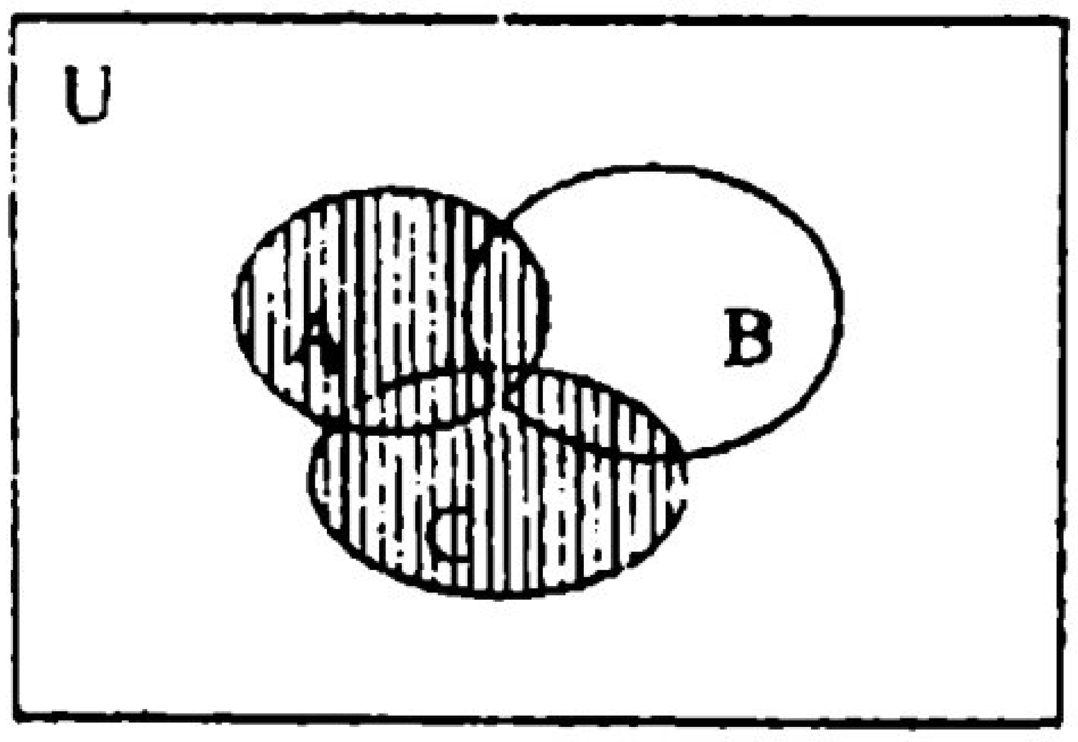
Fig. 1: $A \cup (B \cap C)$ is shown by horizontal line segments in the above figure.

Fig. 2: A u B is shown by horizontal line segments in the above figure.



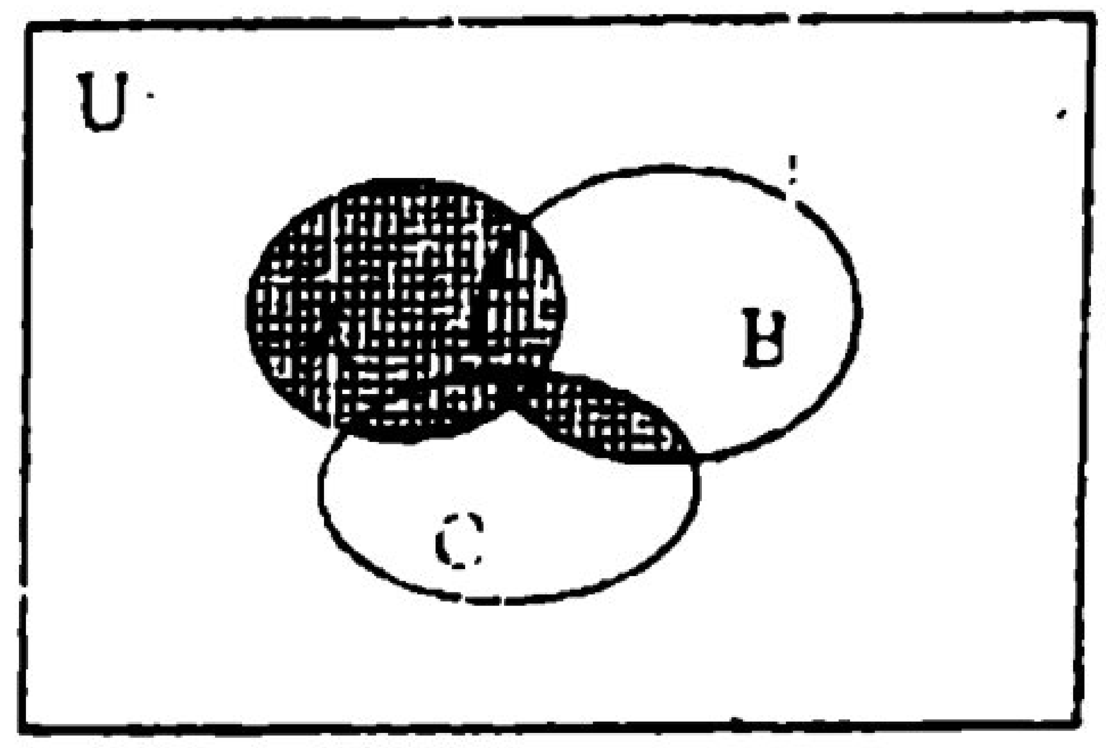
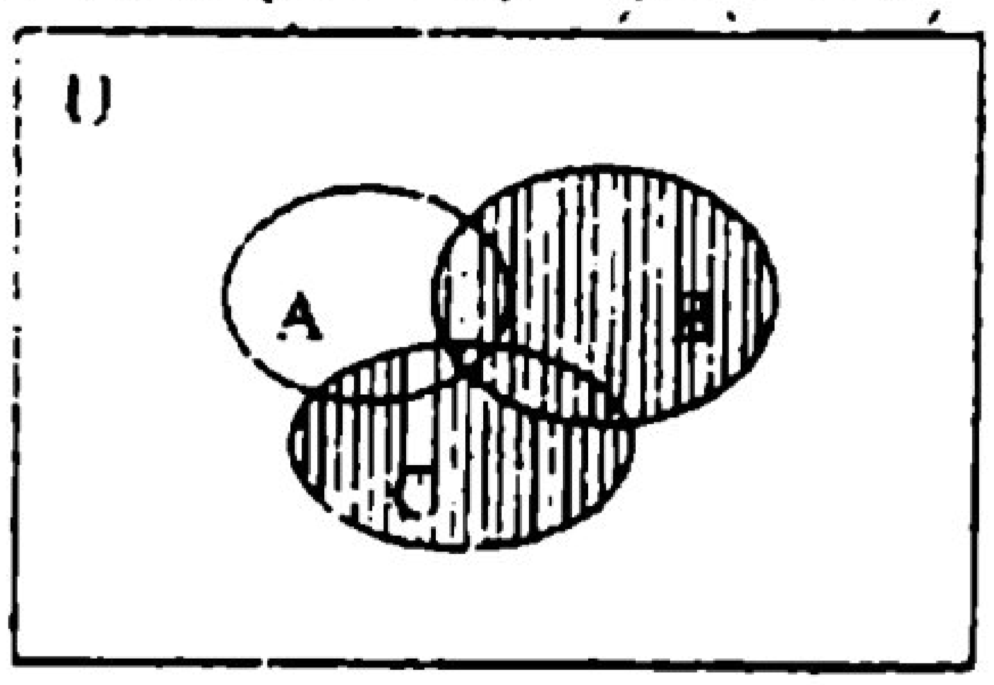


Fig. 3: $A \cup C$ is shown by vertical line segments in Fig. 3,

Fig. 4: $(A \cup B) \cap (A \cup C)$ is shown by double crossing line segments in Fig. 4.

Regions shown in Fig. 1 and Fig. 4 are equal.

Thus $A \cup (B \cap C) = (A \cup B') \cap (A \cup C)$



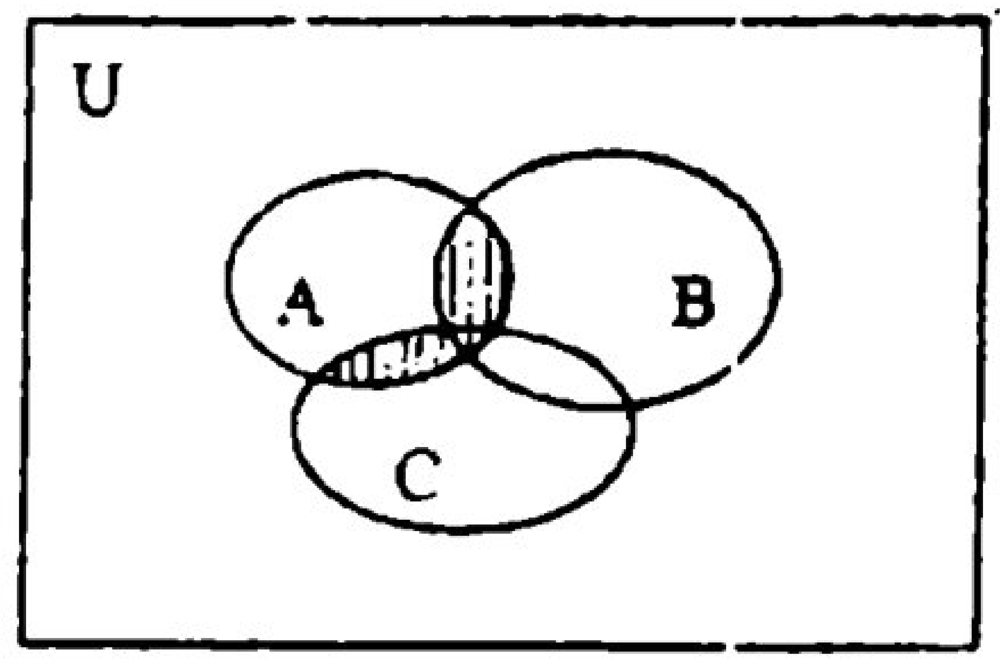


Fig. 5: $B \cup C$ is shown by vertical line segments in Fig. 5.

Fig. 6: $A \cap (B \cup C)$ is shown in Fig. 6 by vertical line segments.

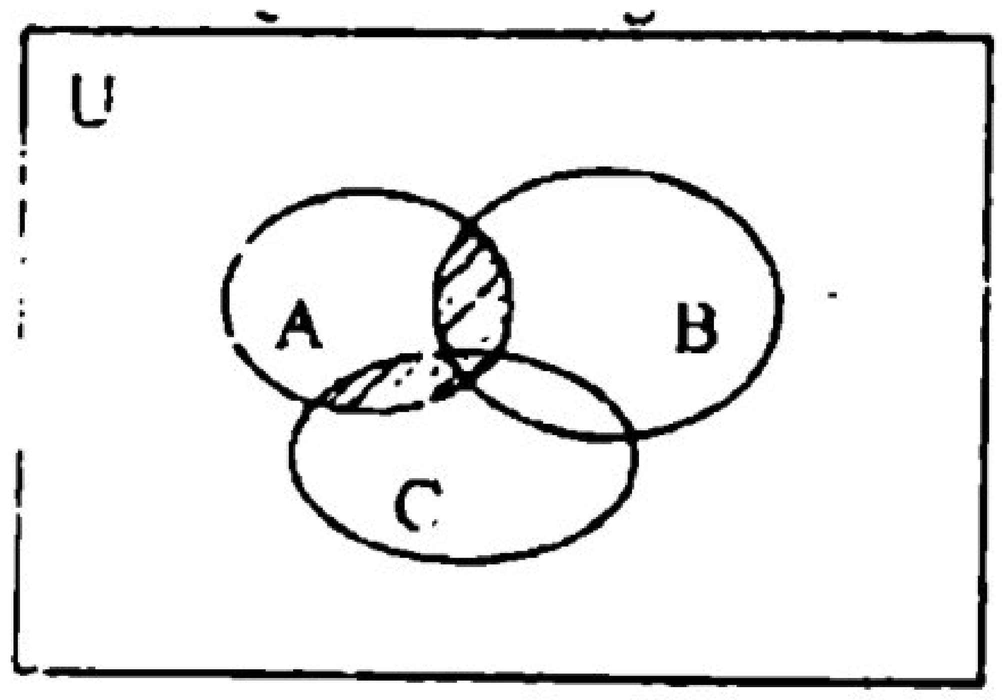


Fig. 7: $(A \cap B) \cup (A \cap C)$ is shown in Fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig, 7 are equal.

Thus $A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$

SOLVED EXERCISE 5.3

1. If $U = \{1, 2, 3, 4, \ldots, 10\}$

$A = \{1, 3, 5, 7, 9\}$

$B = \{1, 4, 7, 10\}$ then verify the following questions,

(i) $A - B = A \cap B'$

L.H.S. $= A - B$

$= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$

$= \{3, 5, 9\}$ ________(i)

R.H.S. $= A \cap B'$

$= A \cap (\cup - B)$
$= \{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 4, \ldots, 10\} - \{1, 4, 7, 10\})$
$= \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{3, 5, 9\}$ _________(ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(ii) $B - A = B \cap A'$

L.H.S. $= B - A$
$= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$ _________(i)
R.H.S. $= B \cap A'$
$= B \cap (\cup - A)$
$= \{1, 4, 7, 10\} \cap \{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 4, 5, 7, 9\})$
$= \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 10\}$ _________(ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(iii) $(A \cup B)' = A' \cap B'$

L.H.S. $= (A \cup B)'$
$= \cup - (A \cup B)$
$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\})$
$= \{1, 2, 3, 4, \ldots 10\} - \{1, 3, 4, 5, 7, 9, 10\}$
$= \{2, 6, 8\}$ _________(i)
R.H.S. $= A' \cap B'$
$= (\cup - A) \cap (\cup - B)$
$= (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\}) \cap \{1, 2, 3, 4, \ldots, 10\} - \{1, 4, 7, 10\})$
$= \{2, 4, 6, 8, 10\} \cap \{2,3, 5, 6, 8, 9\}$
$= \{2, 6, 8\}$ _________(ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(iv) $(A \cap B)' = A' \cup B'$

L.H.S. $= (A \cup B)'$
$= \cup - (A \cap B)$
$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\})$
$= \{1, 2, 3, 4, \ldots 10\} - \{1, 7\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$ _________(i)
R.H.S. $= A' \cup B'$
$= (\cup - A) \cup (\cup - B)$
$= (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, 4, \ldots, 10\} - \{1, 4, 7, 10\})$

= {2, 4, 6, 8, 10} ∪ {2,3, 5, 6, 8, 9}
= {2, 3, 4, 5, 6, 8, 9, 10} _________(ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(v) (A – B)′ = A′ ∪ B

L.H.S. = (A – B)′
= ∪ – (A – B)
= {1, 2, 3, 4, ..., 10} – {1, 3, 5, 7, 9} – {1, 4, 7, 10})
= {1, 2, 3, 4, ... 10} – {3, 5, 9}
= {1, 2, 3, 6, 7, 8, 10} _________(i)

R.H.S. = A′ ∪ B′
= (∪ – A) ∪ B
= ({1, 2, 3, 4, ..., 10} – {1, 3, 5, 7, 9}) ∪ {1, 4, 7, 10}
= {2, 4, 6, 8, 10} ∪ {1,4, 7, 10}
= {1, 2, 4, 6, 7, 8, 10} _________(ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(vi) (B – A)′ = B′ ∪ A

