Since 1 and 3 are the root of the equation

$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Then by synthetic division, we get

	1	2	−13	−14	24
3	↓	3	15	6	−24
	1	5	2	−8	0
−4	↓	−4	−4	8	
	1	1	−2	0	

The depressed equation is

$$x^2 + x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$(x+1)(x-2) = 0$$

Either $x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $\quad$ $x = 2$

Thus, − 4, −1, 2 and 3 are the roots of given equation.

Simultaneous equations:

A system of equations having a common solution is called a system of simultaneous equations.

The set of all the ordered pairs (x, y), .which satisfies the system of equations is called the solution set of the system.

SOLVED EXERCISE 2.7

Solve the following simultaneous equations.

1. $\mathbf{x + y = 5}$; $\mathbf{x^2 - 2y - 14 = 0}$

Solution:

$x + y = 5$ ________ (i)

$x^2 - 2y - 14 = 0$ ________ (ii)

From (i), we have

$y = 5 - x$ ________ (iii)

Put value of y in eq. (ii), we get

$$x^2 - 2(5 - x) - 14 = 0$$
$$x^2 - 10 + 2x - 14 = 0$$
$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x+6) - 4(x+6) = 0$$
$$(x-4)(x+6) = 0$$

Either $x - 4 = 0$ or $x + 6 = 0$

$x = 4$ $\quad$ $x = -6$

Put x = 4 in eq. (iii), we get $\quad$ Put x = − 6 in eq (iii), we get

Since 1 and 3 are the root of the equation

$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Then by synthetic division, we get

$$\begin{array}{r|rrrrr} & 1 & 2 & -13 & -14 & 24 \\ 3 & \downarrow & 3 & 15 & 6 & -24 \\ \hline & 1 & 5 & 2 & -8 & \big|\,0 \\ -4 & \downarrow & -4 & -4 & 8 & \\ \hline & 1 & 1 & -2 & \big|\,0 & \end{array}$$

The depressed equation is

$$x^2 + x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$(x+1)(x-2) = 0$$

Either $x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $\quad$ $x = 2$

Thus, $-4, -1, 2$ and 3 are the roots of given equation.

Simultaneous equations:

A system of equations having a common solution is called a system of simultaneous equations.

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SOLVED EXERCISE 2.7

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1. $\mathbf{x + y = 5 \quad ; \quad x^2 - 2y - 14 = 0}$

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$$x^2 - 2(5 - x) - 14 = 0$$
$$x^2 - 10 + 2x - 14 = 0$$
$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x+6) - 4(x+6) = 0$$
$$(x-4)(x+6) = 0$$

Either $x - 4 = 0$ or $x + 6 = 0$

$x = 4$ $\quad$ $x = -6$

Put $x = 4$ in eq. (iii), we get $\qquad$ Put $x = -6$ in eq (iii), we get

$y = 5 - 4$
$= 1$

$y = 5 - (-6)$
$= 5 + 6$
$= 11$

The ordered pairs are (4, 1) are (– 6, 11).
Thus, solution set = {(4, 1), (– 6, 11)}

2. $\mathbf{3x - 2y = 1}$; $\mathbf{x^2 + xy - y^2 = 1}$

Solution:

$3x - 2y = 1$ ________ (i)
$x^2 + xy - y^2 = 1$ ________ (ii)
From (i), we hae
$2y = 3x - 1$
$$y = \frac{1}{2}(3x - 1)$$
$$y = \frac{3}{2}x - \frac{1}{2} \quad \text{\_\_\_\_\_\_\_\_ (iii)}$$
Put value of y in eq. (ii), we get
$$x^2 + x\left(\frac{3}{2}x - \frac{1}{2}\right) - \left(\frac{3}{2}x - \frac{1}{2}\right)^2 = 1$$
$$x^2 + \frac{3}{2}x^2 - \frac{1}{2}x - \frac{9}{4}x^2 + \frac{3}{2}x - \frac{1}{4} = 1$$
Multiplying both sides by '4', we get
$4x^2 + 6x^2 - 2x - 9x^2 + 6x - 1 = 4$
$4x^2 + 6x^2 - 9x^2 - 2x + 6x - 1 - 4 = 0$
$x^2 + 4x - 5 = 0$
$x^2 + 5x - x - 5 = 0$
$x(x + 5) - 1(x + 5) = 0$
$(x - 1)(x + 5) = 0$
Either $x - 1 = 0$ or $x + 5 = 0$
$x = 1$ $x = -5$

