6. If w varies inversely as the cube of u, and w = 5 when y = 3. Find w, when u = 6.

Solution:

Given that v varies directly as u^3.

Therefore $W \propto \frac{1}{u^3}$

$\Rightarrow$ $W = \frac{k}{u^3}$ ________ (i)

Put w = 5 and u = 3, in eq. (i), we get

$$5 = \frac{K}{(3)^3}$$

$$K = 27 \times 5 = 135$$

Put K= 135 in eq. (i), we get

$$W = \frac{135}{u^3} \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

Put u = 6, in eq. (ii), we get

$$W = \frac{135}{(6)^3}$$

$$= \frac{135}{216} = \frac{5}{8}$$

K-Method:

3.4 (i) Use k - method to prove conditional equalities involving proportions.

If a : b :: c : d is a proportion, then putting each ratio equal to k

i.e., $\frac{a}{b} = \frac{c}{d} = k$

$\frac{a}{b} = k$ and $\frac{c}{d} = k$

a = bk and c = dk

Using the above equations, we can solve certain problems relating to proportions more easily. This method is known as A-method. We illustrate the A-method through the following examples.

SOLVED EXERCISE 3.6

1. If a : b = c : d, (a, b, c, d ≠ 0), then show that

(i) $\frac{4a - 9b}{4b + 9b} = \frac{4c - 9d}{4c + 9d}$

Solution:

Given a : b = c : d

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k \qquad$ and $\qquad \frac{c}{d} = k$

$a = bk \qquad\qquad c = dk$

$$\text{L.H.S.} = \frac{4a-9b}{4b+9b} \qquad\qquad \text{R.H.S.} = \frac{4c-9d}{4c+9d}$$

$$= \frac{4bk-9b}{4bk+9} \qquad\qquad = \frac{4dk-9d}{4dk+9d}$$

$$= \frac{b(4k-9)}{b(4k-9)} \qquad\qquad = \frac{d(4k-9)}{d(4k+9)}$$

$$= \frac{4k-9}{4k+9} \text{ ——— (i)} \qquad\qquad = \frac{4k-9}{4k+9} \text{ ——— (ii)}$$

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

(ii) $\mathbf{\frac{6a-5b}{6b+5b} = \frac{4c-5d}{4c+5d}}$

Solution:

Given a : b = c : d

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k \qquad$ and $\qquad \frac{c}{d} = k$

$a = bk \qquad\qquad c = dk$

$$\text{L.H.S.} = \frac{6a-5b}{6b+5b} \qquad\qquad \text{R.H.S.} = \frac{4c-5d}{4c+5d}$$

$$= \frac{6bk-5b}{6bk+5b} \qquad\qquad = \frac{6dk-5d}{6dk+5d}$$

$$= \frac{b(6k-5)}{b(6k-5)} \qquad\qquad = \frac{d(6k-5)}{d(6k+5)}$$

$$= \frac{6k-5}{6k+5} \text{ ——— (i)} \qquad\qquad = \frac{6k-5}{6k+5} \text{ ——— (ii)}$$

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $\frac{6a-5b}{6b+5b} = \frac{4c-5d}{4c+5d}$

(iii) $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Solution:

Given a : b = c : d

Let

$\Rightarrow \quad \frac{a}{b}=k \quad$ and $\quad \frac{c}{d}=k$

$a = bk \quad\quad c = dk$

L.H.S. $=\frac{a}{b}$

$=\frac{bk}{b}$

$=k$ ______ (i)

R.H.S. $=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

$=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$=\sqrt{k^2}$

$=k$ ______ (ii)

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$

Solution:

Given a : b = c : d

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k \quad$ and $\quad \frac{c}{d}=k$

$a = bk \quad\quad c = dk$

L.H.S. $= a^6 + c^6 : b^6 + d^6$

$= \dfrac{a^6 + c^6}{b^6 + d^6}$

$= \dfrac{b^6k^6 + d^6k^6}{b^6 + d^6}$

$= \dfrac{k^6\left(b^6 + d^6\right)}{b^6 + d^6}$

$= k^6$ ______(i)

