

SOLVED EXERCISE 2.8

1. The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Let the numbers be $x, x + 1$

Then, $(x)(x + 1) = 182$

$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x + 14) - 13(x + 14) = 0$$

$$(x - 13)(x + 14) = 0$$

$$x - 13 = 0 \quad \text{gives } x = 13$$

Numbers are: $x = 13$

$$x + 1 = 13 + 1 = 14$$

Now, $x + 14 = 0$ gives $x = -14$

We ignore this value.

2. The sum of the squares of three positive consecutive numbers is 77. Find them.

Solution:

Let the numbers be $x, x + 1, x + 2$

Applying the given condition.

$$(x)^2 + (x + 1)^2 + (x + 2)^2 = 77$$

$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 77$$

$$3x^2 + 6x + 5 = 77$$

$$3x^2 + 6x + 5 - 77 = 0$$

$$3x^2 + 6x - 72 = 0$$

$$x^2 + 2x - 24 = 0$$

(Dividing by 3)

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x + 6) - 4(x + 6) = 0$$

$$(x - 4)(x + 6) = 0$$

$$x - 4 = 0 \quad \text{gives } x = 4$$

Numbers are: $x = 4$

$$x + 1 = 4 + 1 = 5$$

$$x + 2 = 4 + 2 = 6$$

4, 5, 6

$$x + 6 = 0$$

$x = -6$, we ignore it.

3. The sum of five times a number and the square of the number is 204. Find the number.

Solution:

Let the number be x

According to the given condition.

$$\begin{aligned}
 x^2 + 5x &= 204 \\
 x^2 + 5x - 204 &= 0 \\
 x^2 + 17x - 12x - 204 &= 0 \\
 x(x + 17) - 12(x + 17) &= 0 \\
 (x + 17)(x - 12) &= 0 \\
 x - 12 &= 0 && \text{gives } x = 12 \\
 \text{or} &&& \\
 x + 17 &= 0 && \text{gives } x = -17
 \end{aligned}$$

Number is 12 or -17.

4. **The product of five less than three times a certain number and one less than four times the number is 7. Find the number.**

Solution:

Let the number be x .

According to the given condition.

$$\begin{aligned}
 (3x - 5)(4x - 1) &= 7 \\
 12x^2 - 23x + 5 &= 7 \\
 12x^2 - 23x + 5 - 7 &= 0 \\
 12x^2 - 23x - 2 &= 0 \\
 12x^2 - 24x + x - 2 &= 0 \\
 12x(x - 2) + 1(x - 2) &= 0 \\
 (12x + 1)(x - 2) &= 0 \\
 x - 2 &= 0 && \text{gives } x = 2 \\
 \text{and} &&& \\
 12x + 1 &= 0 && \text{gives } x = -\frac{1}{12}
 \end{aligned}$$

Number is 2 or $-\frac{1}{12}$

5. **The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.**

Find the number.

Solution:

Let the number be x .

$$\begin{aligned}
 \text{Then,} &&& x - \frac{1}{x} = \frac{15}{4} \\
 &&& 4x^2 - 4 = 15x && \text{(Multiplying by } 4x) \\
 &&& 4x^2 - 15x - 4 = 0 \\
 &&& 4x^2 - 16x + x - 4 = 0 \\
 &&& 4x(x - 4) + 1(x - 4) = 0 \\
 &&& (x - 4)(4x + 1) = 0 \\
 &&& x - 4 = 0 && \text{gives } x = 4 \\
 \text{and} &&& 4x + 1 = 0 && \text{gives } x = -\frac{1}{4}
 \end{aligned}$$

Number is 4 and $-\frac{1}{4}$

6. **The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.**

Solution:

Let xy be the number, where unit digit is y and tens digit is x .
According to the given condition.

$$x^2 + y^2 = 65 \dots\dots\dots(A)$$

$$\text{Number} = y + 10x$$

$$y + 10x = 9(x + y)$$

$$y + 10x = 9x + 9y$$

$$10x - 9x = 9y - y$$

$$x = 8y \dots\dots\dots(B)$$

Putting

$$x = 8y \quad \text{in (A)}$$

$$(8y)^2 + y^2 = 65$$

$$64y^2 + y^2 = 65$$

$$65y^2 = 65$$

$$y^2 = 1$$

$$y = \pm 1$$

$$y = 1 \quad \text{(taking +ve value)}$$

put $y = 1$ in (B)

$$x = 8(1) = 8$$

Therefore, number is 8, 1

7. **The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.**

Solution:

Let $P(x, y)$ be the point.

