

## SOLVED EXERCISE 3.1

1. Express the following as a ratio  $a : b$  and as a fraction in its simplest (lowest) form.

(i) Rs.750, Rs. 1250

*Solution:*

$$\begin{aligned} & 750 : 1250 \\ &= \frac{750}{10} : \frac{1250}{10} \quad (\text{Divided by } 10) \\ &= 75 : 125 \\ &= \frac{75}{10} : \frac{125}{10} \quad (\text{Divided by } 5) \\ &= 15 : 25 \\ &= \frac{15}{5} : \frac{25}{5} \quad (\text{Divided by } 5) \\ &= 3 : 5 \\ &= \frac{3}{5} \end{aligned}$$

(ii) 450cm, 3 m

*Solution:*

$$\begin{aligned} 450 \text{ cm} : 3 \text{ m} &= 450 \text{ cm} : 300 \text{ cm} && 1 \text{ m} = 100 \text{ cm} \\ &= 450 : 300 \\ &= \frac{450}{10} : \frac{300}{10} && (\text{Divided by } 10) \\ &= 45 : 30 \\ &= \frac{45}{5} : \frac{30}{5} && (\text{Divided by } 5) \\ &= 9 : 6 \\ &= \frac{9}{3} : \frac{6}{3} && (\text{Divided by } 3) \\ &= 3 : 2 \\ &= \frac{3}{2} \end{aligned}$$

(iii) 4kg, 2kg 750gm

*Solution:*

$$4 \text{ kg} : 2 \text{ kg } 750 \text{ gm} = 4000 \text{ g} : 2750 \text{ g} \quad 1 \text{ kg} = 1000 \text{ g}$$

$$\begin{aligned}
&= 4000 : 2750 \\
&= \frac{400}{10} : \frac{2750}{10} && \text{(Divided by 10)} \\
&= 400 : 275 \\
&= \frac{400}{5} : \frac{275}{5} && \text{(Divided by 5)} \\
&= 80 : 55 \\
&= \frac{80}{5} : \frac{55}{5} && \text{(Divided by 5)} \\
&= 16 : 11 \\
&= \frac{16}{11}
\end{aligned}$$

**(iv) 27 min 30 sec, 1 hour**

**Solution:**

$$\begin{aligned}
27 \text{ min. } 30 \text{ sec} : 1 \text{ hour} &= (27 \times 60 + 30) \text{ sec} : (1 \times 60 \times 60) \text{ sec} \\
&= 1650 : 3600 \\
&= \frac{1650}{10} : \frac{3600}{10} && \text{(Divided by 10)} \\
&= 165 : 360 \\
&= \frac{165}{5} : \frac{360}{5} && \text{(Divided by 5)} \\
&= 33 : 72 \\
&= \frac{33}{3} : \frac{72}{3} && \text{(Divided by 3)} \\
&= 11 : 24 \\
&= \frac{11}{24}
\end{aligned}$$

**(v) 75°, 225°**

**Solution:**

$$\begin{aligned}
75^\circ : 225^\circ &= \frac{75}{5} : \frac{225}{5} && \text{(Divided by 5)} \\
&= 15 : 45 \\
&= \frac{15^\circ}{5} : \frac{45}{5} && \text{(Divided by 5)} \\
&= 3 : 15 \\
&= \frac{3}{3} : \frac{15}{3} && \text{(Divided by 3)}
\end{aligned}$$



$$= 1:5 \Rightarrow = \frac{1}{5}$$

2. In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

(i) Boys to total students

*Solution:*

$$\text{Total students} = 60$$

$$\text{Number of girls students} = 25$$

$$\text{Number of boys students} = 60 - 25 = 35$$

$$\begin{aligned} 25 : 60 &= \frac{25}{5} : \frac{60}{5} && \text{(Divided by 5)} \\ &= 7:12 \end{aligned}$$

(ii) Boys to girls

*Solution:*

$$\begin{aligned} 35 : 25 &= \frac{35}{5} : \frac{25}{5} && \text{(Divided by 5)} \\ &= 7:5 \end{aligned}$$

