$$\frac{(x+5)^3}{(x-3)^3} = (3)^3$$

Taking power '$\frac{1}{3}$' on both sides, we get.

$$\frac{\left[(x+5)^3\right]^{\frac{1}{3}}}{\left[(x+5)^3\right]^{\frac{1}{3}}} = \left[(3)^3\right]^{\frac{1}{3}}$$

$$\frac{x+5}{x-3} = 3$$

$$3(x-3) = x+5$$

$$3x - 9 = x + 5$$

$$3x - x = 9 + 5$$

$$2x = 14$$

$$\Rightarrow \quad x = 7$$

Thus, solution set = $\{7\}$

(i) Joint variation

A combination of direct and inverse variations of one or more than one variables forms joint variation,

If a variable y varies directly as x and varies inversely as z.

Then $y \propto x$ and $y \propto \frac{1}{z}$

In joint variation, we write it as

$$y \propto \frac{x}{z}$$

i.e., $\quad y = k\frac{x}{z}$

Where $k \neq 0$ is the constant of variation.

SOLVED EXERCISE 3.5

1. If s varies directly as u^2 and inversely as v and s = 7 when M = 3, v = 2. Find the value of s when u = 6 and v = 10.

Solution:

Given that s varies directly as u^2, so

$$S \propto u^2$$

Also given that S varies inversely as V, so

$$S \propto \frac{1}{V}$$

In joint variation, we can write

$$S \propto \frac{u^2}{V}$$

$$\Rightarrow \quad S = K\frac{u^2}{V} \quad \_\_\_\_\_\_\_\_ (i)$$

Put S = 7, u = 3 and V = 2 in eq. (i), we get

$$7 = K\frac{(3)^2}{2}$$

$$7 = \frac{9K}{2}$$

or $$\frac{9K}{2} = 7$$

multiplying both sides by $\frac{2}{9}$, we get

$$K = 7 \times \frac{2}{9}$$

$$K = \frac{14}{9}$$

Put $K = \frac{14}{9}$ in eq. (i), we get

$$S = \frac{14}{9} \quad \_\_\_\_\_\_\_\_ (ii)$$

Put u = 6 and V = 10 in eq. (ii), we get

$$S = \frac{14 \times u^2}{9v}$$

$$= \frac{14 \times 36}{9 \times 10} = \frac{504}{90} = \frac{28}{5}$$

2. If w varies jointly as x, y^2 and 2 and w = 5 when x == 2, .y = 3, z = 10. Find w when x = 4, y = 7 and z = 3.

Solution:

Given that s varies directly as x, y^2 and z.

Therefore $W \propto xy^2z$

$$\Rightarrow \quad W = K\,xy^2z \quad \_\_\_\_\_\_\_\_ (i)$$

Put W = 5, x = 2, y = 3 and z = 10 in eq. (i), we get

$$5 = k(2)(3)^2(10)$$

$$5 = k(2)(9)(10)$$

$$5 = 180k$$

or $180k = 5$

$$k = \frac{5}{180} = \frac{1}{36}$$

Put $k = \frac{1}{36}$ in eq. (i), we get

$$W = \frac{1}{36}xy^2 = \text{\_\_\_\_\_\_\_\_} \quad (ii)$$

Put x = 4, y = 7 and z = 3 in eq. (ii), we get

$$W = \frac{1}{36}(4)(7)^2(3)$$

$$= \frac{1}{36}(4)(49)(3)$$

$$= \frac{588}{36} = \frac{49}{3}$$

3. If y varies directly as x^3 and inversely as z^2 and t, and y = 16 when x = 4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

Solution:

Given that s varies directly as x^2.

Therefore $\quad y \propto x^2$

Also given that y varies inversely as z^2 and t.

Therefore $\quad y \propto \frac{1}{z^2 t}$

In joint variation, we can write

$$y \propto \frac{x^3}{z^2 t}$$

$$\Rightarrow \quad y = K\frac{x^3}{z^2 t} \quad \text{\_\_\_\_\_\_\_\_} \quad (i)$$

Put y = 16, x = 4, z = 2, t = 3 in eq. (i), we get

$$16 = k\frac{(4)^3}{(2)^2(3)}$$

$$16 = k\frac{64}{4 \times 3}$$

$$16 = \frac{64}{12}$$

or $\quad = \frac{64}{12}K = 16$

$$K = 16 \times \frac{12}{64}$$

$$K = 3$$

Put K= 2 in eq. (i), we get

$$y = \frac{3x^3}{z^2 t} \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

Put x = 2, z = 3 and t = 4 in eq. (ii), we get

$$y = \frac{3(2)^3}{(3)^2(4)}$$

$$= \frac{3 \times 8}{9 \times 4} = \frac{2}{3}$$

4. If u varies directly as x^2 and inversely as the product yz^3, and u = 2 when x = 8, y = 7, z = 2. Find the value of u when x = 6, y = 3, z = 2.