Put x = 1 in eq. (iii), we get
$$y = \frac{3}{2}(1) - \frac{1}{2}$$
$$= \frac{3}{2} - \frac{1}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

Put x = – 5 in eq (iii), we get
$$y = \frac{3}{2}(-5) - \frac{1}{2}$$
$$= -\frac{15}{2} - \frac{1}{2}$$
$$= -\frac{16}{2}$$
$$= -8$$

The ordered pairs are (1, 1) are (–5, –8).
Thus, solution set = {(1, 1), (– 5, –8)}

3. $x - y = 7$; $\frac{2}{x} - \frac{5}{y} = 2$

Solution:

$$x - y = 7 \quad \text{\_\_\_\_\_\_\_\_ (i)}$$

$$\frac{2}{x} - \frac{5}{y} = 2$$

$$\frac{2y - 5x}{xy} = 2$$

$$2y - 5x = 2xy \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

From (i), we hae

$$y = x - 7 \quad \text{\_\_\_\_\_\_\_\_ (iii)}$$

Put value of y in eq. (ii), we get

$$2(x-7) - 5x = 2x(x-7)$$

$$2x - 14 - 5x = 2x^2 - 14x$$

$$-3x - 14 = 2x^2 - 14x$$

or $2x^2 - 14x + 3x + 14 = 0$

$$2x^2 - 11x + 14 = 0$$

$$2x^2 - 7x - 4x + 14 = 0$$

$$x(2x - 7) - 2(2x - 7) = 0$$

$$(x - 2)(2x - 7) = 0$$

Either $x - 2 = 0$, $x = 2$ or $2x - 7 = 0$, $2x = 7$, $x = \frac{7}{2}$

Put $x = 2$ in eq. (iii), we get

$$y = 2 - 7$$

$$= -5$$

Put $x = \frac{7}{2}$ in eq (iii), we get

$$y = \frac{7}{2} - 7$$

$$y = -\frac{7}{2}$$

The ordered pairs are $(2,-5), \left(\frac{7}{2}, -\frac{7}{2}\right)$

Thus, solution set $= \left\{(2,-5), \left(\frac{7}{2}, -\frac{7}{2}\right)\right\}$

4. $x + y = a - b$; $\frac{a}{x} - \frac{b}{y} = 2$

Solution:

$$x + y = a - b \quad \text{\_\_\_\_\_\_\_\_ (i)}$$

$$\frac{a}{x}-\frac{b}{y}=2$$

$$\frac{ay-bx}{xy}=2 \qquad \text{\_\_\_\_\_\_\_\_\_ (ii)}$$

From eq. (i), we have

$y = a - b - x$ _________ (iii)

Put value of y in eq. (ii),, we get

$a(a-b-x) - bx - 2x(a-b-x)$

$2x^2 - 2ax - ax + 2bx - bx - a^2 - ab = 0$

$2x^2 - 3ax + bx + a^2 - ab = 0$

$2x^2 - (3a-b)x + (a^2 - ab) = 0$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-\left[-(3a-b)\pm\sqrt{\left[-(3a-b)\right]^2-4(2)(a^2-ab)}\right]}{4(2)}$$