L.H.S. = (B ∪ A)′
= ∪ – (B – A)
= {1, 2, 3, 4, ..., 10} – {1, 4, 7, 10} – {1, 3, 5, 7, 9})
= {1, 2, 3, 4, ..., 10} – {4, 10}
= {1, 2, 3, 5, 6, 7, 8, 9} _________(i)

R.H.S. = B′ ∪ A
= (∪ – B) ∪ A
= ({1, 2, 3, 4, ..., 10} – {1, 3, 5, 7, 9}) ∪ {1, 3, 5, 7, 9}
= {2, 3, 5, 6, 8, 9} ∪ {1,3, 5, 7, 9}
= {1, 2, 3, 5, 6, 7, 8, 9} _________(ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

2. If U = {1,2, 3,4,-,10}
A = {1,3,5,7,9}; B = {1,4,7,10}; C = {1,5, 8, 10} then verify the following:

Solution:

(i) (A ∪ B) ∪ C = A ∪ (B ∪ C)

L.H.S. = (A ∪ B) ∪ C
= ({1, 3, 5, 7, 9} ∪ {1, 4, 7, 10}) ∪ {1, 5, 8, 10}
= {1, 3, 4, 5, 7, 9, 10} ∪ {1, 5, 8, 10}
= {1, 3, 4, 5, 7, 8, 9, 10} ________ (i)

R.H.S. $= A \cup (B \cup C)$
$= \{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\})$
$= \{1, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8, 10\}$
$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ ________ (ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S. $= (A \cap B) \cap C$
$= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$
$= \{1, 7\} \cap \{1, 5, 8, 10\}$
$= \{1\}$ ________ (i)
R.H.S. $= A \cap (B \cap C)$
$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$
$= \{1, 3, 5, 7, 9\} \cap \{1, 10\}$
$= \{1\}$ ________ (ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(iii) $A \cup (B \cup C) = (A \cup B) \cap (A \cup C)$

L.H.S. $= A \cup (B \cap C)$
$= \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$
$= \{1, 3, 5, 7, 9\} \cup \{1, 10\}$
$= \{1, 3, 5, 7, 9, 10\}$ ________ (i)
R.H.S. $= (A \cup B \cap (A \cup C)$
$= (\{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\}) \cap (\{1, 3, 5, 7, \} \cup \{1, 5, 8, 10\})$
$= \{1, 3, 4, 5, 7, 9, 10\} \cap \{1, 3, 5, 7, 8, 9, 10\}$
$= \{1, 3, 5, 7, 9, 10\}$ ________ (ii)
From (i) and (ii), we have
L.H.S. = R.H.S.
Hence Proved

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. $= A \cap (B \cup C)$
$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\}) \cup \{1, 5, 8, 10\}$
$= \{1, 3, 5, 7, 9\} \cap \{1, 4, 5, 7, 8, 10\}$
$= \{1, 5, 7\}$ ________ (i)
R.H.S. $= (A \cap B \cup (A \cap C)$
$= (\{1, 3. 5, 7, 9\} \cap (\{1, 4, 7, 10\}) \cup (\{1, 3, 5, 7, 9\} \cap \{1, 5, 8, 10\})$
$= \{1, 7\} \cup \{1, 5\}$
$= \{1, 5, 7\}$ ________ (ii)
From (i) and (ii), we have
L.H.S. = R.H.S.

Hence Proved

3. If U = N; then verify De-Morgan's laws by using $A = \phi$ and B = P.

Solution:

$\cup = N$, $A = \phi$, $B = P$

(i) $(A \cap B)' = A' \cup B'$

L.H.S. $= (A \cap B)'$
$= \cup - (A \cap B)$
$= N - (\phi \cap P)$
$= N - \phi$
$= N$ ________ (i)

R.H.S. $= A' = B'$
$= (\cup - A) \cup (\cup - B)$
$= (N - \phi) \cup (N - P)$
$= N \cup (N - P)$
$= N$ ________ (ii)

From (i) and (ii), we have
L. H.S. = R.H.S
Hence Proved

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. $= (A \cup B)'$
$= \cup - (A \cup B)$
$= N - (\phi \cup P)$
$= N - \phi$
$= N$ ________ (i)

R.H.S. $= A' \cap B'$
$= (\cup - A) \cap (\cup - B)$
$= (N - \phi) \cap (N - P)$
$= N \cap (N - P)$
$= N - P$ ________ (ii)

From (i) and (ii), we have
L. H.S. = R.H.S
Hence Proved

4. If U = {1, 2, 3, 4, ... 10}, A – {1, 3, 5, 7, 9} and B = {2, 3, 4, 5, 8} then prove the following questions by Venn diagram:

(i) $A - B = A \cap B'$

Solution:

U = {1, 2, 3, 4,....,10}, A = {1, 3, 5, 7, 9}
B = {2, 3, 4, 5, 8}

(i) A – B = A ∩ B′

L.H.S. $= A - B$

$= \{1, 3, 5, 7, 9\} - \{2, 3, 4, 5, 8\}$

$= \{1, 7, 9\}$ ________ (i)

R.H.S. $= A \cap B'$

$= A \cap (\cup - B)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 2, 3, 4, \ldots, 10\} - \{2, 3, 4, 5, 8\})$

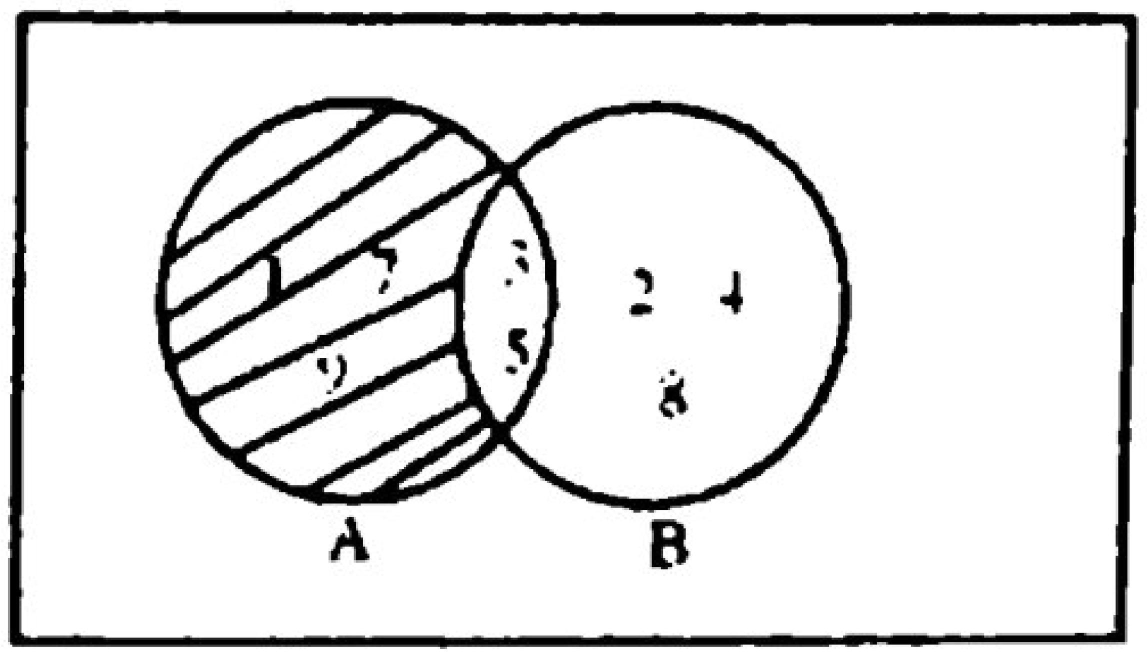
$= \{1, 3, 5, 7, 9\} \cap \{1, 6, 7, 9, 10\}$

$= \{1, 7, 9\}$ ________ (ii)

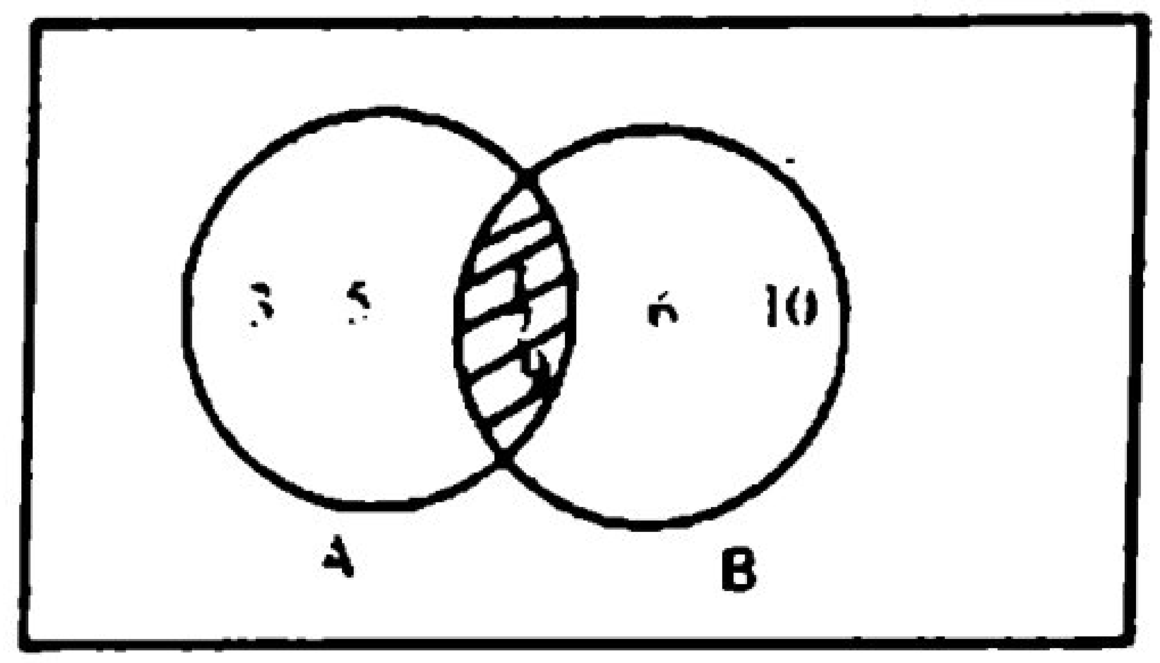
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved



A B



A ∩ B′

(ii) B – A = B ∩ A′

Solution:

L.H.S. $= B - A$

$= \{2, 3, 4, 5, 8\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 8\}$ ________ (i)

R.H.S. $= B \cap A'$

$= B \cap (\cup - A)$

$= \{2, 3, 4, 5, 8\} \cap (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\})$