3. If  $3(4x - 5y) = 2x - 7y$ , find the ratio  $x : y$ .

*Solution:*

$$3(4x - 5y) = 2x - 7y$$

$$12x - 15y = 2x - 7y$$

$$12x - 2x = 15y - 7y$$

$$10x = 8y$$

$$\frac{x}{y} = \frac{8}{10}$$

$$\frac{x}{y} = \frac{4}{5} \quad \text{(Divided by 2)}$$

$$\Rightarrow x : y = 4 : 5$$

4. Find the value of  $p$ , if the ratios  $2p + 5 : 3p + 4$  and  $3 : 4$  are equal.

*Solution:*

As the given ratios are equal, so

$$2p + 5 : 3p + 4 = 3 : 4$$

$$\frac{2p + 5}{3p + 4} = \frac{3}{4}$$

$$3(3p + 4) = 4(2p + 5)$$



$$9p + 12 = 8p + 20$$

$$9p - 8p = 20 - 12$$

$$p = 8$$

5. If the ratios  $3x + 1 : 6 + 4x$  and  $2 : 5$  are equal. Find the value of  $x$ .

*Solution:*

As the given ratios are equal, so

$$3x + 1 : 6 + 4x = 2 : 5$$

$$\frac{3x + 1}{6 + 4x} = \frac{2}{5}$$

$$5(3x + 1) = 2(6 + 4x)$$

$$15x + 5 = 12 + 8x$$

$$15x - 8x = 12 - 5$$

$$7x = 7$$

Dividing throughout by '7', we get

$$x = 1$$

6. Two numbers are in the ratio  $5 : 8$ . If 9 is added to each number, we get a new ratio  $8 : 11$ . Find the numbers,

*Solution:*

Because the ratio of two numbers is  $5 : 8$ .

Multiply each number of the ratio with  $x$ . then the numbers be  $5x$ ,  $8x$  and the ratio becomes  $5x : 8x$ .

Now according to the given condition, we have

$$\frac{5x + 9}{8x + 9} = \frac{8}{11}$$

$$8(8x + 9) = 11(5x + 9)$$

$$64x + 72 = 55x + 99$$

$$64x - 55x = 99 - 72$$

$$9x = 27$$

Dividing both sides by '9', we get

$$x = 3$$

The required numbers are

$$5x = 5(3) = 15$$

$$8x = 8(3) = 24$$

7. If 10 is added in each number of the ratio  $4:13$ , we get a new ratio  $1 : 2$ . What are the numbers?

*Solution:*

Because the ratio of two numbers is  $4 : 13$ .



Multiply each number of the ratio with  $x$ . then the numbers be  $4x$ ,  $13x$  and the ratio becomes  $4x : 13x$ .

Now according to the given condition, we have

$$\frac{4x + 10}{13x + 10} = \frac{1}{2}$$
$$1(13x + 10) = 2(4x + 10)$$
$$13x + 10 = 8x + 20$$
$$13x - 8x = 20 - 10$$
$$5x = 10$$

Dividing both sides by '5', we get

$$x = 2$$

The required numbers are

$$4x = 4(2) = 8$$

$$13x = 13(2) = 26$$

**8. Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250.**

*Solution:*

Let the cost of 8 kg mangoes be  $x$  – rupees.

Then in proportion form, we have

$$8 \text{ kg} : 5 \text{ kg} :: \text{Rs. } x : \text{Rs. } 250$$

Product of means = Product of extremes

$$(5)(x) = (8)(250)$$

$$5x = 8 \times 250$$

$$x = \frac{8 \times 250}{5}$$

$$x = 8 \times 50$$

$$x = \text{Rs. } 400$$

**9. If  $a : b = 7 : 6$ , find the value of  $3a + 5b : 7b - 5a$ .**

*Solution:*

Given that  $a : b = 7 : 6$

or  $\frac{a}{b} = \frac{7}{6}$

$$\begin{aligned} \text{Now } 3a + 5b : 7b - 5a &= \frac{3a + 5b}{7b - 5a} \\ &= \frac{3a + 5b}{7b - 5a} \\ &= \frac{b}{b} \quad (\text{Dividing numerator and denominator by 'b'}) \end{aligned}$$