R.H.S. $= a^3c^3 : b^3d^3$

$= \dfrac{a^3c^3}{b^3d^3}$

$= \dfrac{\left(b^3k^3\right)\left(d^3k^3\right)}{b^3d^3}$

$= \dfrac{b^3d^3k^6}{b^3d^3}$

$= k^6$ ______(ii)

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

(v) $p(a + b) + qb : p(c + d) + qd = a : c$

Solution:

Given $a : b = c : d$

Let $\dfrac{a}{b} = \dfrac{c}{d} = k$

$\Rightarrow \quad \dfrac{a}{b} = k \quad$ and $\quad \dfrac{c}{d} = k$

$a = bk \qquad c = dk$

L.H.S. $= P(a+b) + qb : P(c+d)(qd)$

$= \dfrac{p(a+b)+qb}{p(c+d)+qd}$

$= \dfrac{p(bk+b)+qb}{p(dk+d)+qd}$

$= \dfrac{pbk+pb+qb}{pdk+pd+qd}$

$= \dfrac{b(pk+p+q)}{d(pk+p+q)}$

$= \dfrac{b}{d}$ ______(i)

R.H.S. $= a : c$

$= \dfrac{a}{c}$

$= \dfrac{bk}{dk}$

$= \dfrac{b}{d}$ ______(ii)

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $p(a + b) + qb : p(c + d) + qd = a : c$

(vi) $a^2 + b^2 : \dfrac{a^2}{a+b} = c^2 + d^2 : \dfrac{c^2}{c+d}$

Solution:

Given a : b = c : d

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k \quad$ and $\quad \frac{c}{d}=k$

$a = bk \quad\quad c = dk$

L.H.S. $=\left(a^2+b^2\right)\times\frac{a+b}{a^3}$

$=\left(b^2k^2+b^2\right)\times\frac{bk+b}{b^3k^3}$

$=b^2\left(k^2+1\right)\frac{b(k+1)}{b^3k^3}$

$=\frac{b^3}{b^3k^3}\left(k^2+1\right)(k+1)$

$=\frac{1}{k^3}\left(k^2+1\right)(k+1)$ ______ (i)

R.H.S. $=\left(c^2+d^2\right)\times\frac{c+d}{c^3}$

$=\left(d^2k^2+d^2\right)\times\frac{dk+d}{d^3k^3}$

$=d^2\left(k^1+1\right)\times\frac{d(k+1)}{d^3k^3}$

$=\frac{d^3}{d^3k^3}\left(k^2+1\right)(k+1)$

$=\frac{1}{k^3}\left(k^2+1\right)(k+1)$ ______ (ii)

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $a^2+b^2:\frac{a^2}{a+b}=c^2+d^2:\frac{c^2}{c+d}$

(vii) $\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$

Solution:

Given a : b = c : d

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k \quad$ and $\quad \frac{c}{d}=k$

$a = bk \quad\quad c = dk$

L.H.S. $=\frac{a}{a-b}:\frac{a+b}{b}$

$=\frac{a}{a-b}\times\frac{b}{a+b}$

$=\frac{bk}{bk-b}\times\frac{b}{bk+b}$

R.H.S. $=\frac{c}{c-d}:\frac{c+d}{d}$

$=\frac{c}{c-d}\times\frac{d}{c+d}$

$=\frac{dk}{dk-d}\times\frac{d}{dk+d}$

$$= \frac{bk}{b(k-1)} \times \frac{b}{b(k+1)} \qquad = \frac{dk}{d(k-1)} \times \frac{d}{d(k+1)}$$

$$= \frac{k}{k^2-1} \text{ ______ (i)} \qquad = \frac{k}{k^2-1} \text{ ______ (ii)}$$