According to the given conditions.

$$x + y = 9 \dots\dots\dots(A)$$

and $x^2 + y^2 = 45 \dots\dots\dots(B)$

From A $x = 9 - y \dots\dots\dots(C)$

Putting $x = 9 - y$ in (B)

$$(9 - y)^2 + y^2 = 45$$

$$81 - 18y + y^2 + y^2 = 45$$

$$2y^2 - 18y + 81 - 45 = 0$$

$$2y^2 - 18y + 36 = 0$$

$$y^2 - 9y + 18 = 0 \quad \text{(Dividing by 2)}$$

$$y^2 - 6y - 3y + 18 = 0$$

$$y(y - 6) - 3(y - 6) = 0$$

$$(y - 6)(y - 3) = 0$$

$$y - 6 = 0 \quad \text{gives } y = 6$$

Then from (C)

$$x = 9 - 6 = 3$$

Point is P (3, 6)

When $y - 3 = 0$ then $y = 3$

From (C)

$$x = 9 - 3 = 6$$

Point is (6, 3)

8. Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution:

Let the integers be x, y .

Then, $x + y = 9$ (A)

and $x^2 - y^2 = 9$ (B)

From A

$$x = 9 - y$$

Putting $x = 9 - y$ in (B)

$$(9 - y)^2 - y^2 = 9$$

$$81 - 18y + y^2 - y^2 = 9$$

$$- 18y = 9 - 81$$

$$- 18y = - 72$$

$$y = - \frac{72}{18}$$

$$y = 4$$

Putting

$$y = 4 \text{ in (A)}$$

$$x + 4 = 9$$

$$x = 9 - 4 = 5$$

Integers are 5, 4

9. Find two integers whose difference is 4 and whose squares differ by 72.

Solution:

Let the integers be x and

according to the given conditions.

$$x - y = 4 \text{(A)}$$

and

$$x^2 - y^2 = 72 \text{(B)}$$

$$x - y = 4 \text{ from (A)}$$

$$x = 4 + y$$

Putting

$$x = 4 + y \text{ in (B), we get}$$

$$(4 + y)^2 - y^2 = 72$$

$$16 + y^2 + 8y - y^2 = 72$$

$$8y = 72 - 16$$

$$8y = 56$$

$$y = \frac{56}{8}$$

$$y = 7$$

put

$$y = 7 \text{ in (A)}$$

$$x - 7 = 4$$

$$x = 4 + 7 = 11$$

Integers are: 11 and 7

10. Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375cm².

Solution:

Let x and y be the length and width respectively of the rectangle.

According to the given conditions.

Perimeter: $2x + 2y = 80$

or $x + y = 40 \dots\dots\dots\text{(A)}$

Area: $xy = 375 \dots\dots\dots\text{(B)}$

$$x = 40 - y \text{ (from A)}$$

Putting $x = 40 - y$ in (B)

$$(40 - y)y = 375$$

$$40y - y^2 = 375$$

$$\Rightarrow y^2 - 40y + 375 = 0$$

$$y^2 - 25y - 15y + 375 = 0$$

$$y(y - 25) - 15(y - 25) = 0$$

$$(y - 15)(y - 25) = 0$$

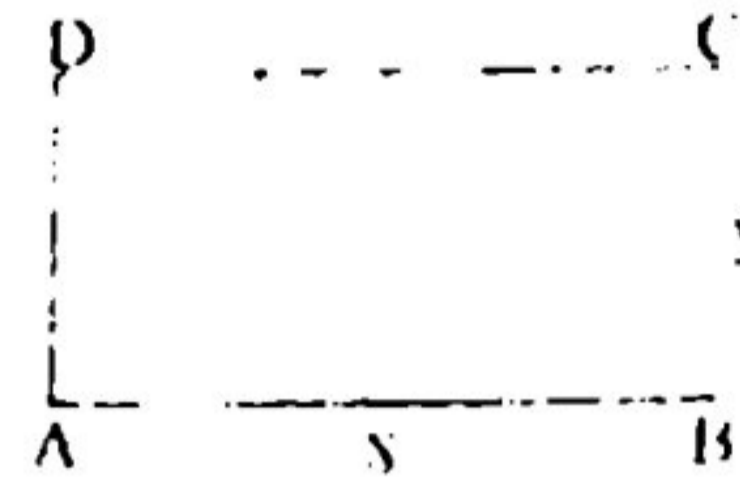
$$y - 15 = 0 \quad \text{gives} \quad y = 15$$

Putting $y = 15$ in (A)

$$x + 15 = 40$$

$$x = 40 - 15 = 25$$

Length = 25cm, Breadth = 15 cm.



SOLVED MISCELLANEOUS EXERCISE - 2

1. Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) If α, β are the roots of $3x^2 + 5x - 2 = 0$, then $\alpha + \beta$ is

(a) $\frac{5}{3}$

(b) $\frac{3}{5}$

(c) $\frac{-5}{3}$

(d) $\frac{-2}{3}$

(ii) If α, β are the roots of $7x^2 - x + 4 = 0$, then $\alpha\beta$ is

(a) $\frac{-1}{7}$

(b) $\frac{4}{7}$

(c) $\frac{7}{4}$

(d) $\frac{-4}{7}$

(iii) Roots of the equation $4x^2 - 5x + 2 = 0$ are