4 - 10x^2 + 9 = 0$
Then by synthetic division, we get

	1	0	−10	0	9
1	↓	1	1	−9	−9
	1	1	−9	−9	0
3	↓	3	12	9	
	1	4	3	0	

The depressed equation is

$$x^2 + 4x + 3 = 0$$
$$x^2 + 3x + x + 3 = 0$$
$$x(x + 3) + 1(x + 3) = 0$$
$$(x + 1)(x + 3) = 0$$

Either $x + 1 = 0$ or $x + 3 = 0$

$x = -1$ $\quad$ $x = -3$

Thus, $-3, -1, 1$ and 3 are the roots of given equation.

(ii) 3 and −4 are the roots of the equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Solution:

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Since 1 and 3 are the root of the equation

$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Then by synthetic division, we get

	1	2	−13	−14	24
3	↓	3	15	6	−24
	1	5	2	−8	0
−4	↓	−4	−4	8	
	1	1	−2	0	

The depressed equation is

$$x^2 + x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$(x+1)(x-2) = 0$$

Either $x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $\quad$ $x = 2$

Thus, − 4, −1, 2 and 3 are the roots of given equation.

Simultaneous equations:

A system of equations having a common solution is called a system of simultaneous equations.

The set of all the ordered pairs (x, y), .which satisfies the system of equations is called the solution set of the system.

SOLVED EXERCISE 2.7

Solve the following simultaneous equations.

1. $\mathbf{x + y = 5}$; $\mathbf{x^2 - 2y - 14 = 0}$

Solution:

$x + y = 5$ ________ (i)

$x^2 - 2y - 14 = 0$ ________ (ii)

From (i), we have

$y = 5 - x$ ________ (iii)

Put value of y in eq. (ii), we get

$$x^2 - 2(5 - x) - 14 = 0$$
$$x^2 - 10 + 2x - 14 = 0$$
$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x+6) - 4(x+6) = 0$$
$$(x-4)(x+6) = 0$$

Either $x - 4 = 0$ or $x + 6 = 0$

$x = 4$ $\quad$ $x = -6$

Put x = 4 in eq. (iii), we get $\qquad$ Put x = − 6 in eq (iii), we get