$$\begin{aligned}
&= \frac{3\left(\frac{a}{b}\right) + 5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right) - 5\left(\frac{a}{b}\right)} \\
&= \frac{3\left(\frac{7}{6}\right) + 5}{7 - 5\left(\frac{7}{6}\right)} \\
&= \frac{\frac{21}{6} + 5}{7 - \frac{35}{6}} = \frac{\frac{21 + 5 \times 6}{6}}{\frac{7 \times 6 - 35}{6}} \\
&= \frac{\frac{21 + 30}{6}}{\frac{42 - 35}{6}} = \frac{\frac{51}{6}}{\frac{7}{6}} \\
&= \frac{51}{6} \times \frac{6}{7} = \frac{51}{7}
\end{aligned}$$

Hence,

$$3a + 5b : 7b - 5a = 51 : 7$$

**10. Complete the following:**

(i) If  $\frac{24}{7} = \frac{6}{x}$ , then  $4x =$  \_\_\_\_\_

*Solution:*

$$\begin{aligned}
\frac{24}{7} &= \frac{6}{x} \\
(24)(x) &= (6)(7) \\
(6 \times 4)(x) &= (6)(7) \\
4x &= \frac{6 \times 7}{6} \\
4x &= 7
\end{aligned}$$

(ii) If  $\frac{5a}{3x} = \frac{15b}{y}$ , then  $ay =$  \_\_\_\_\_

*Solution:*

$$\frac{5a}{3x} = \frac{15b}{y}$$



$$(5a)(y) = (3x)(15b)$$

$$(5)(ay) = (3x)(15b)$$

$$ay = \frac{(3x)(15b)}{5}$$

$$ay = 9bx$$

(iii) If  $\frac{9pq}{2lm} = \frac{18p}{5m}$ , then  $5q = \underline{\hspace{2cm}}$

**Solution:**

$$(5m)(9pq) = (2lm)(18p)$$

$$(9mp)(5q) = (2lm)(18p)$$

$$5q = (2lm)(18p)$$

$$5q = (2l)(2)$$

$$5q = 4l$$

**11. Find x in the following proportions.**

(i)  $3x - 2 : 4 :: 2x + 3 : 7$

**Solution:**

$$3x - 2 : 4 :: 2x + 3 : 7$$

$$3x - 2 : 4 = 2x + 3 : 7$$

∴ Product of extremes = Product of means

$$(3x - 2)(7) = (4)(2x + 3)$$

$$21x - 14 = 8x + 12$$

$$13x = 26$$

Dividing both sides by '13', we get

$$x = 2$$

(ii)  $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

**Solution:**

$$\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$$

$$\frac{3x-1}{7} : \frac{3}{5} = \frac{2x}{3} : \frac{7}{5}$$

Multiplying throughout by '105' (L.C.M. of 3, 5, 7), we get

$$105 \times \frac{3x-1}{7} : 105 \times \frac{3}{5} = 105 \times \frac{2x}{3} : 105 \times \frac{7}{5}$$

$$15 \times (3x-1) : 21 \times 3 = 35 \times 2x : 21 \times 7$$

$$(45x-15) : 63 = 70x : 147$$

$\therefore$  Product of extremes = Product of means

$$(45x-15)(147) = (63)(70x)$$

$$6615x - 2205 = 4410x$$

$$6615x - 4410x = 2205$$

$$2205x = 2205$$

Dividing both sides by '2205', we get

$$(iii) \quad \frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$$

**Solution:**

$$\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$$

$$\frac{x-3}{2} : \frac{5}{x-1} = \frac{x-1}{3} : \frac{4}{x+4}$$

$\therefore$  Product of extremes = Product of means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right) = \left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$

