

$$\frac{2}{3} + C = 1 \quad \therefore A = \frac{2}{3}$$

$$C = 1 - \frac{2}{3}$$

$$C = \frac{1}{3}$$

Thus required partial fractions are $\frac{2/3}{x+1} + \frac{1/3x + 1/3}{x^2 - x + 1}$

$$\text{Hence, } \frac{x^2 + 1}{x^3 + 1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2 - x + 1)}$$

Resolution of a fraction when D (x) has repeated irreducible quadratic factors.

Rule IV:

If a quadratic factor $(ax^2 + bx + c)$ with $a \neq 0$, occurs twice in the denominator, the corresponding partial fractions are

$$\frac{Ax + B}{(ax^2 + bx + c)} + \frac{Cx + D}{(ax^2 + bx + c)^2}$$

The constants A, B, C and D are found in the usual way.

SOLVED EXERCISE 4.4

Resolve into partial fractions.

$$(1) \frac{x^3}{(x^2 + 4)^2}$$

Solution:

$$\text{Let } \frac{x^3}{(x^2 + 4)^2} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2}$$

Multiplying both sides by $(x^2 + 4)^2$, we get

$$x^3 = (Ax + B)(x^2 + 4) + Cx + D \quad \text{_____ (1)}$$

$$x^3 = Ax^3 + 4Ax + Bx^2 + 4B + Cx + D$$

To find A, B, C and D, equating coefficient of x^3 , x^2 , x and constant on both sides of eq. (2),

We get.

$$\begin{array}{ll} \text{Coefficient of } x^3: & A = 1 \\ \text{Coefficient of } x^2: & B = 0 \\ \text{Coefficient of } x: & 4A + C = 0 \quad \text{_____ (2)} \end{array}$$

Constant: $4B + D = 0$ _____ (3)

Put $A = 1$ in eq. (2), we get

$$4(1) + C = 0$$

$$4 + C = 0$$

$$C = -4$$

Put $B = 0$ in eq. (3), we get

$$4(0) + D = 0$$

$$D = 0$$

Thus required partial fractions are $\frac{(1)x + 0}{x^2 + 4} + \frac{(-4)x + 0}{(x^2 + 4)^2}$

Hence, $\frac{x^3}{(x^3 + 4)^2} = \frac{x}{x^2 + 4} - \frac{4x}{3(x^2 + 4)}$

(2) $\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}$

Solution:

Let $\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$

Multiplying both sides by $(x + 1)(x^2 + 1)^2$, we get

$$x^4 + 3x^2 + x + 1 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \text{ --- (1)}$$

$$x^4 + 3x^2 + x + 1 = A(x^4 + 2x^2 + 1) + (Bx + C)(x^3 + x^2 + x + 1) + Dx^2 + Dx + Ex + E$$

$$x^4 + 3x^2 + x + 1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^3 + Bx^2 + Bx + Cx^3 + Cx^2$$

$$+ Cx + C + Dx^2 + Dx + Ex + E$$

$$x^4 + 3x^2 + x + 1 = Ax^4 + Bx^4 + Bx^3 + Cx^3 + 2Ax^2 + Bx^2 + Cx^2$$

$$+ Dx^2 + Bx + Cx + Dx + Ex + A + C + E \text{ --- (2)}$$

To Find A, we put $x + 1 = 0 \Rightarrow x = -1$ in eq. (1), we get

$$(-1)^4 + 3(-1)^2(-1) + 1 = A((-1)^2 + 1)^2$$

$$1 + 3 - 1 + 1 = A(1 + 1)^2$$

$$4 = A(2)^2$$

$$4 = 4A$$

Or $4A = 4$

$\Rightarrow A = 1$

To find B, C, D and E, equating coefficient of x^4 , x^3 , x^2 and x on both sides of eq. (2), we get.

$$\text{Coefficient of } x^4: \quad A + B = 1 \quad \underline{\hspace{1cm}} (3)$$

$$\text{Coefficient of } x^3: \quad B + C = 0 \quad \underline{\hspace{1cm}} (4)$$

$$\text{Coefficient of } x^2: \quad 2A + B + C + D = 3 \quad \underline{\hspace{1cm}} (5)$$

$$\text{Coefficient of } x: \quad B + C + D + E = 1 \quad \underline{\hspace{1cm}} (6)$$