$$=\frac{(3a-b)\pm\sqrt{9a^2-6ab+b^2-8a^2+8ab}}{4}$$

$$=\frac{(3a-b)\pm\sqrt{a^2+2ab+b^2}}{4}$$

$$=\frac{(3a-b)\pm\sqrt{(a+b)^2}}{4}=\frac{(3a-b)\pm(a+b)}{4}$$

Either $x=\dfrac{(3a-b)\pm(a+b)}{4}$ or $x=\dfrac{(3a-b)\pm(a+b)}{4}$

$$=\frac{3a-b+a+b}{4} \qquad\qquad =\frac{3a-b-a-b}{4}$$

$$=\frac{4a}{4} \qquad\qquad =\frac{2a-2b}{4}$$

$$=a \qquad\qquad =\frac{2(a-b)}{4}=\frac{a-b}{2}$$

Put $x = a$ in eq. (iii), we get

$y = a - b - a$

$= -b$

Put $x=\dfrac{a-b}{2}$ in eq. (iii), we get

$$y=(a-b)-\frac{a-b}{2}$$

$$=\frac{2a-2b-a+b}{2}$$

$$=\frac{a-b}{2}$$

$\therefore$ The ordered pairs are (a, - b), $\left(\dfrac{a-b}{2}, \dfrac{a-b}{2}\right)$

Thus, solution set = $\left\{(a,-b), \left(\dfrac{a-b}{2}, \dfrac{a-b}{2}\right)\right\}$

5. $\mathbf{x^2 + (y-1)^2 = 10}$; $\mathbf{x^2 + y^2 + 4x = 1}$

Solution:

$x^2 + (y-1)^2 = 10$

$x^2 + y^2 - 2y + 1 = 10$

$x^2 + y^2 - 2y = 10 - 1$

$x^2 + y^2 - 2y = 9$ ________ (i)

$x^2 + y^2 + 4x = 1$ ________ (ii)

Subtract eq. (ii) from eq. (i), we have

$$\begin{array}{r} x^2 + y^2 - 2y = 9 \\ \underline{\pm x^2 \pm y^2 \pm 4x = \pm 1} \\ -4x - 2y = 8 \end{array}$$

$-2(2x + y) = 8$

$\Rightarrow$ $2x + y = -4$

$y = -2x - 4$ ________ (iii)

Put the value of y in eq. (ii). We have

$x^2 + (-2x - 4)^2 + 4x = 1$

$x^2 + \{-(2x + 4)^2 + 4x = 1$

$x^2 + 4x^2 + 16x + 16 + 4x - 1 = 0$

$5x^2 + 20x + 15 = 0$

$5(x^2 + 4x + 3) = 0$

$\Rightarrow$ $x^2 + 4x + 3 = 0$

$x^2 + 3x + x + 3 = 0$

$x(x + 3) + 1(x + 3) = 0$

$(x + 1)(x + 3) = 0$

Either $x + 3 = 0$ or $x + 1 = 0$

$x = -3$ $x = -1$

Put $x = -3$ in eq (iii) we get,

$y = -2(-3) - 4$

$= -6 - 4$

$= -10$

Put $x = -1$ in eq (ii)

$y = -2(-1) - 4$

$= -2 - 4$

$= -6$

$\therefore$ The ordered pairs are $(-3, -10)$, $(-1, -6)$

Thus, solution set = $\{(-3, -10), (-1, -6)\}$

6. $\mathbf{(x+1)^2 + (y+1)^2 = 5}$; $\mathbf{(x+2)^2 + y^2 = 5}$

Solution:

$(x + 1)^2 + (y + 1)^2 = 5$

$x^2 + 2x + 1 + y^2 + 2y + 1 = 5$
$x^2 + y^2 + 2x + 2y = 5 - 1 - 1$
$x^2 + y^2 + 2x + 2y = 3$ ______ (i)
$(x + 2)^2 + y^2 = 5$
$x^2 + 4x + 4 + y^2 = 5$
$x^2 + y^2 + 4x = 5 - 4$
$x^2 + y^2 + 4x = 1$ _______ (ii)
Subtract eq. (ii) from eq. (i), we have

$$\begin{array}{l} x^2 + y^2 + 2x + 2y = 3 \\ \underline{\pm x^2 \pm y^2 \pm 4 \ = \pm 1} \\ \quad -2x + 2y = 2 \end{array}$$