Solution:

Given that u varies directly as x^2.

Therefore $u \propto x^2$

Also given that u varies inversely as yz^3.

Therefore $u \propto \frac{1}{yz^3}$

In joint variation, we can write

$$u \propto \frac{x^2}{yz^3}$$

$$\Rightarrow \quad u = K\frac{x^2}{yz^3} \quad \text{\_\_\_\_\_\_\_\_ (i)}$$

Put u = 2, x = 8, y = 7, and z = 2, we get

$$2 = k\frac{(8)^2}{(7)(2)^3}$$

$$2 = \frac{64}{56}K$$

or $$K = 2 \times \frac{56}{64} = \frac{7}{4}$$

Put K= $\frac{7}{4}$ in eq. (i), we get

$$u = \frac{7x^3}{4yz^2} \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

Put x = 6, y = 3 and z = 2 in eq. (ii), we get

$$u = \frac{7(6)^2}{4(3)(2)^3}$$

$$= \frac{7 \times 36}{4 \times 3 \times 8} = \frac{252}{96} = \frac{21}{8}$$

5. If v varies directly as the product xy^3 and inversely as z^2 and v = 27 when x = 7, y = 6, z = 7. Find the value of v when x = 6, y = 2, z = 3.

Solution

Given that v varies directly as xy^3.

Therefore $v \propto xy^3$

Also given that u varies inversely as z^2.

Therefore $v \propto \frac{1}{z^2}$

In joint variation, we can write

$$v \propto \frac{xy^2}{z^2}$$

$$\Rightarrow \quad v = K\frac{xy^2}{z^2} \quad \text{\_\_\_\_\_\_\_\_ (i)}$$

Put v = 27, x = 7, y = 6, and z = 7, we get

$$27 = \frac{K(7)(6)^3}{(7)^2}$$

$$27 = \frac{K(7)(216)}{49}$$

$$27 = \frac{1512}{49}$$

or $$K = 27 \times \frac{49}{1512} = \frac{7}{8}$$

$$K = \frac{1323}{1512} = \frac{7}{8}$$

Put $K = \frac{7}{8}$ in eq. (i), we get

$$v = \frac{7xy^3}{8z^2} \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

Put x = 6, y = 2 and z = 3 in eq. (ii), we get

$$v = \frac{7(6)(2)^3}{8(3)^2}$$

$$v = \frac{336}{72} = \frac{14}{3}$$

6. If w varies inversely as the cube of u, and w = 5 when y = 3. Find w, when u = 6.

Solution:

Given that v varies directly as u^3.

Therefore $W \propto \frac{1}{u^3}$

$\Rightarrow$ $W = \frac{k}{u^3}$ ________ (i)

Put w = 5 and u = 3, in eq. (i), we get

$$5 = \frac{K}{(3)^3}$$

$$K = 27 \times 5 = 135$$

Put K= 135 in eq. (i), we get

$$W = \frac{135}{u^3} \text{ ________ (ii)}$$

Put u = 6, in eq. (ii), we get

$$W = \frac{135}{(6)^3}$$

$$= \frac{135}{216} = \frac{5}{8}$$

K-Method:

3.4 (i) Use k - method to prove conditional equalities involving proportions.

If a : b :: c : d is a proportion, then putting each ratio equal to k

i.e., $\frac{a}{b} = \frac{c}{d} = k$

$\frac{a}{b} = k$ and $\frac{c}{d} = k$

a = bk and c = dk

Using the above equations, we can solve certain problems relating to proportions more easily. This method is known as A-method. We illustrate the A-method through the following examples.

SOLVED EXERCISE 3.6

1. If a : b = c : d, (a, b, c, d ≠ 0), then show that

(i) $\frac{4a - 9b}{4b + 9b} = \frac{4c - 9d}{4c + 9d}$

Solution: