

$$\begin{aligned} \Rightarrow x \in A \text{ and } x \in B \cap C \\ \Rightarrow x \in A \text{ and } [x \in B \text{ or } x \in C] \\ \Rightarrow [x \in A \text{ and } x \in B] \text{ or } [x \in A \text{ and } x \in C] \\ \Rightarrow [x \in A \cap B] \text{ or } [x \in A \cap C] \\ \Rightarrow x \in (A \cap B) \cup (A \cap C) \end{aligned}$$

Hence by def. of subsets

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad (i)$$

$$\text{Similarly } (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \quad (ii)$$

From (i) and (ii), we have, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) De-Morgan's laws

For any two sets A and B, prove that

$$(i) (A \cup B)' = A' \cap B'$$

Proof: Let $x \in (A \cup B)'$

$$\Rightarrow x \notin A \cup B \quad (\text{by definition of complement of set})$$

$$\Rightarrow x \notin A \text{ and } x \notin B$$

$$\Rightarrow x \in A' \text{ and } x \in B'$$

$$\Rightarrow x \in A' \cap B' \quad (\text{by definition of intersection of sets})$$

$$\Rightarrow (A \cup B)' \subseteq (A' \cap B') \quad (i)$$

$$\text{Similarly } A' \cap B' \subseteq (A \cup B)' \quad (ii)$$

Using (i) and (ii), we have $(A \cup B)' = A' \cap B'$

$$(ii) \text{ Let } x \in (A \cap B)'$$

$$\Rightarrow x \notin A \cap B$$

$$\Rightarrow x \notin A \text{ or } x \notin B$$

$$\Rightarrow x \in A' \text{ or } x \in B'$$

$$\Rightarrow x \in A' \cup B'$$

$$\Rightarrow (A \cap B)' \subseteq A' \cup B' \quad (i)$$

$$\text{Let } y \in A' \cup B'$$

$$\Rightarrow y \in A' \text{ or } y \in B'$$

$$\Rightarrow y \notin A \text{ or } y \notin B$$

$$\Rightarrow y \notin A \cap B$$

$$\Rightarrow y \in (A \cap B)'$$

$$\Rightarrow (A' \cup B)' \subseteq (A \cap B)' \quad (ii)$$

From (i) and (ii) we have proved that

$$(A \cap B)' = A' \cup B'$$

SOLVED EXERCISE 5.2

1. If $X = \{1, 3, 5, 7, \dots, 19\}$, $Y = \{0, 2, 4, 6, 8, \dots, 20\}$
 $Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, then find the following.

$$(i) X \cup (Y \cap Z)$$

Solution:

$$\begin{aligned}
Y \cup Z &= \{0, 2, 4, 6, 8, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\
&= \{0, 2, 3, 4, \dots, 17, 19, 20, 23\} \\
X \cup (Y \cup Z) &= \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 3, 4, \dots, 17, 19, 20, 23\} \\
&= \{0, 1, 2, 3, \dots, 30, 33\}
\end{aligned}$$

(ii) $(X \cup Y) \cup Z$

Solution:

$$\begin{aligned}
X \cup Y &= \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\} \\
&= \{0, 1, 2, 3, \dots, 19, 20\} \\
(X \cup Y) \cup Z &= \{0, 1, 2, 3, \dots, 19, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\
&= \{0, 1, 2, 3, \dots, 20, 23\}
\end{aligned}$$

(iii) $X \cap (Y \cap Z)$

Solution:

$$\begin{aligned}
Y \cap Z &= \{0, 2, 4, 6, 8, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\
&= \phi \\
X \cap (Y \cap Z) &= X \cap \phi \\
X \cap Y &= \{1, 3, 5, 7, \dots, 19\} \cap \phi \\
&= \phi
\end{aligned}$$

(iv) $(X \cap Y) \cap Z$

Solution:

$$\begin{aligned}
X \cap Y &= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\} \\
&= \phi \\
(X \cap Y) \cap Z &= \phi \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\
&= \phi
\end{aligned}$$

(v) $X \cup (Y \cap Z)$

Solution:

$$\begin{aligned}
Y \cap Z &= \{0, 2, 4, 6, 8, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\
&= \{2\} \\
X \cup (Y \cap Z) &= \{1, 3, 5, 7, \dots, 19\} \cup \{2\} \\
&= \{1, 2, 3, 5, 7, \dots, 19\}
\end{aligned}$$

(vi) $(X \cup Y) \cap (X \cup Z)$

Solution:

$$\begin{aligned}
X \cup Y &= \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\} \\
&= \{0, 1, 2, 3, \dots, 19, 20\} \\
X \cup Z &= \{1, 3, 5, 7, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\
&= \{1, 2, 3, 5, 7, \dots, 17, 19, 23\} \\
(X \cup Y) \cap (X \cup Z) &= \{0, 1, 2, 3, \dots, 19, 20\} \cap \{1, 2, 3, 5, 7, \dots, 17, 19, 23\} \\
&= \{1, 2, 3, 5, 7, \dots, 19\}
\end{aligned}$$

(vii) $X \cap (Y \cup Z)$

Solution:

$$Y \cup Z = \{0, 2, 4, 6, 8, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{0, 2, 3, 4, 5, 6, \dots, 19, 20, 23\}$$

$$X \cap (Y \cup Z) = \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 3, 4, 5, 6, \dots, 19, 20\}$$

$$= \{3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\}$$

$$= \phi$$

$$X \cap Z = \{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

(viii) $(X \cap Y) \cup (X \cap Z)$

Solution:

$$X \cap Y = \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\}$$

$$= \phi$$

$$X \cap Z = \{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

2. If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 4, 8\}$.
Prove the following identities:

(i) $A \cap B = B \cap A$

Solution:

$$\text{L.H.S.} = A \cap B$$

$$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$$

$$= \{2, 4, 6\} \quad \text{_____ (i)}$$

$$\text{R.H.S.} = B \cap A$$

$$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$$

$$= \{2, 4, 6\} \quad \text{_____ (ii)}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence proved.

(ii) $A \cup B = B \cup A$

Solution:

$$\text{L.H.S.} = A \cup B$$

$$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \quad \text{_____ (i)}$$

$$\text{R.H.S.} = B \cup A$$

$$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \quad \text{_____ (ii)}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence proved.

(iii) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

Solution:

$$\begin{aligned}
\text{L.H.S.} &= A \cap (B \cup C) \\
&= \{1, 2, 3, 4, 5, 6\} \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\}) \\
&= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\} \\
&= \{1, 2, 3, 4, 5, 6\} \quad \text{(i)}
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S.} &= (A \cap B) \cup (A \cap C) \\
&= (\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}) \cup (\{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}) \\
&= \{2, 4, 6\} \cup \{1, 4\} \\
&= \{1, 2, 3, 4, 5, 6\} \quad \text{(ii)}
\end{aligned}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence Proved.

$$\text{(iv) } A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution:

$$\begin{aligned}
\text{L.H.S.} &= A \cup (B \cap C) \\
&= \{1, 2, 3, 4, 5, 6\} \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\}) \\
&= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\} \\
&= \{1, 2, 3, 4, 5, 6, 8\} \quad \text{(i)}
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S.} &= (A \cup B) \cap (A \cup C) \\
&= (\{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}) \cap (\{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}) \\
&= \{1, 2, 3, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\} \\
&= \{1, 2, 3, 4, 5, 6, 8\} \quad \text{(ii)}
\end{aligned}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence Proved.

3. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, then verify the De-Morgan's Laws

$$\text{i.e., } (A \cap B)' = A' \cup B' \quad \text{and} \quad (A \cup B)' = A' \cap B'$$

Solution:

$$\begin{aligned}
\text{L.H.S.} &= A' \cup B' \\
&= U - (A \cap B) \\
&= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}) \\
&= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 5, 7\} \\
&= \{1, 2, 4, 6, 8, 9, 10\} \quad \text{(i)}
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S.} &= A' \cap B' \\
&= [U - A] \cap [U - B] \\
&= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}) \\
&\quad \cap (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\}) \\
&= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\} \\
&= \{1, 2, 4, 6, 8, 9, 10\} \quad \text{(ii)}
\end{aligned}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

$$(ii) (A \cup B)' = A' \cap B'$$

$$\text{L.H.S.} = A' \cup B'$$

$$= \cup - (A \cup B)$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 5, 7, 9\}$$

$$= \{4, 6, 8, 9, 10\} \text{ _____ (i)}$$

$$\text{R.H.S.} = A' \cap B'$$

$$= [\cup - A] \cap [\cup - B]$$

$$= (\{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, \dots, 10\} - \{2, 3, 5, 7\})$$

$$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$$

$$= \{4, 6, 8, 9, 10\} \text{ _____ (ii)}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

4. If $U = \{1, 2, 3, \dots, 20\}$, $X = \{1, 3, 7, 9, 15, 18, 20\}$ and $Y = \{1, 3, 5, \dots, 17\}$, then show that

$$(i) X - Y = X \cap Y'$$

Solution:

$$\text{L.H.S.} = X - Y$$

$$= \{1, 3, 5, 7, 9, 15, 18, 20\} \cap (\cup - Y)$$

$$= \{1, 2, 5, 7, 9, 15, 18, 20\} \cap (\{1, 2, 3, \dots, 20\} - \{1, 3, 5, \dots, 17\})$$

$$= \{1, 3, 5, 7, 9, 15, 18, 20\} \cap \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$= \{18, 20\} \text{ _____ (ii)}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence Proved.

$$(ii) Y - X = Y \cap X'$$

Solution:

$$\text{L.H.S.} = Y - X$$

$$= \{1, 3, 5, \dots, 17\} - \{1, 2, 5, 7, 9, 15, 18, 20\}$$

$$= \{5, 11, 13, 17\} \text{ _____ (i)}$$

$$\text{R.H.S.} = Y \cap X'$$

$$= Y \cap (\cup - X)$$

$$= \{1, 3, 5, \dots, 17\} \cap (\{1, 2, 3, \dots, 20\} - \{1, 3, 5, 7, 9, 15, 18, 20\})$$

$$= \{1, 3, 5, \dots, 17\} \cap \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 19\}$$

$$= \{5, 11, 13, 17\} \text{ _____ (ii)}$$

From (i) and (ii), we have

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence Proved.

Verify the fundamental properties for given sets:

(a) A and B are any two subsets of U, then $A \cup B = B \cup A$ (commutative law).

For example

$$\begin{aligned} & A = \{1, 3, 5, 7\} \text{ and } B = \{2, 3, 5, 7\} \\ \text{then } & A \cup B = \{1, 3, 5, 7\} \cup \{2, 3, 5, 7\} = \{1, 2, 3, 5, 7\} \\ \text{and } & B \cup A = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7\} = \{1, 2, 3, 5, 7\} \\ \text{Hence, verified that } & A \cup B = B \cup A. \end{aligned}$$

(b) Commutative property of intersection

For example $A = \{1, 3, 5, 7\}$ and $B = \{2, 3, 5, 7\}$

$$\begin{aligned} \text{Then } & A \cap B = \{1, 3, 5, 7\} \cap \{2, 3, 5, 7\} = \{3, 5, 7\} \\ \text{and } & B \cap A = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7\} = \{3, 5, 7\} \\ \text{Hence, verified that } & A \cap B = B \cap A. \end{aligned}$$

(c) If A, B and C are the subsets of U, then $(A \cup B) \cup C = A \cup (B \cup C)$.

(Associative law)

$$\begin{aligned} \text{Suppose } & A = \{1, 2, 4, 8\}; \quad B = \{2, 4, 6\} \\ \text{And } & C = \{3, 4, 5, 6\} \\ \text{Then L.H.S. } & = (A \cup B) \cup C \\ & = (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cup \{3, 4, 5, 6\} \\ & = \{1, 2, 4, 6, 8\} \cup \{3, 4, 5, 6\} \\ & = \{1, 2, 3, 4, 5, 6, 8\} \\ \text{and R.H.S. } & = A \cup (B \cup C) \\ & = \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cup \{3, 4, 5, 6\}) \\ & = \{1, 2, 4, 8\} \cup \{2, 3, 4, 5, 6\} \\ & = \{1, 2, 3, 4, 5, 6, 8\} \end{aligned}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence, union of Sets is associative.

(d) If A, B and C are the subsets of U, then $(A \cap B) \cap C = A \cap (B \cap C)$

(Associative Law).

$$\begin{aligned} \text{Suppose ., } & A = \{1, 2, 4, 8\}; \quad B = \{2, 4, 6\} \text{ and } C = \{3, 4, 5, 6\} \\ \text{then L.H.S. } & = (A \cap B) \cap C \\ & = (\{1, 2, 4, 8\} \cap \{2, 4, 6\}) \cap \{3, 4, 5, 6\} \\ & = \{2, 4\} \cap \{3, 4, 5, 6\} = \{4\} \\ \text{and R.H.S. } & = A \cap (B \cap C) \\ & = \{1, 2, 4, 8\} \cap (\{2, 4, 6\} \cap \{3, 4, 5, 6\}) \\ & = \{1, 2, 4, 8\} \cap \{4, 6\} = \{4\} \end{aligned}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence, intersection of sets is associative.

Distributive laws

(e) Union is distributive over intersection of sets

If A, B and C are the subsets of universal set U, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution: Suppose $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6\}$ and $C = \{3, 4, 5, 6\}$

$$\begin{aligned} \text{then L.H.S } & = A \cup (B \cap C) \\ & = \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cap \{3, 4, 5, 6\}) \\ & = \{1, 2, 4, 8\} \cup \{4, 6\} = \{1, 2, 4, 6, 8\} \end{aligned}$$

$$\begin{aligned}
\text{and R.H.S} &= (A \cup B) \cap (A \cup C) \\
&= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cap (\{1, 2, 4, 8\} \cup \{3, 4, 5, 6\}) \\
&= \{1, 2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\} \\
&= \{1, 2, 4, 6, 8\} \\
\text{L.H.S} &= \text{R.H.S}
\end{aligned}$$

(f) Intersection is distributive over union of sets

To prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$\begin{aligned}
\text{Suppose } A &= \{1, 2, 3, 4, 5, \dots, 20\} \\
B &= \{5, 10, 15, 20, 25, 30\} \\
C &= \{3, 9, 15, 21, 27, 33\}
\end{aligned}$$

$$\begin{aligned}
\text{L.H.S.} &= A \cap (B \cup C) \\
&= \{1, 2, 3, 4, 5, \dots, 20\} \cap (\{5, 10, 15, 20, 25, 30\} \cup \{3, 9, 15, 21, 27, 33\}) \\
&= \{1, 2, 3, 4, 5, \dots, 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\} \\
&= \{3, 5, 9, 10, 15, 20\}
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S.} &= (A \cap B) \cup (A \cap C) \\
&= (\{1, 2, 3, 4, \dots, 20\} \cap \{5, 10, 15, 20, 25, 30\}) \\
&\quad \cup (\{1, 2, 3, 4, 5, \dots, 20\} \cap \{3, 9, 15, 21, 27, 33\}) \\
&= \{5, 10, 15, 20\} \cup \{3, 9, 15\} = \{3, 5, 9, 10, 15, 20\}
\end{aligned}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

(g) De Morgan's Laws $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

$$\begin{aligned}
\text{Suppose } U &= \{1, 2, 3, 4, \dots, 10\} \\
A &= \{2, 4, 6, 8, 10\} \Rightarrow A' = \{1, 3, 5, 7, 9\} \\
B &= \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}
\end{aligned}$$

$$\begin{aligned}
\text{Now consider } A \cap B &= \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6\} \\
&= \{2, 4, 6\}
\end{aligned}$$

$$\begin{aligned}
\text{Then L.H.S.} &= (A \cap B)' = U - (A \cap B) \\
&= \{1, 2, 3, 4, \dots, 10\} - \{2, 4, 6\} \\
&= \{1, 3, 5, 7, 8, 9, 10\}
\end{aligned}$$

$$\begin{aligned}
\text{and R.H.S.} &= A' \cup B' \\
&= \{1, 3, 5, 7, 9\} \cup \{7, 8, 9, 10\} \\
&= \{1, 3, 5, 7, 8, 9, 10\}
\end{aligned}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$(A \cup B)' = A' \cap B'$$

$$\begin{aligned}
\text{Suppose } U &= \{1, 2, 3, 4, \dots, 10\} \\
A &= \{2, 4, 6, 8, 10\} \Rightarrow A' = \{1, 3, 5, 7, 9\} \\
B &= \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}
\end{aligned}$$

$$\begin{aligned}
\text{Now consider } A \cup B &= \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5, 6\} \\
&= \{1, 2, 3, 4, 5, 6, 8, 10\}
\end{aligned}$$

$$\begin{aligned}
\text{L.H.S.} &= (A \cup B)' = U - (A \cup B) \\
&= \{1, 2, 3, 4, \dots, 10\} - \{1, 2, 3, 4, 5, 6, 8, 10\} \\
&= \{7, 9\}
\end{aligned}$$

$$\text{and R.H.S } A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{7, 8, 9, 10\}$$

$$= \{7,9\}$$

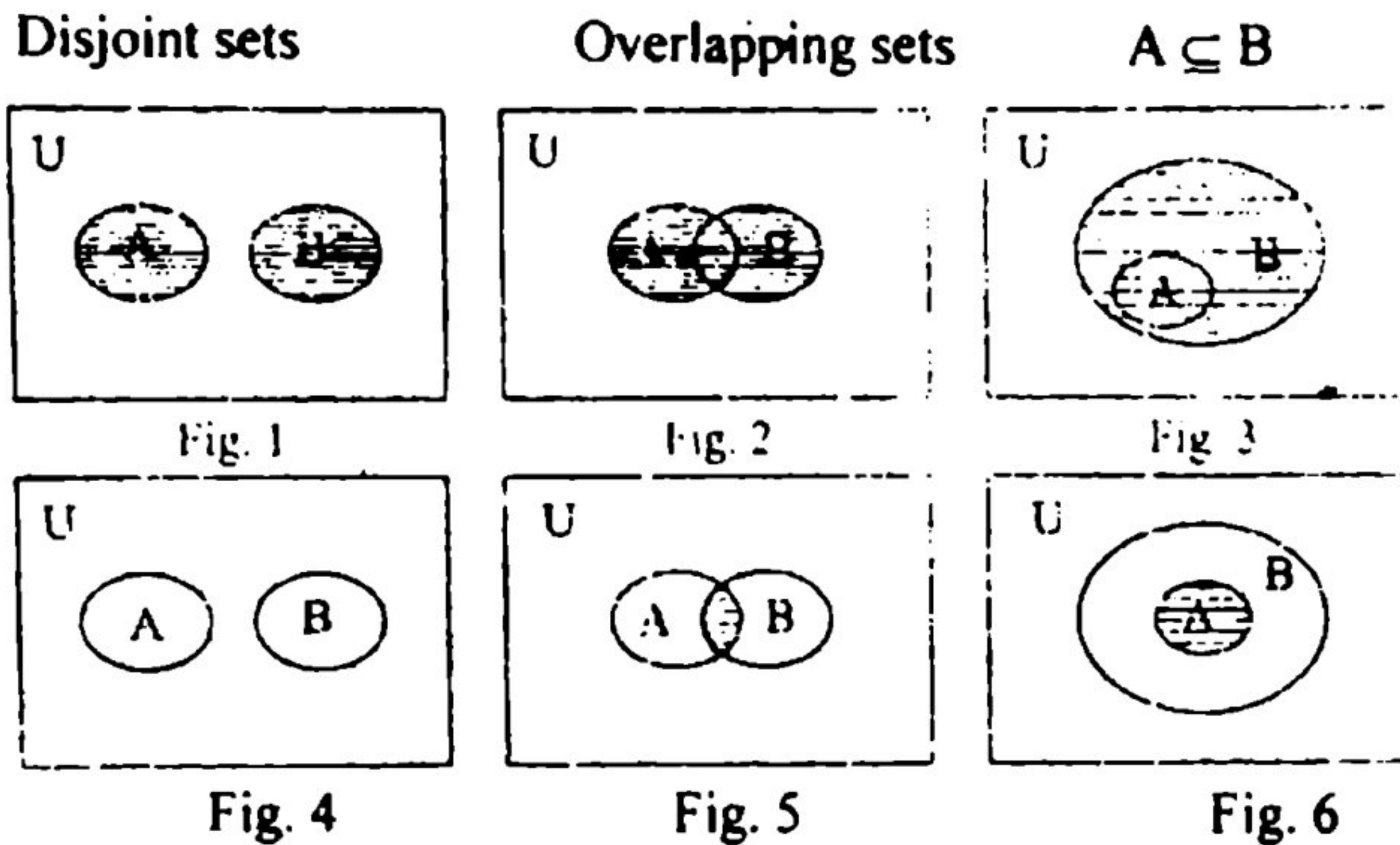
$$\text{L.H.S.} = \text{R.H.S.}$$

Venn Diagram:

British mathematician John Venn (1834 – 1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

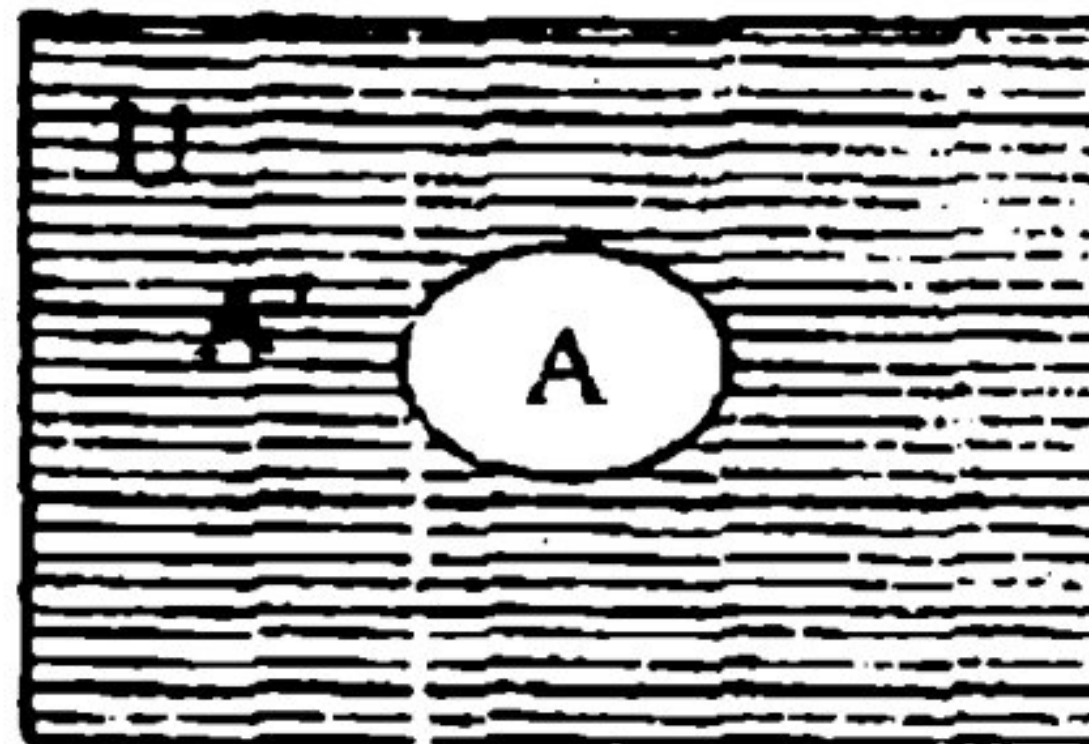
Use Venn diagrams to represent:

(a) Union and intersection of sets



(Regions shown by horizontal line segments in figures 1 to 6.)

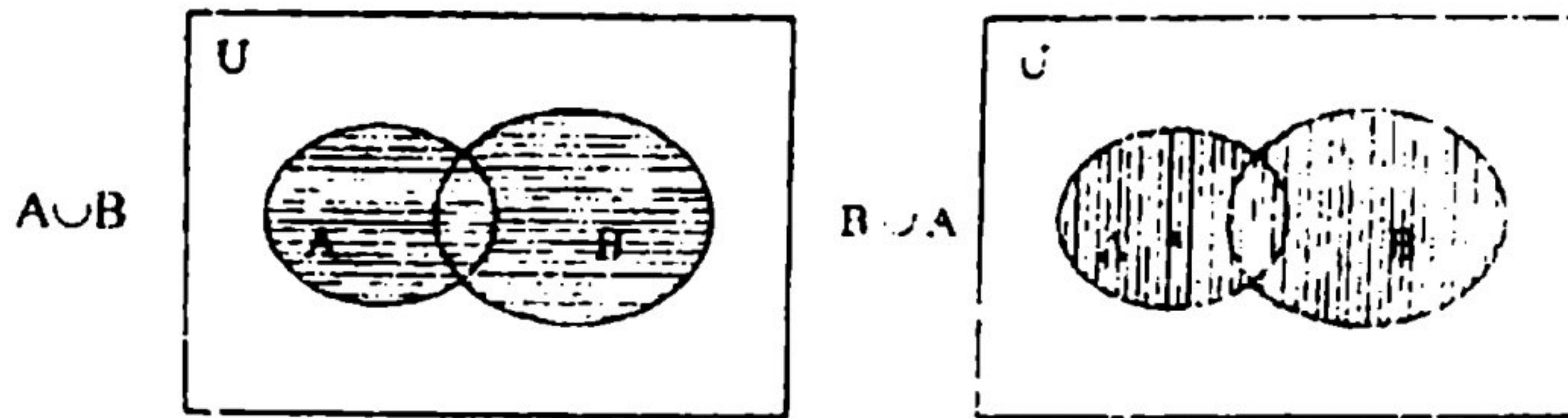
(b) Complement of a set



$U - A = A'$ is shown by horizontal line segments.

Use Venn diagram to verify:

(a) Commutative law for union and intersection of sets.

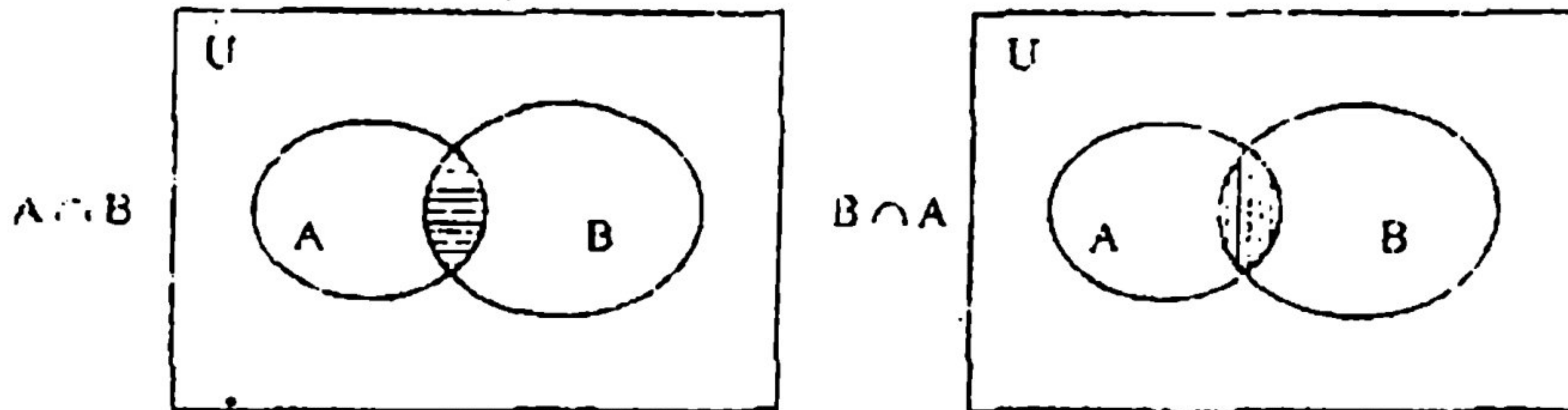


$A \cup B$ is shown by horizontal line segments,

$B \cup A$ is shown by vertical line segments.

The regions shown in both cases are equal.

Thus $A \cup B = B \cup A$.



$A \cap B$ is shown by horizontal line segments.

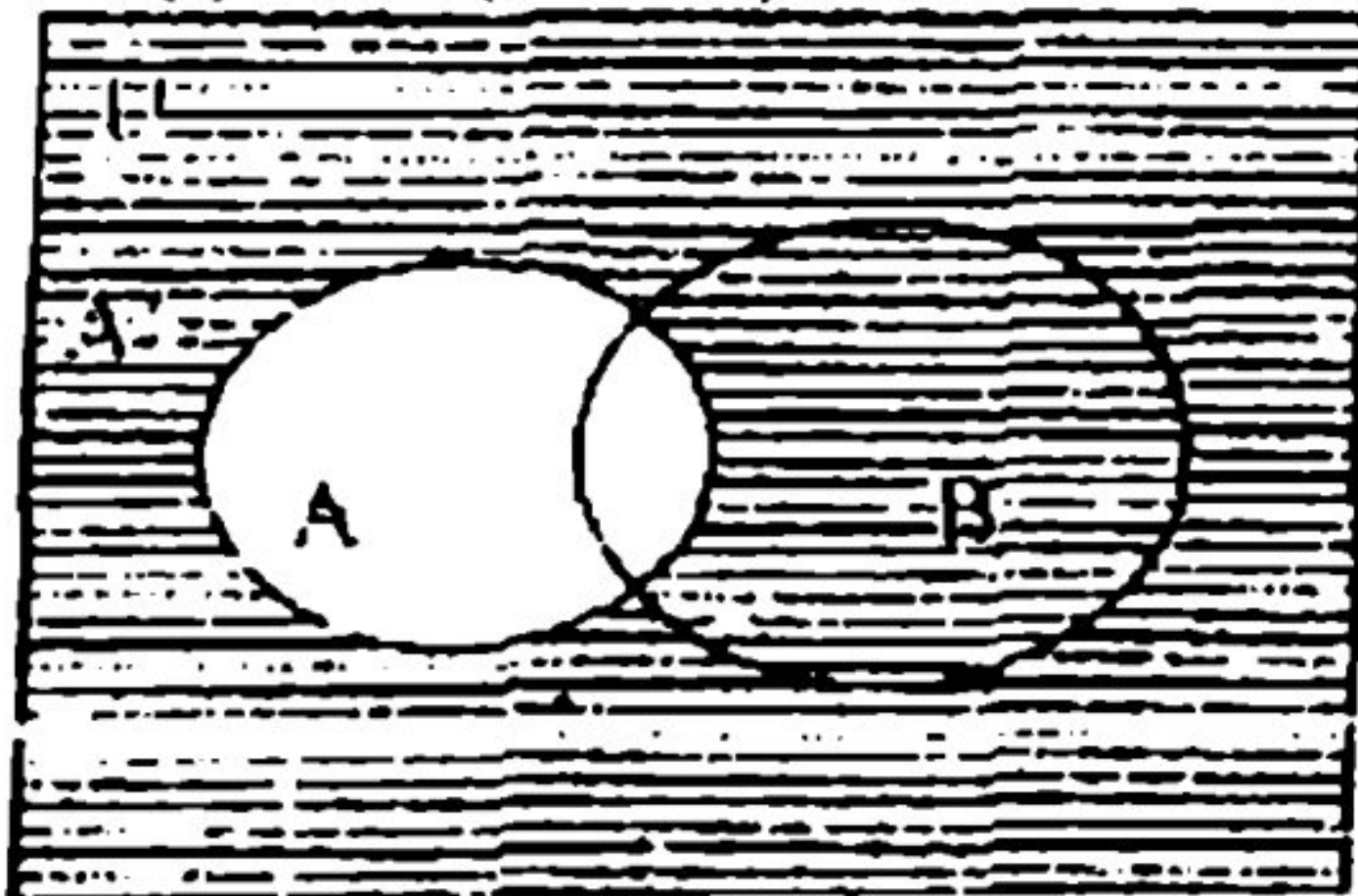
$B \cap A$ is shown by vertical line segments.

The regions shown in both cases are equal.

Thus $A \cap B = B \cap A$.

(b) De Morgan's laws

(i) $(A \cup B)' = A' \cap B'$



(ii) $(A \cap B)' = A' \cup B'$

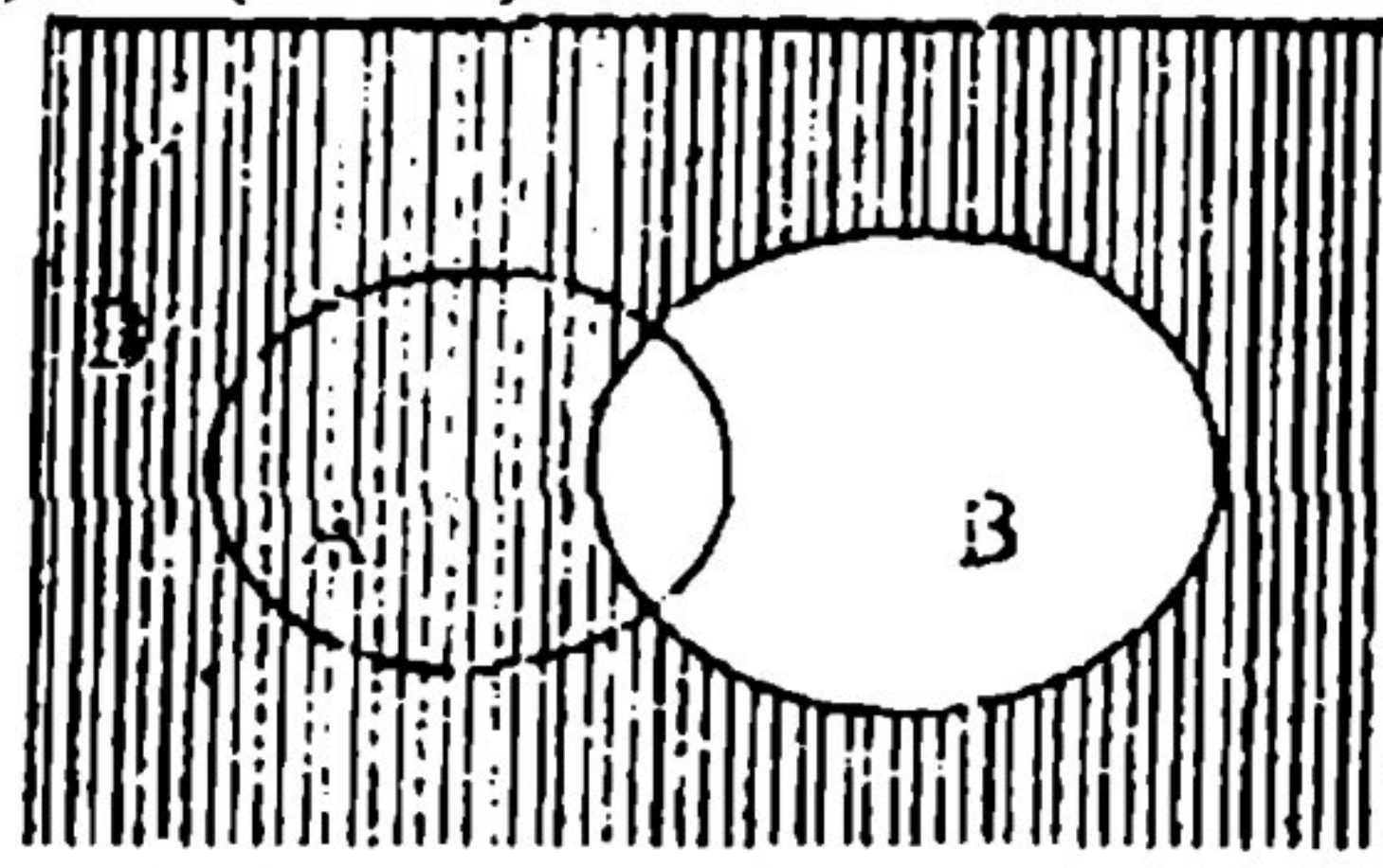


Fig. 1: A' is shown by horizontal line segments Fig. 2: B' is shown by vertical line segments

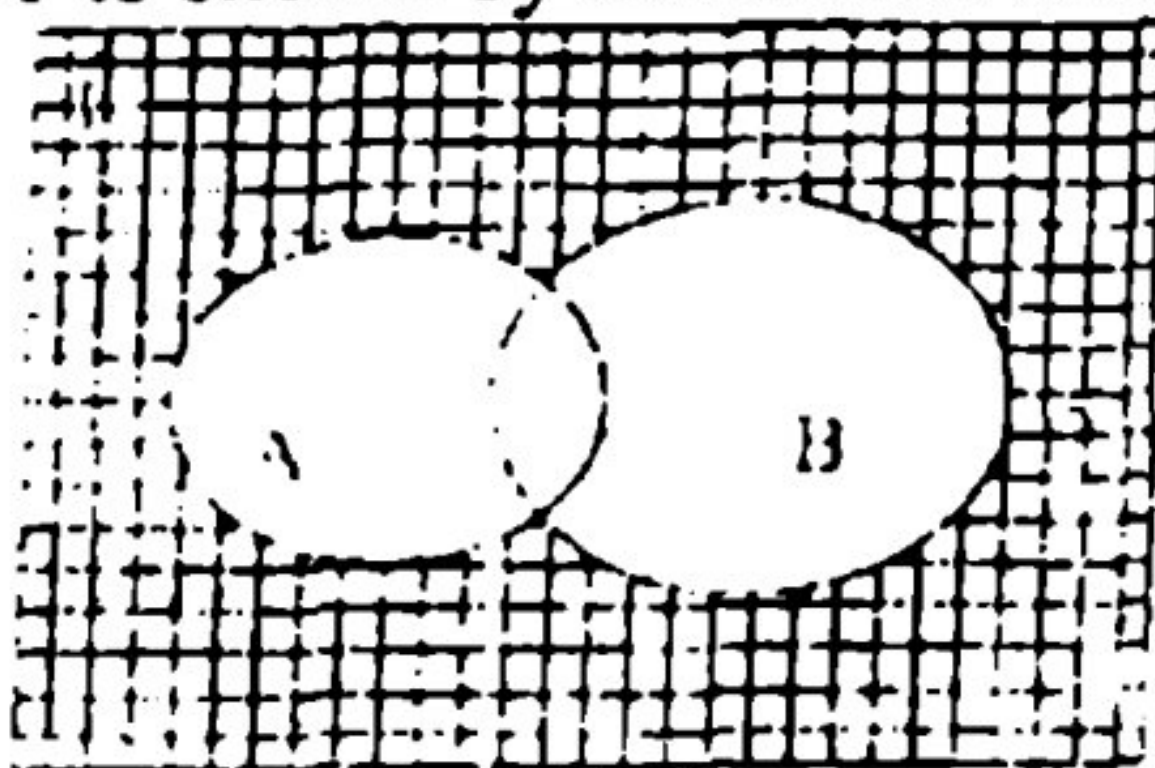


Fig. 3: $A' \cap B'$ is shown by squares



Fig. 4: $(A \cup B)'$ is shown by slanting line segments

Regions shown in Fig. 3 and Fig. 4 are equal.

Thus $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

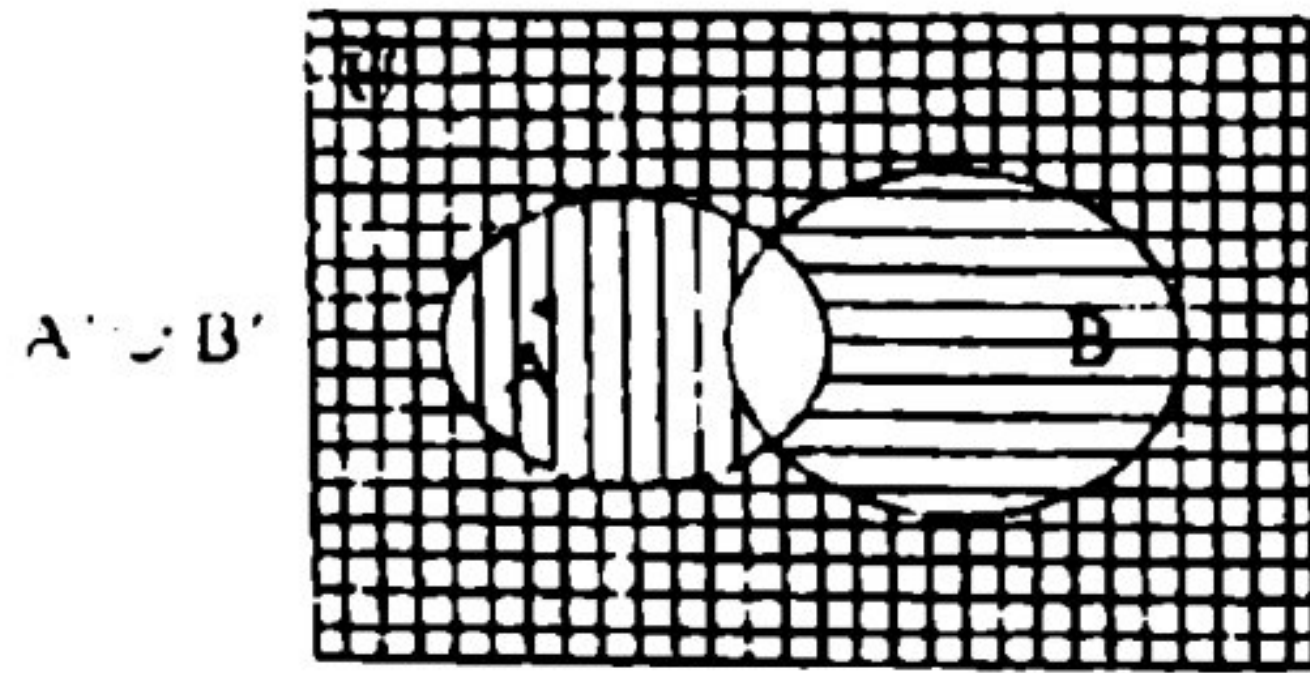


Fig. 5 $A' \cap B'$ is shown by squares, horizontal and vertical line segments.

Regions shown in Fig. 5 and Fig. 6 are equal.

Thus $(A \cap B)' = A' \cup B'$

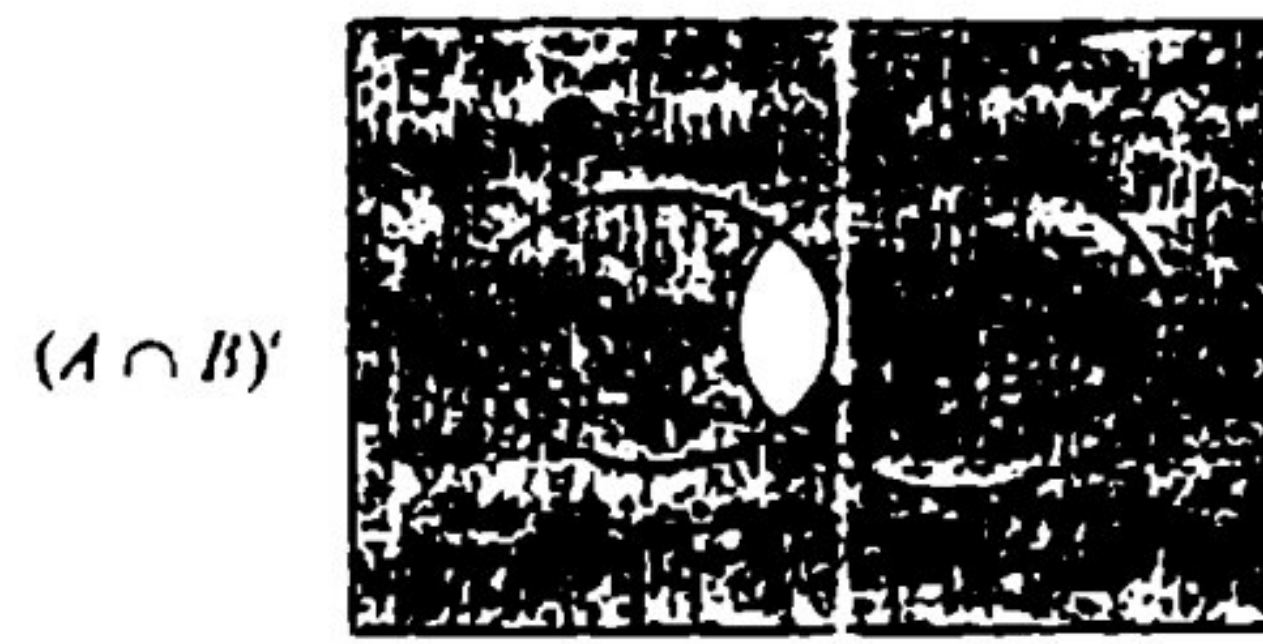


Fig. 6 $U - (A \cap B) = (A \cap B)'$ is shown by squares, horizontal and vertical line segments.

(c) Associative law:

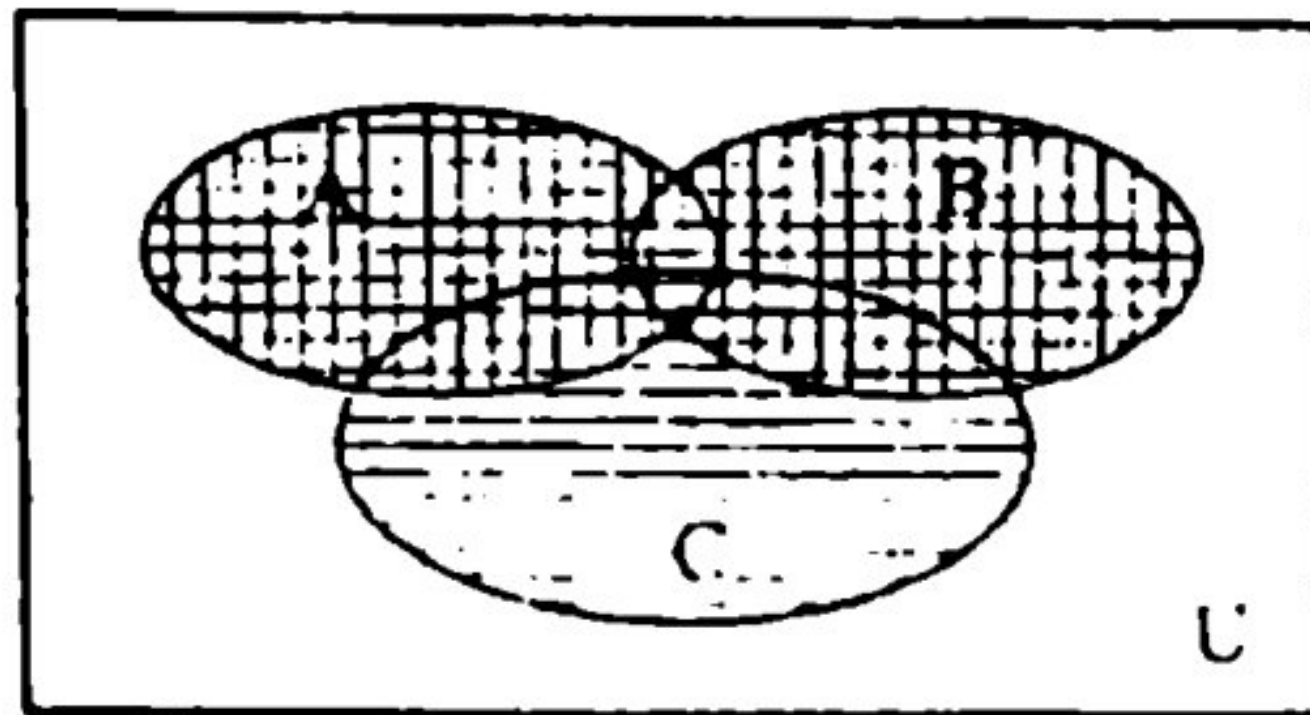


Fig. 1
 $(A \cup B) \cup C$ is shown in the above figure.

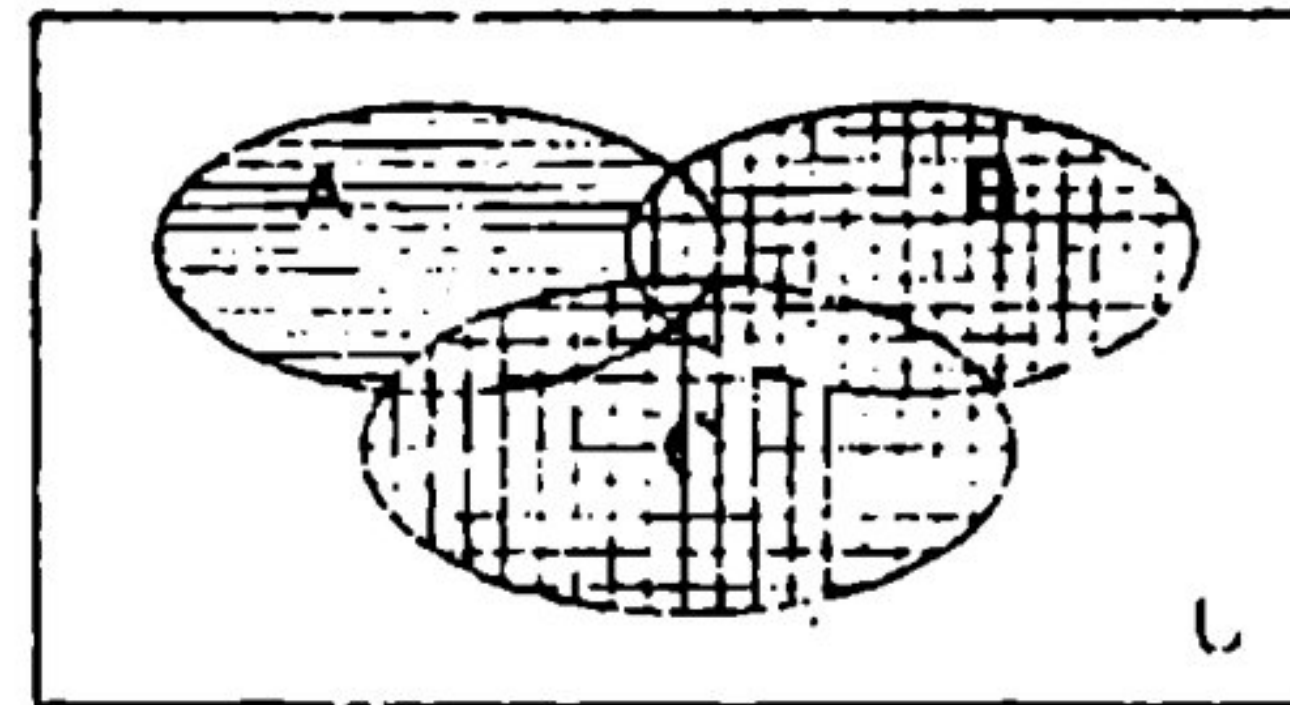


Fig. 2
 $A \cup (B \cup C)$ is shown in the above figure.

Regions shown in fig. 1 and fig. 2 by different ways are equal.

Thus $(A \cup B) \cup C = A \cup (B \cup C)$

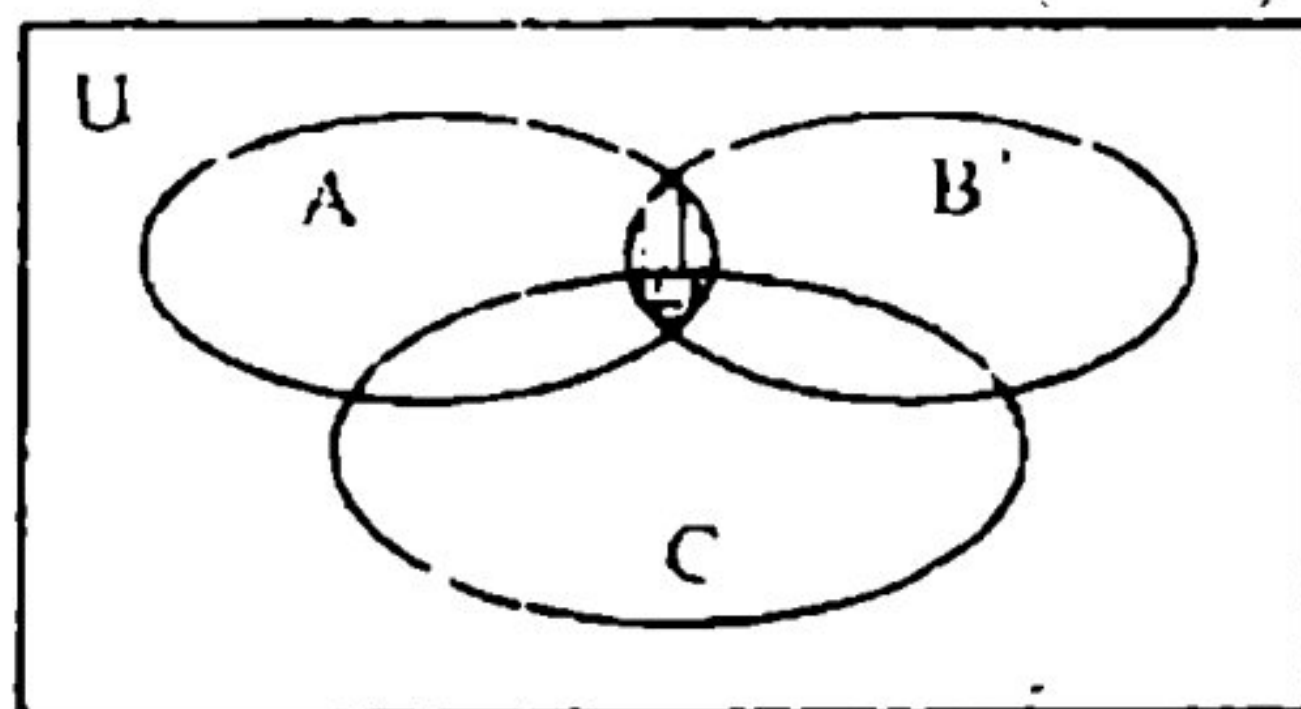


Fig. 3

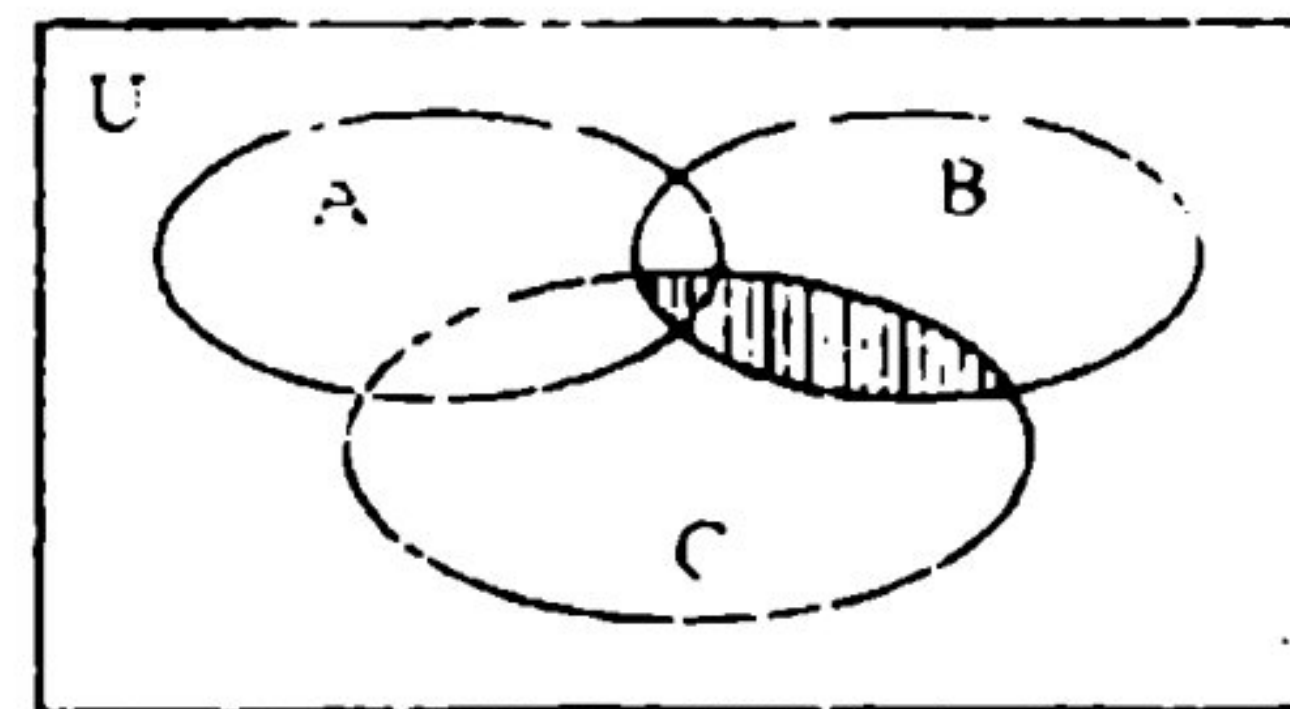


Fig. 4

$(A \cap B) \cap C$ is shown in figure 3 by double horizontal line segments

$A \cap (B \cap C)$ is shown in figure 4 by double crossing line segments

Regions shown in Fig. 3 and fig. 4 are equal.