$2(-x + y) = 2$

$\Rightarrow \quad -x + y = 1$

$y = x + 1$ ______(iii)

Put the value of y in eq. (ii), we have

$x^2 + (x + 1)^2 + 4x = 1$

$x^2 + x^2 + 2x + 1 + 4x = 1$

$2x^2 + 6x + 1 - 1 = 0$

$2x^2 + 6x + 0 = 0$

$2x^2 + 6x = 0$

$2x(x + 3) = 0$

Either $2x = 0$ or $x + 3 = 0$

Put $x = 0$ in eq. (iii), we get
$y = 0 + 1$
$= 0$

Put $x = -3$ in eq. (iii), we get
$y = -3 + 1$
$= -2$

$\therefore$ The ordered pairs are (0, 1), (– 3, –2)

Thus, solution set = $\{(0,1), (-3,-2)\}$

7. $\mathbf{x^2 + 2y^2 = 22}$; $\mathbf{5x^2 + y^2 = 29}$

Solution:

$x^2 + 2y^2 = 22$ _______ (i)
$5x^2 + y^2 = 29$ _______ (ii)

Multiply eq. (ii) by '2' then subtract eq. (ii) from eq. (i) we get

$$\begin{array}{l} \underline{\pm 10x^2 \pm 2y^2 = \pm 58} \\ \quad -9x^2 = -36 \end{array}$$

$\Rightarrow \quad x^2 = 4$

Put the value of $x^2 = 4$ in eq. (i), we get

$4 + 2y^2 = 22$

$2y^2 = 22 - 4$

$2y^2 = 18$

$\Rightarrow \quad y^2 = 9$

$y = \pm 3$

Thus, solution set = $\{(\pm 2, \pm 3)\}$

8. $4x^2 - 5y^2 = 6$; $3x^2 + y^2 = 14$

Solution:

$4x^2 - 5y^2 = 6$ ________ (i)

$3x^2 + y^2 = 14$ ________ (ii)

Multiply eq. (ii) by 5 then add eq. (i) and (ii) we get.

$3x^2 - 5y^2 = 6$

$\underline{15x^2 + 5y^2 = 70}$

$19x^2 = 76$

$\Rightarrow \quad x^2 = 4$

Put the value of $x^2 = 4$ in eq. (ii), we get

$3(4) + y^2 = 14$

$12 + y^2 = 14$

$y2 = 14 - 12$

$y2 = 2$

$y = \pm \sqrt{2}$

Thus, solution set = $\{(\pm 2, \pm\sqrt{2})\}$

9. $7x^2 - 3y^2 = 4$; $2x^2 + 5y^2 = 7$

Solution:

$7x^2 - 3y^2 = 4$ ________ (i)

$2x^2 + 5y^2 = 7$ ________ (ii)

Multiply eq. (i) by '5' and eq. (ii) by add eq. (i) and (ii), we get

$35x^2 - 15y^2 = 20$

$\underline{6x^2 + 15y = 21}$

$41x^2 = 41$

$\Rightarrow \quad x^2 = 1$

$x = \pm 1$

Put the value of $x^2 = 1$ in eq. (ii), we get

$2(1) + 5y^2 = 7$

$2 + 5y^2 = 7$

$5y^2 = 7 - 2$

$\Rightarrow \quad y^2 = 1$

$y = \pm 1$

Thus, solution set $= \{(\pm 1, \pm 1)\}$

10. $x^2 + 2y^2 = 3 \quad ; \quad x^2 + 4xy - 5y^2 = 0$

Solution:

$x^2 + 2y^2 = 3$ ________ (i)