$$\frac{2(x-3) \cdot 5}{x+4} = \frac{5}{3}$$

$$\frac{2x-6}{x+4} = \frac{5}{3}$$

$$3(2x-6) = 5(x+4)$$

$$6x-18 = 5x+20$$

$$6x-5x = 18+20$$

$$x = 38$$

$$(iv) \quad p^2 + pq + q^2 : x :: \frac{p^3 - q^3}{p+q} : (p-q)^2$$

**Solution:**

$$p^2 + pq + q^2 : x :: \frac{p^3 - q^3}{p+q} : (p-q)^2$$



$$p^2 + pq + q^2 : x = \frac{(p^3 - q^3)}{p + q} : (p - q)^2$$

∴ Product of means = Product of extremes

$$x \times \frac{p^3 - q^3}{p + q} = (p - q)^2 (p^2 + pq + q^2)$$

$$x = \frac{(p - q)(p - q)(p^2 + pq + q^2)(p + q)}{(p - q)(p^2 + pq + q^2)}$$

$$x = p^2 - q^2$$

(v)  $8 - x : 11 - x :: 16 - x : 25 - x$

**Solution:**

$$8 - x : 11 - x :: 16 - x : 25 - x$$

$$8 - x : 11 - x = 16 - x : 25 - x$$

∴ Product of extremes = Product of means

$$(8 - x)(25 - x) = (11 - x)(16 - x)$$

$$200 - 8x - 25x + x^2 = 176 - 11x - 16x + x^2$$

$$200 - 33x + x^2 = 176 - 27x + x^2$$

$$200 - 33x = 176 - 27x$$

$$-33x + 27x = 176 - 200$$

$$-6x = -24$$

$$\Rightarrow x = 4$$

(c) **Variation:**

The word variation is frequently used in all sciences. There are two types of variations:

(i) Direct variation (ii) Inverse variation.

(i) **Direct Variation**

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

In other words, if a quantity varies directly with regard to a quantity  $x$ . We say that  $y$  is

directly proportional to  $x$  and is written as  $y \propto x$  or  $y = kx$ . i.e.,  $\frac{y}{x} = k$ ,  $k \neq 0$ .

The sign read as "varies as" is called the sign of proportionality or variation, while  $k \neq 0$  is known as constant of variation. e.g.,

(i) Faster the speed of a car, longer the distance it covers.

(ii) The smaller the radius of the circle, smaller the circumference is.

(ii) **Inverse Variation**

If two quantities are related in such a way that when one quantity increases, the other decreases is called inverse variation.



In other words, if a quantity  $y$  varies inversely with regard to quantity  $x$ . We say that  $y$  is inversely proportional to  $x$  or  $y$  varies inversely as  $x$  and is written as  $y \propto \frac{1}{x}$  or

$$y = \frac{k}{x}.$$

i.e.,  $xy = k$ , where  $k \neq 0$  is the constant of variation.

## SOLVED EXERCISE 3.2

1. If  $y$  varies directly as  $x$ , and  $y = 8$  when  $x = 1$ , find

(i)  $y$  in terms of  $x$

*Solution:*

Given that  $y$  varies directly as  $x$ .

Therefore  $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where  $k$  is constant of variation.

Put  $x = 1$  and  $y = 8$  in eq. (i), we have

$$8 = k(1)$$

$$\text{or } 2k = 8$$

$$\Rightarrow k = 4$$

Put  $k = 4$  in eq. (i), we get

$$y = 4x.$$

(ii)  $y$  when  $x = 5$

*Solution:*

Given that  $y$  varies directly as  $x$ .

Therefore  $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where  $k$  is constant of variation.

Put  $x = 1$  and  $y = 8$  in eq. (i), we have

$$8 = k(1)$$

$$\text{or } 2k = 8$$

$$\Rightarrow k = 4$$

Put  $k = 4$  and  $x = 5$  in eq. (i), we get

$$y = (5)(4) = 20$$

(iii)  $x$  when  $y = 28$

*Solution:*

Given that  $y$  varies directly as  $x$ .

Therefore  $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where  $k$  is constant of variation.

Put  $x = 1$  and  $y = 8$  in eq. (i), we have