From (i) and (ii), we have

L. H. S. = R. H. S.

Hence $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (a, b, c, d, e, f ≠ 0), then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$ and $\frac{e}{f} = k$

$a = bk \qquad c = dk \qquad e = fk$

$$\text{L.H.S.} = \frac{a}{b}$$

$$= \frac{bk}{b}$$

$$= k \text{ ________ (i)}$$

$$\text{R.H.S.} = \sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$$

$$= \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$

$$= \sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$$

$$= \sqrt{k^2}$$

$$= k \text{ ________ (ii)}$$

From (i) and (ii), we have

L.H.S = R.H.S

Hence $\frac{a}{b} = \sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

(ii) $\dfrac{ac+ce+ea}{bd+df+fb}=\left[\dfrac{ace}{bdf}\right]^{2/3}$

Solution:

Let $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$

$\Rightarrow \quad \dfrac{a}{b}=k \quad \text{and} \quad \dfrac{c}{d}=k \quad \text{and} \quad \dfrac{e}{f}=k$

$a=bk \qquad c=dk \qquad e=fk$

$$\text{L.H.S.} =\frac{ac+ce+ca}{bd+df+fb}$$

$$=\frac{(bk)(dk)+(dk)(fk)+(fk)(bk)}{bd+df+fb}$$

$$=\frac{bdk^2+dfk^2+fbk^2}{bd+df+fb}$$

$$=\frac{k^2(bd+df+bf)}{bd+df+bf}$$

$$=k^2 \quad \text{\_\_\_\_\_\_ (i)}$$

$$\text{R.H.S.} =\left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$$

$$=\left[\frac{(bk)(dk)(fk)}{bdf}\right]^{\frac{2}{3}}$$

$$=\left[\frac{bdfk^3}{bdf}\right]^{\frac{2}{3}}$$

$$=\left[k^3\right]^{\frac{2}{3}}$$

$$=k^2 \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

From (i) and (ii), we have

L.H.S = R.H.S

Hence $\dfrac{ac+ce+ea}{bd+df+fb}=\left[\dfrac{ace}{bdf}\right]^{2/3}$

(iii) $\dfrac{ac}{bd}+\dfrac{ce}{df}+\dfrac{ca}{fb}=\dfrac{a^2}{b^2}+\dfrac{c^2}{d^2}+\dfrac{e^2}{f^2}$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$\Rightarrow \quad \frac{a}{b}=k \quad \text{and} \quad \frac{c}{d}=k \quad \text{and} \quad \frac{e}{f}=k$

$a = bk \qquad c = dk \qquad e = fk$

L.H.S. $=\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}$

$=\frac{(bk)(dk)}{bd}+\frac{(dk)(fk)}{df}+\frac{(fk)(bk)}{bf}$

$=\frac{bdk^2}{bk}+\frac{dfk^2}{df}+\frac{bfk^2}{fb}$

$=k^2+k^2+k^2$

$=3k^2$ ________(i)

L.H.S. $=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

$=\frac{b^2k^2}{b^2}+\frac{d^2k^2}{d^2}+\frac{f^2k^2}{f^2}$

$=k^2+k^2+k^2$

$=3k^2$ ________(ii)

From (i) and (ii), we have

L.H.S = R.H.S

Hence $\frac{ac}{bd}+\frac{ce}{df}+\frac{ca}{fb}=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

SOLVED EXERCISE 3.7

1. The surface area A of a cube varies directly as the square of the length l of an edge and A = 27 square units when $l = 3$ units.
Find (i) A when $l = 4$ units (ii) l when A = 12 sq. units.

Solution:

Given that $\quad A \propto l^2$

$\Rightarrow \quad A = kl^2$ ______(i)

Put A = 27 and l = 3 in eq. (i), we get

$27 = k(3)^2$

$27 = 9k$

or $\quad 9k = 27$

$k = \frac{27}{9} = 3$