$x^2 + 4xy - 5y^2 = 0$

$x^2 + 5xy - xy - 5y^2 = 0$

$x(x + 5y) - y(x + 5y) = 0$

$(x - y)(x + 5y) = 0$

$x - y = 0$ or $x + 5y = 0$

or $y = x$ $\quad$ $x = -5y$

Put $y = x$ in eq. (i), we get

$x^2 + 2x^2 = 3$

$3x^2 = 3$

$\Rightarrow \quad x^2 = 1$

$x = \pm 1$

Put $x = \pm 1$ in $y = x$, we get

$y = \pm 1$

Put $x = -5y$ in eq. (i), we get

$(-5y)^2 + 2y^2 = 3$

$25y^2 + 2y^2 = 3$

$27y^2 = 3$

$\Rightarrow \quad y^2 = \frac{1}{9}$

$y = \pm \frac{1}{3}$

Put $y = \pm \frac{1}{3}$ in $x = -5y$, we get

$x = -5\left(\pm \frac{1}{3}\right)$

$x = \mp \frac{5}{3}$

$\therefore$ The ordered pairs are $(1, 1)$, $(-1, 1)$ $\quad$ $\therefore$ The ordered pairs are $\left(\frac{5}{3}, -\frac{1}{3}\right), \left(-\frac{5}{3}, \frac{1}{3}\right)$

Thus, solution set $= \left\{(-1, 1), (-1, 1), \left(\frac{5}{3}, -\frac{1}{3}\right), \left(-\frac{5}{3}, \frac{1}{3}\right)\right\}$

11. $3x^2 - y^2 = 26 \quad ; \quad 3x^2 - 5xy - 2y^2 = 0$

Solution:

$3x^2 - y^2 = 26$ ________ (i)

$3x^2 - 5xy - 2y^2 = 0$

$$3x^2 - 9xy - 4xy - 12y^2 = 0$$
$$3x(x - 3y) + 4y(x - 3y) = 0$$
$$(3x + 4y)(x - 3y) = 0$$

Either $3x + 4y = 0$ or $x - 3y = 0$

$3x = -4y$ $\quad$ $x = 3y$

$x = -\frac{4}{3}y$

Put $x = -\frac{4}{3}y$ in eq. (i), we get

Put $x = 3y$ in eq. (i), we get

$$3(3y)^2 - y^2 = 26$$
$$3(9x)^2 - y^2 = 26$$
$$27y2 - y^2 = 26$$
$$26y^2 = 26$$
$$\Rightarrow \quad y^2 = 1$$
$$y = \pm 1$$

Put $y = \pm 1$ in $x = 2y$, we get

$$x = 3(\pm 1)$$
$$x = \pm 3$$

$\therefore$ The ordered pairs are $(3, 1), (-3, 1)$

$$3\left(-\frac{4}{3}y\right)^2 - y^2 = 26$$
$$3\left(\frac{16}{9}y^2\right) - y^2 = 26$$
$$\frac{16}{9}y^2 - y^2 = 26$$
$$\frac{16y^2 - 3y^2}{3} = 26$$
$$13y^2 = \frac{26 \times 3}{13}$$
$$y^2 = 6$$
$$y = \pm\sqrt{6}$$

Put $y = \pm\sqrt{6}$ in $x = -\frac{4}{3}y$, we get

$$x = -\frac{4}{3}\left(\pm\sqrt{6}\right)$$
$$x = \mp\frac{4\sqrt{3}}{3}$$

$\therefore$ The ordered pairs are $\left(\frac{-4\sqrt{3}}{3}, \sqrt{6}\right), \left(\frac{4\sqrt{3}}{3}, -\sqrt{6}\right)$

Thus, solution set =

12. $x^2 + xy = 5 \quad ; \quad y^2 + xy = 3$

Solution:

$x^2 + xy = 5$ ________ (i)

$y^2 + xy = 3$ ________ (ii)

Multiply eq. (i) by '3' and eq. (ii) by '5' then subtract eq. (ii) from eq. (i), we get.