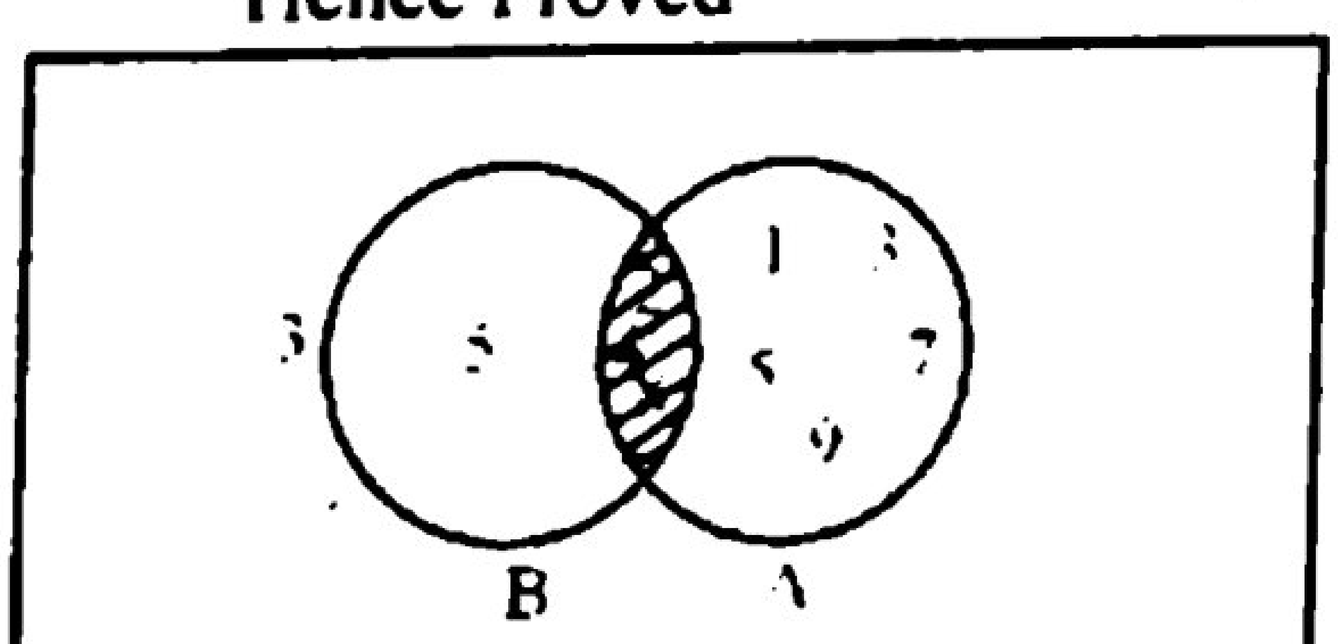
$= \{2, 3, 4, 5, 8\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 8\}$ ________ (ii)

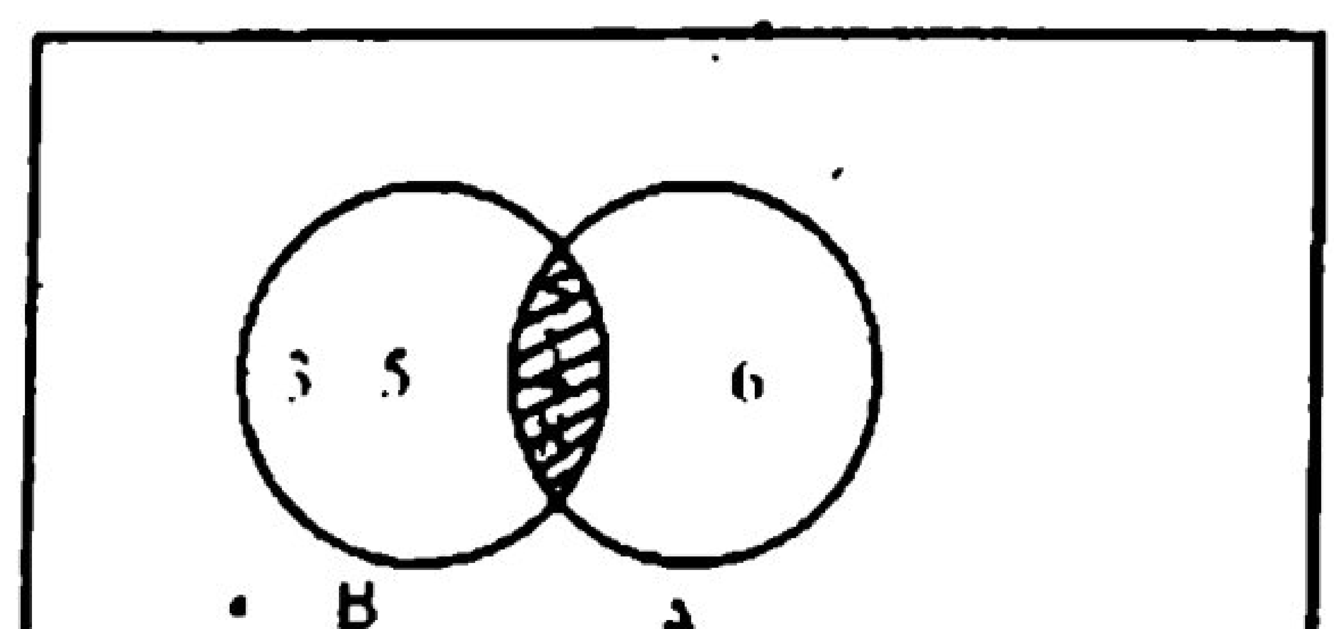
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved



B – A



B ∩ A′

(iii) (A ∩ B)′ = A′ ∩ B′

Solution:

L.H.S. $= (A \cup B)'$

$= \cup - (A \cup B)$

$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} \cup \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 2, 3, 4, 5, 7, 8, 9\}$

$= \{6, 10\}$ _______ (i)

R.H.S. $= A' \cap B'$

$= (\cup - A) \cap (\cup - B)$

$= (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\}) \cap \{2, 3, 4, 5, 8\} - (\{1, 2, 3, 4, \ldots, 10\})$

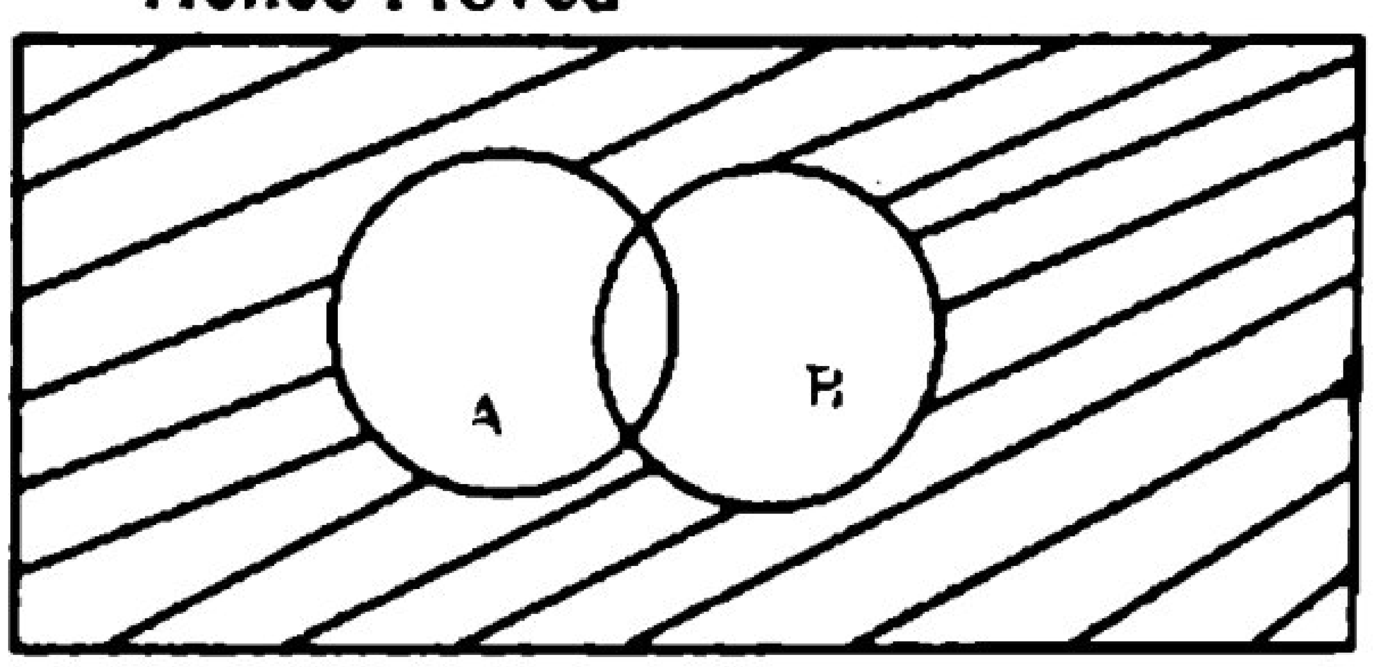
$= \{2, 4, 6, 8, 10\} \cap \{1, 6, 10\}$

$= \{6, 10\}$ _______ (ii)