Thus $(A \cap B)' \cap C' = A \cap (B \cap C)$

(d) Distributive law:

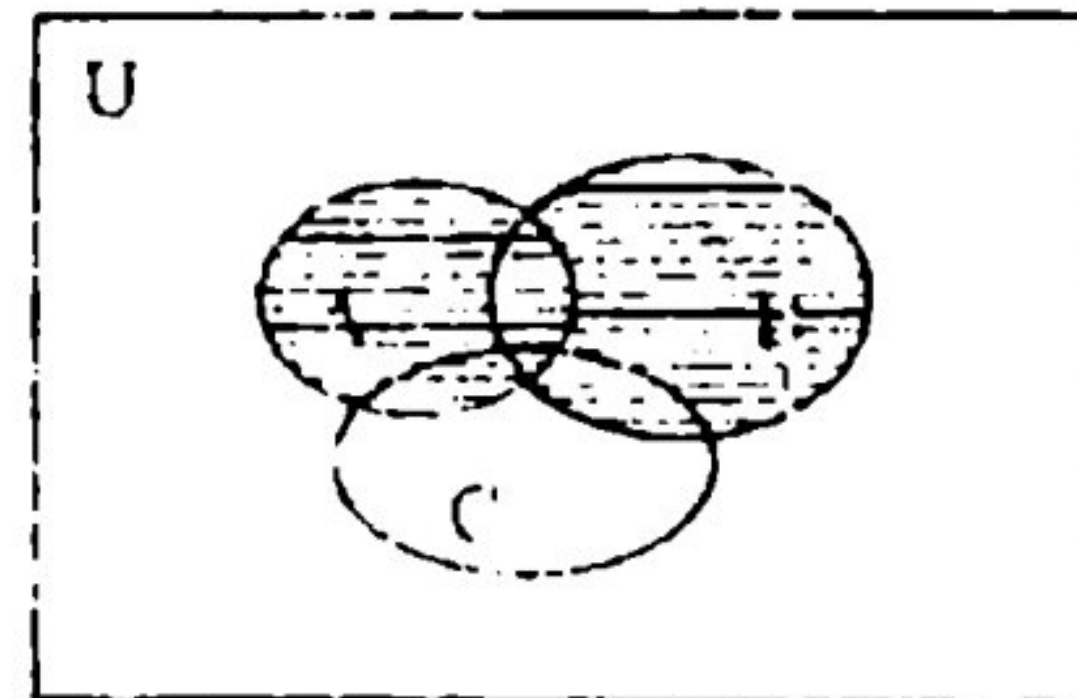
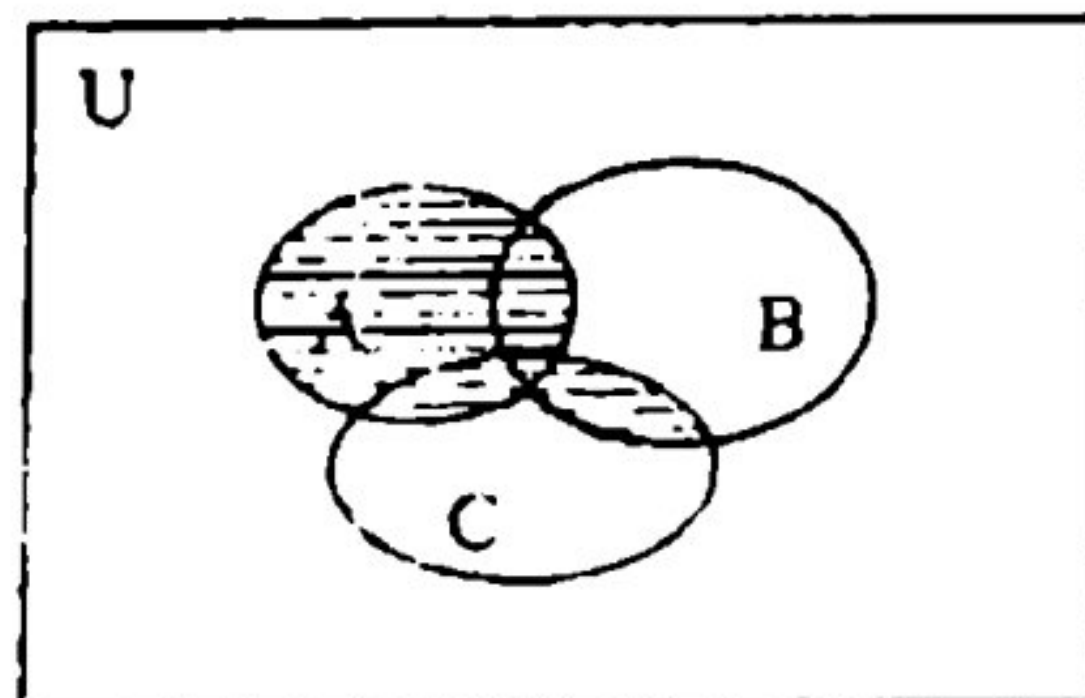


Fig. 1: $A \cup (B \cap C)$ is shown by horizontal line segments in the above figure.

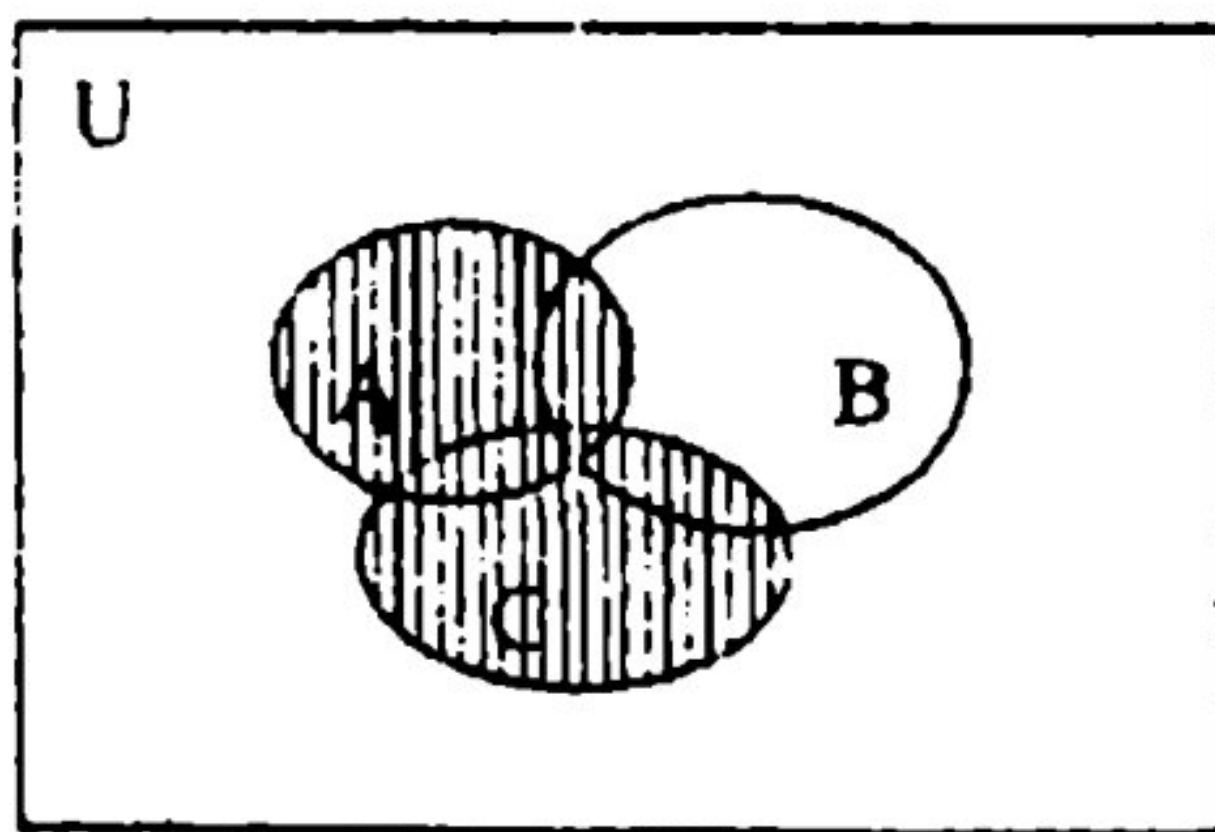


Fig. 2: $A \cup B$ is shown by horizontal line segments in the above figure.

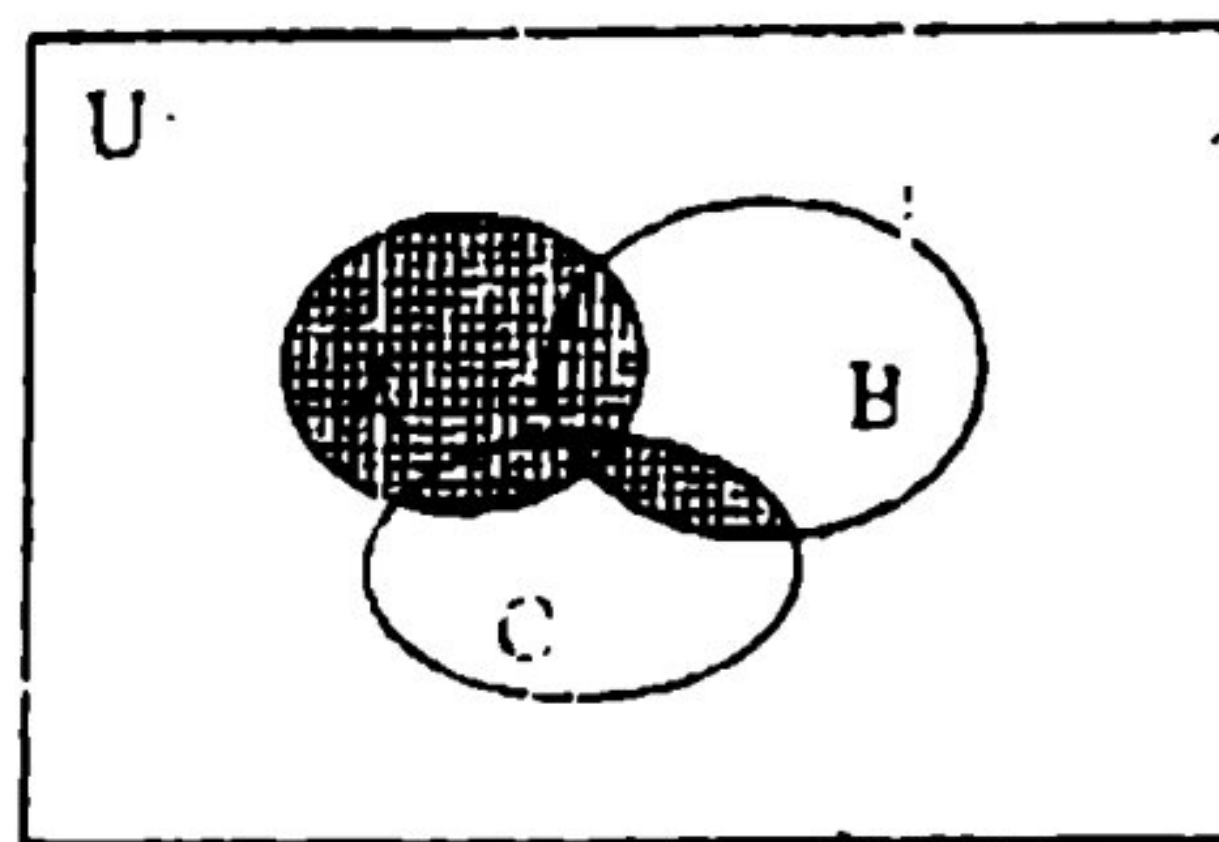


Fig. 3: $A \cup C$ is shown by vertical line segments in Fig. 3,
Regions shown in Fig. 1 and Fig. 4 are equal.
Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