$$3x^2 + 3xy = 15$$
$$\underline{\pm 5y^2 \pm 5xy = \pm 15}$$
$$3x^2 - 2xy - 5y^2 = 0$$
$$3x^2 - 5xy + 3xy - 5y^2 = 0$$
$$x(3x - 5y) + y(3x - 5y) = 0$$
$$(x + y)(3x - 5y) = 0$$

Either $x + y = 0$ or $3x - 5y = 0$

$y = -x$; $3x = 5y$

$x = \frac{5}{3}y$

Put $x = -x$ in eq. (i), we get

$$x^2 + x(-x) = 5$$
$$x^2 - x^2 = 5$$
$$0 = 5$$

which is not possible

Put $x = \frac{5}{3}y$ in eq. (i), we get

$$\left(\frac{5}{3}y\right)^2 + \left(\frac{5}{3}y\right)y = 5$$
$$\frac{25}{9}y^2 + \frac{5}{3}y^2 = 5$$
$$\frac{25y^2 + 15y^2 = 5}{9}$$
$$\frac{40y^2}{9} = 5$$
$$40y^2 = 45$$
$$y^2 = \frac{45}{40}$$
$$y^2 = \frac{9}{8}$$
$$y = \pm\frac{3}{2\sqrt{2}}$$

Put $y = \pm\dfrac{3}{2\sqrt{2}}$ in $x = \dfrac{5}{3}y$, we get

$$x = \frac{5}{3}\left(\pm\frac{3}{2\sqrt{2}}\right)$$

$$x \pm \frac{5}{2\sqrt{2}}$$

$\therefore$ The ordered pairs are $\left(\dfrac{5}{2\sqrt{2}}, -\dfrac{3}{2\sqrt{2}}\right), \left(-\dfrac{5}{2\sqrt{2}}, -\dfrac{3}{2\sqrt{2}}\right)$

Thus, solution set $= \left\{\left(\dfrac{5}{2\sqrt{2}}, -\dfrac{3}{2\sqrt{2}}\right), \left(-\dfrac{5}{2\sqrt{2}}, -\dfrac{3}{2\sqrt{2}}\right)\right\}$

13. $x^2 - 2xy = 7$; $xy + 3y^2 = 2$

Solution:

$x^2 - 2xy = 7$ ________ (i)

$xy + 3y^2 = 2$ ________ (ii)

Multiply eq (i) by '2' and eq. (ii) by '7' then subtract eq. (ii) from eq (i), we get

$$2x^2 - 4xy = 14$$

$$\underline{\pm 21y^2 \pm 7xy = \pm 14}$$

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(2x + 3y)(x - 7y) = 0$$

Either $2x + 3y = 0$ or $x - 7y = 0$

$2x = -3y$ $x = 7t$

Put $x = -\dfrac{3}{2}y$ in eq. (ii), we get Put $x = 7y$ in eq. (ii), we get

$x = -\dfrac{3}{2}\left(\pm\dfrac{2}{\sqrt{3}}\right)$ $= \pm\dfrac{7}{\sqrt{5}}$

$= \mp\sqrt{3}$

$\therefore$ The ordered pairs are $\left(-\sqrt{3}, \dfrac{2}{\sqrt{3}}\right), \left(\sqrt{3}, -\dfrac{2}{\sqrt{3}}\right)$

$\therefore$ The ordered pairs are $\left(\dfrac{7}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\right), \left(\dfrac{7}{\sqrt{5}}, -\dfrac{1}{\sqrt{5}}\right)$

Thus, solution set $= \left\{\left(\dfrac{7}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\right), \left(\dfrac{7}{\sqrt{5}}, -\dfrac{1}{\sqrt{5}}\right), \left(-\sqrt{3}\,\dfrac{2}{\sqrt{3}}\right), \left(\sqrt{3} - \dfrac{2}{\sqrt{3}}\right)\right\}$