Put $A = 1$ in eq. (3), we get

$$1 + B = 1$$

$$B = 1 - 1$$

$$B = 0$$

Put $B = 0$ in eq. (4), we get

$$0 + C = 0$$

$$C = 0$$

Put $A = 1, B = 0, C = 0$ in eq. (5), we get

$$2(1) + 0 + 0 + D = 3$$

$$2 + D = 3$$

$$D = 3 - 2$$

$$D = 1$$

Put $B = 0, C = 0, D = 1$ in eq. (6) we get

$$0 + 0 + 1 + E = 1$$

$$1 + E = 1$$

$$E = 1 - 1$$

$$E = 0$$

Thus required partial fractions are $\frac{1}{x+1} + \frac{(0)x+0}{x^2+1} + \frac{(1)x+0}{(x^2+1)^2}$

$$\text{Hence, } \frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2} = \frac{1}{x+1} + \frac{x}{(x^2+1)^2}$$

$$(3) \quad \frac{x^2}{(x+1)(x^2+1)^2}$$

Solution:

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

Multiplying both sides by $(x-1)(x^2+1)^2$, we get

$$x^2 = A(x^2 + 1)^2 + (Bx + C)(x^2 + 1)(x - 1) + (Dx + E)(x - 1) \quad (1)$$

$$x^2 = A(x^4 + 2x^2 + 1) + (Bx + C)(x^3 - x^2 + x - 1)$$

$$+ Dx^2 - Dx + Ex - E$$

$$x^2 = Ax^4 + 2Ax^2 + A + Bx^4 - Bx^3 + Bx^2 - Bx + Cx^3$$

$$- Cx^2 + Cx - C + Dx^2 - Dx + Ex - E$$

$$x^2 = Ax^4 + Bx^4 - Bx^3 + Cx^3 + 2Ax^2 + Bx^2 - Cx^2 + Dx^2$$

$$- Bx + Cx - Dx + Ex + A - C - E$$

____ (2)

To find A, we put $x - 1 = 0 \Rightarrow x = 1$ in eq. (1), we get

$$(-1)^2 = A((-1)^2 + 1)^2$$

$$1 = A(1 + 1)^2$$

$$1 = A(2)^2$$

$$1 = 4A$$

$$\Rightarrow A = \frac{1}{4}$$

To find B, C, D and E, equating coefficient of x^4 , x^3 , x^2 and x on both sides of eq. (2), we get.

$$\text{Coefficient of } x^4: \quad A + B = 0 \quad \text{____ (3)}$$

$$\text{Coefficient of } x^3: \quad -B + C = 0 \quad \text{____ (4)}$$

$$\text{Coefficient of } x^2: \quad 2A + B - C + D = 1 \quad \text{____ (5)}$$

$$\text{Coefficient of } x: \quad -B + C - D + E = 0 \quad \text{____ (6)}$$

Put $A = \frac{1}{4}$ in eq. (4), we get

$$\frac{1}{4} + B = 0$$

$$B = -\frac{1}{4}$$

Put $B = -\frac{1}{4}$, in eq. (4), we get

$$-\left(-\frac{1}{4}\right) + C = 0$$

$$\frac{1}{4} + C = 0$$

$$C = -\frac{1}{4}$$

Put $A = \frac{1}{4}$, $B = -\frac{1}{4}$, $C = -\frac{1}{4}$, in eq. (5), we get

$$2\left(\frac{1}{4}\right) + \left(-\frac{1}{4}\right) - \left(-\frac{1}{4}\right) + D = 1$$

$$\frac{2}{4} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$\frac{1}{2} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$D = \frac{1}{2}$$

Put $B = -\frac{1}{4}$, $C = -\frac{1}{4}$, $D = \frac{1}{2}$ in eq. (6) we get

$$-\left(-\frac{1}{4}\right) + \left(-\frac{1}{4}\right) - \frac{1}{2} + E = 0$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$E = 0\frac{1}{2}$$

Thus, required partial fractions are $\frac{1/4}{x-1} + \frac{(-1/4)x + (-1/4)}{x^2+1} + \frac{(1/2)x + (1/2)}{(x^2+1)^2}$

$$\text{Hence, } \frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

$$(4) \frac{x^2}{(x-1)(x^2+1)^2}$$

Solution:

$$(5) \frac{x^4}{(x^2+2)^2}$$

Solution:

$$\frac{x^4}{(x^2+2)^2} = \frac{x^4}{x^4 + 4x^2 + 4}$$

By long division, we have

$$\begin{array}{r}
 1 \\
 x^4 + 4x^2 + 4 \overline{) x^4} \\
 \underline{\pm x^4 \pm 4x^2 \pm 4} \\
 -4x^2 - 4 \\
 -(4x^2 + 4)
 \end{array}$$

$$\frac{x^4}{(x^2 + 2)^2} = 1 - \frac{4x^2 + 4}{(x^2 + 2)^2}$$

Let $\frac{4x^2 + 4}{(x^2 + 2)^2} = \frac{Ax + B}{(x^2 + 2)} + \frac{Cx + D}{(x^2 + 2)^2}$

Multiply both sides by $(x^2 + 2)^2$, we get

$$4x^2 + 4 = A(Ax + B)(x^2 + 2) + (Cx + D)$$

$$4x^2 + 4 = Ax^3 + 2Ax + Bx^2 + 2B + Cx + D$$

$$4x^2 + 4 = Ax^3 + Bx^2 + 2Ax + Cx + 2B + D \quad \text{--- (1)}$$

To Find A, B, C, and D, equating coefficient of x^3 , x^2 , x and constant on both sides of eq. (1), we get.

Coefficient of x^3 : $A = 0$

Coefficient of x^2 : $B = 4$

Coefficient of x : $2A + C = 0$ --- (2)

Constant: $2B + D = 0$ --- (3)

Put $A = 0$ in eq. (2), we get.

$$2(0) + C = 0$$

$$C = 0$$

Put $B = 4$ in eq. (3), we get.

$$2(4) + D = 0$$

$$8 + D = 0$$

$$D = -8$$

$$D = -8$$

Thus, required partial fractions are $\frac{(0)x + 4}{x^2 + 2} + \frac{(0)x + (-8)}{(x^2 + 2)^2}$

$$\begin{aligned}
 \text{Hence, } \frac{x^4}{(x^2 + 2)^2} &= 1 - \left[\frac{4}{x^2 + 2} - \frac{8}{(x^2 + 2)^2} \right] \\
 &= 1 - \frac{4}{x^2 + 2} + \frac{8}{(x^2 + 2)^2}
 \end{aligned}$$

$$(6) \frac{x^5}{(x^2+1)^2}$$

Solution:

$$\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4 + 2x^2 + 1}$$

By long division, we have

$$\begin{array}{r} x \\ x^4 + 2x^2 + 1 \overline{) x^5} \\ \underline{\pm x^5 \pm 2x^3 \pm x} \\ -2x^3 - x \\ \underline{-(4x^3 + x)} \end{array}$$

$$\frac{x^5}{(x^2+1)^2} = x - \frac{2x^3 + x}{(x^2+1)^2}$$

Let
$$\frac{2x^3 + x}{(x^2+1)^2} = \frac{Ax + B}{x^2+1} + \frac{Cx + D}{(x^2+1)^2}$$

Multiply both sides by $(x^2+1)^2$, we get

$$2x^3 + x = (Ax + B)(x^2 + 1) + (Cx + D)$$

$$2x^3 + x = Ax^3 + Ax + Bx^2 + B + Cx + D$$

$$2x^3 + x = Ax^3 + Bx^2 + Ax + Cx + B + D \quad \text{--- (1)}$$

To Find A, B, C, and D, equating coefficient of x^3 , x^2 , x and constant on both sides of eq. (1), we get.

Coefficient of x^3 : $A = 2$

Coefficient of x^2 : $B = 0$

Coefficient of x : $A + C = 1$ --- (2)

Constant: $B + D = 0$ --- (3)

Put $A = 2$ in eq. (2), we get.

$$A + C = 1$$

$$2 + C = 1$$

$$C = 1 - 2$$

$$C = -1$$

Put $B = 0$ in eq. (3), we get.

$$0 + D = 0$$

$$D = 0$$

Thus, required partial fractions are
$$\frac{2x + 0}{x^2 + 1} + \frac{(-1)x + 0}{(x^2 + 1)^2}$$

$$\begin{aligned}
 \text{Hence, } \frac{x^5}{(x^2+1)^2} &= x - \left[\frac{2x}{x^2+1} + \frac{-x}{(x^2+1)^2} \right] \\
 &= x - \left[\frac{2x}{x^2+1} - \frac{x}{(x^2+1)^2} \right] \\
 &= x - \left[\frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2} \right]
 \end{aligned}$$

SOLVED MISCELLANEOUS EXERCISE - 4

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

- (i) The identity $(5x + 4)^2 = 25x^2 + 40x + 16$ is true for
 (a) one value of x (b) two values of x
 (c) all values of x (d) none of these
- (ii) A function of the form $f(x) = \frac{N(x)}{D(x)}$, with $D(x) \neq 0$, where $N(x)$ and $D(x)$ are polynomials in x is called
 (a) an identity (b) an equation
 (c) a fraction (c) none of these
- (iii) A fraction in which the degree of the numerator is greater or equal the degree of denominator is called:
 (a) a proper fraction (b) an improper fraction
 (c) an equation (d) algebraic relation
- (iv) A fraction in which the degree of numerator is less than the degree of the denominator is called
 (a) an equation (b) an improper fraction
 (c) an identity (d) a proper fraction
- (v) $\frac{2x+1}{(x+1)(x-1)}$ is:
 (a) an improper fraction (b) an equation
 (c) a proper fraction (d) none of these
- (vi) $(x+3)^2 = x^2 + 6x + 9$ is:
 (a) a linear equation (b) an equation
 (c) an identity (d) none of these