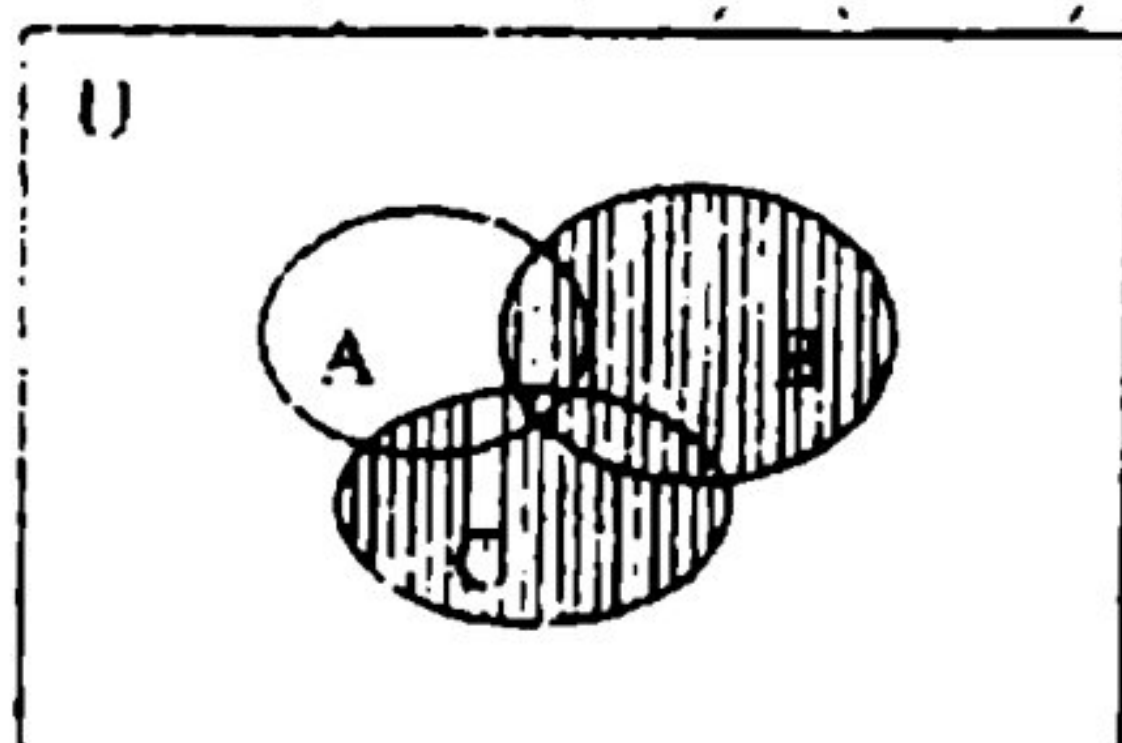


Fig. 4: $(A \cup B) \cap (A \cup C)$ is shown by double crossing line segments in Fig. 4.

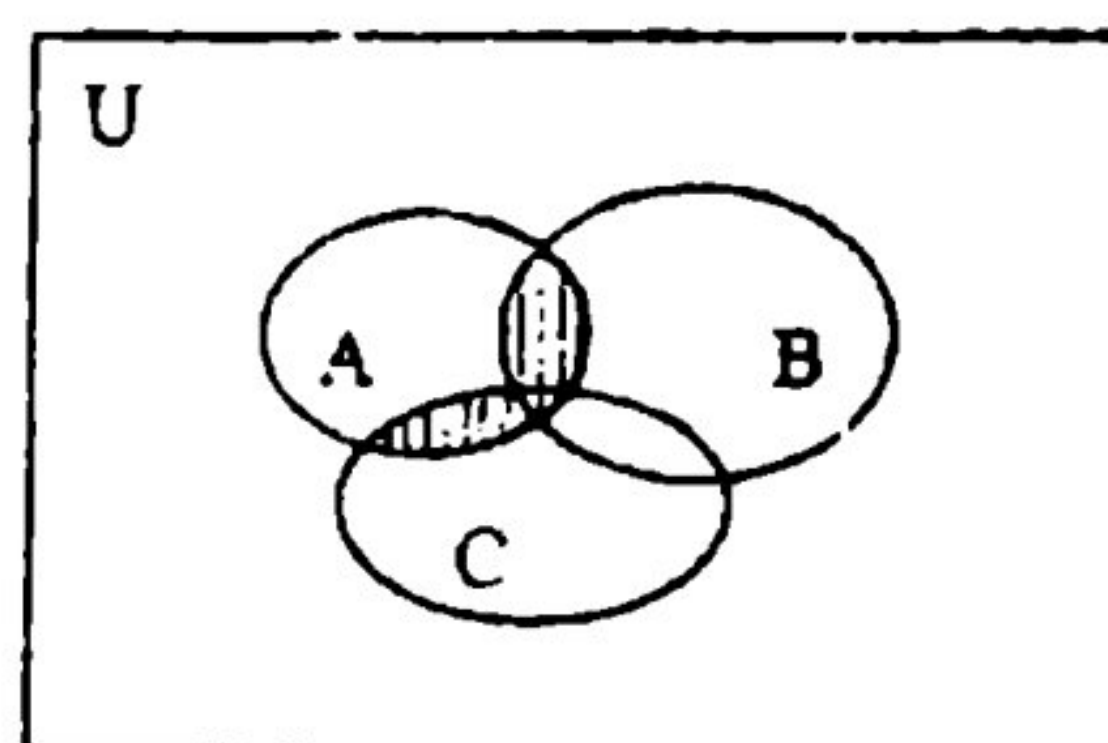


Fig. 5: $B \cup C$ is shown by vertical line segments in Fig. 5.

Fig. 6: $A \cap (B \cup C)$ is shown in Fig. 6 by vertical line segments.

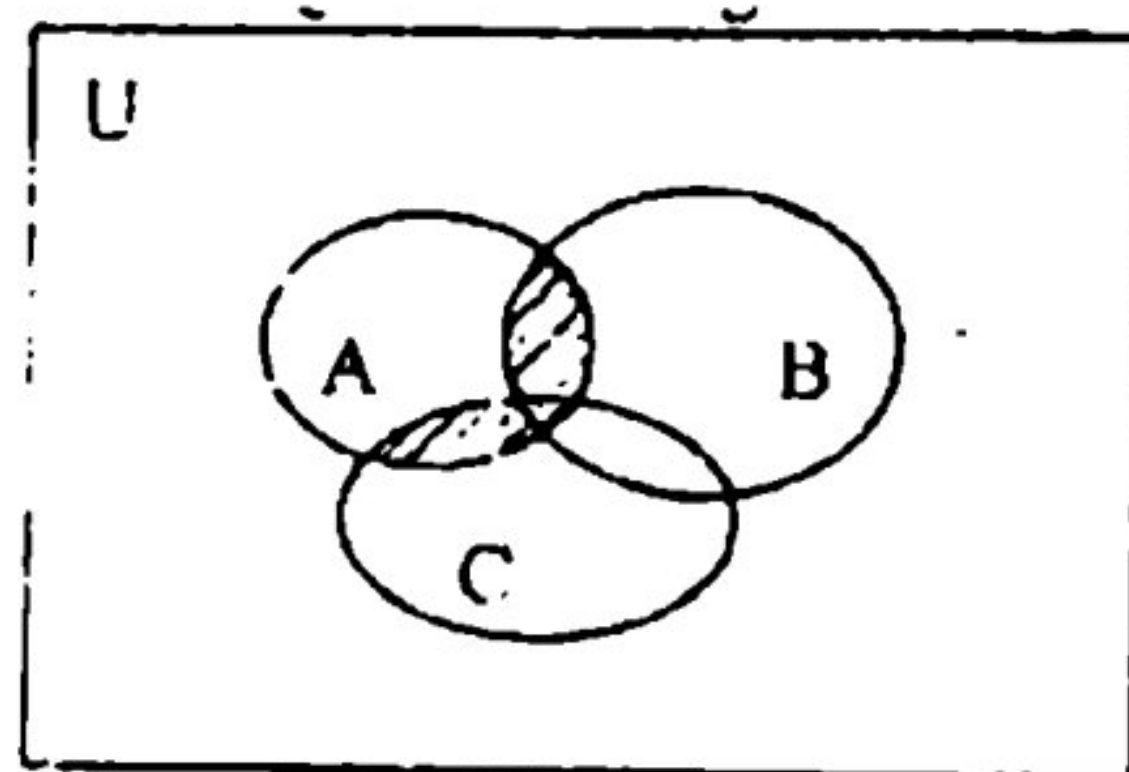


Fig. 7: $(A \cap B) \cup (A \cap C)$ is shown in Fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig. 7 are equal.

Thus $A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$

SOLVED EXERCISE 5.3

1. If $U = \{1, 2, 3, 4, \dots, 10\}$

$A = \{1, 3, 5, 7, 9\}$

$B = \{1, 4, 7, 10\}$ then verify the following questions,

(i) $A - B = A \cap B'$

L.H.S. = $A - B$

= $\{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$

= $\{3, 5, 9\}$ _____ (i)

R.H.S. = $A \cap B'$