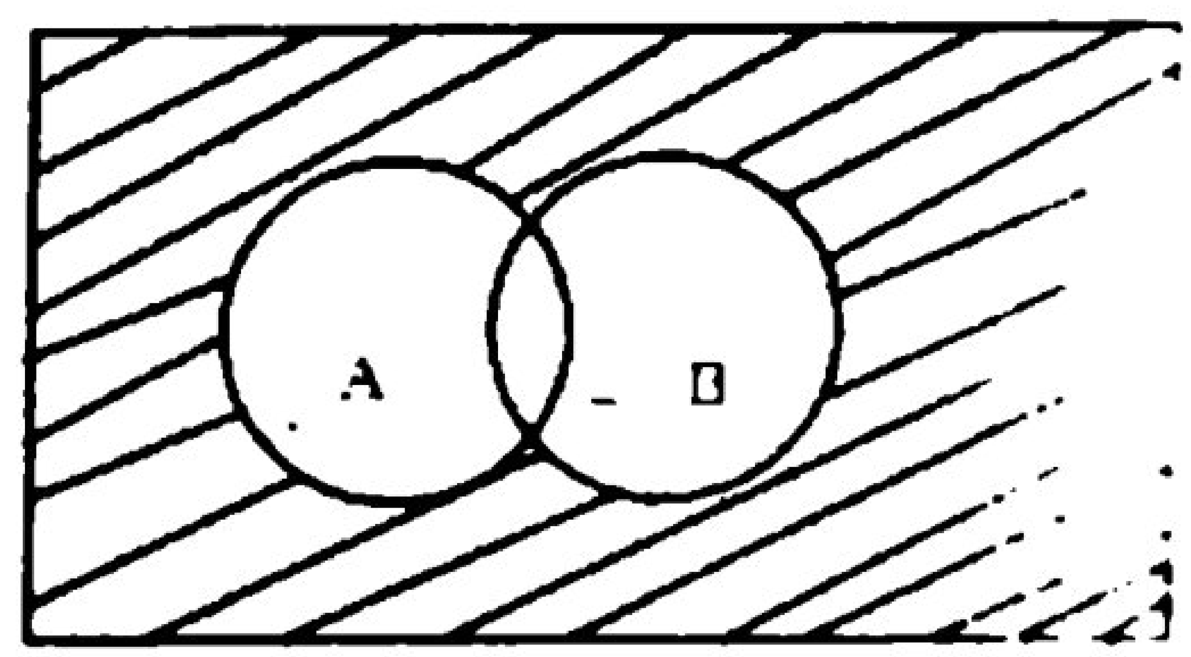
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved



$(A \cup B)'$



$A' \cap B'$

(iv) $(A \cap B)' = A \cup B'$

Solution:

L.H.S. $= (A \cap B)'$

$= \cup - (A \cap B)$

$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots, 10\} - \{3, 5\}$

$= \{1, 2, 4, 6, 7, 8, 9, 10\}$ _______ (i)

R.H.S. $= A' \cup B'$

$= (\cup - A) \cup (\cup - B)$

$= (\{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} \cup (\{1, 2, 3, 4, \ldots, 10\} - \{2, 3, 4, 5, 8\})$

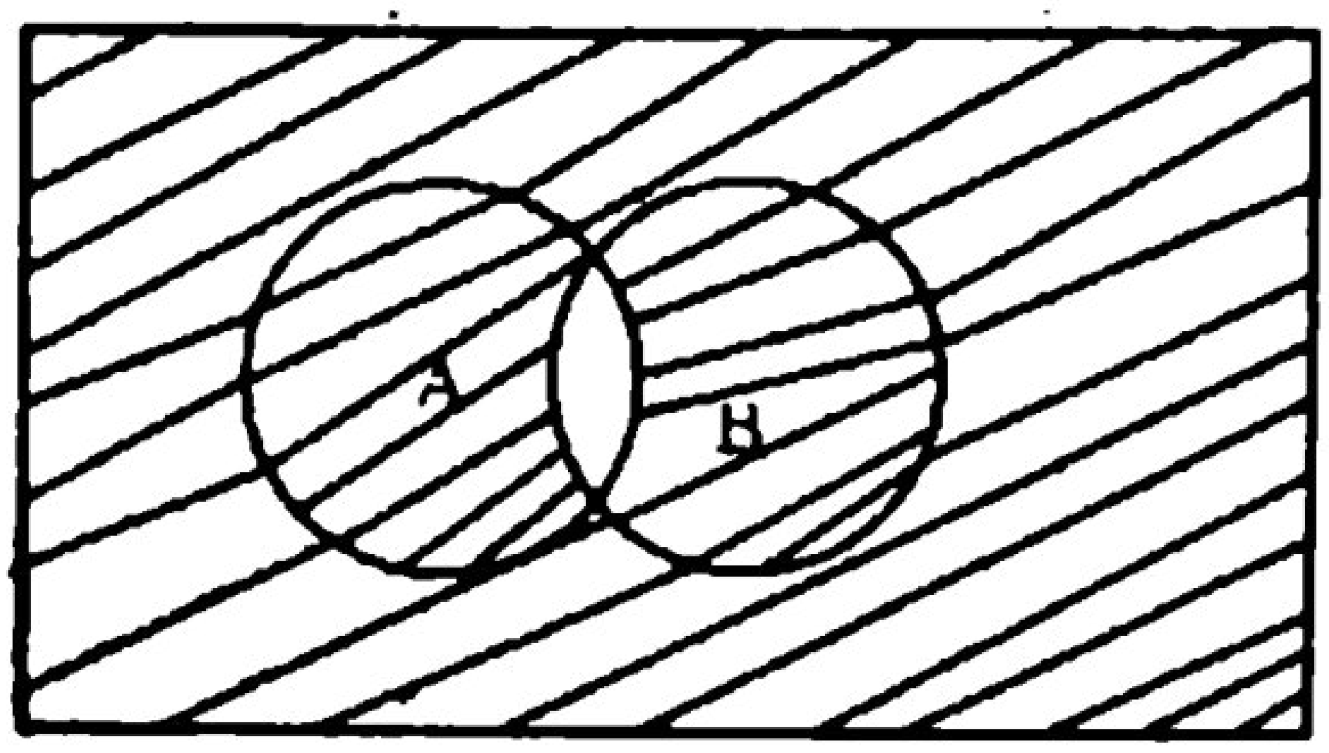
$= \{2, 4, 6, 8, 10\} - \{1, 6, 7, 9, 10\}$

$= \{1, 2, 4, 6, 7, 8, 9, 10\}$ _______ (i)

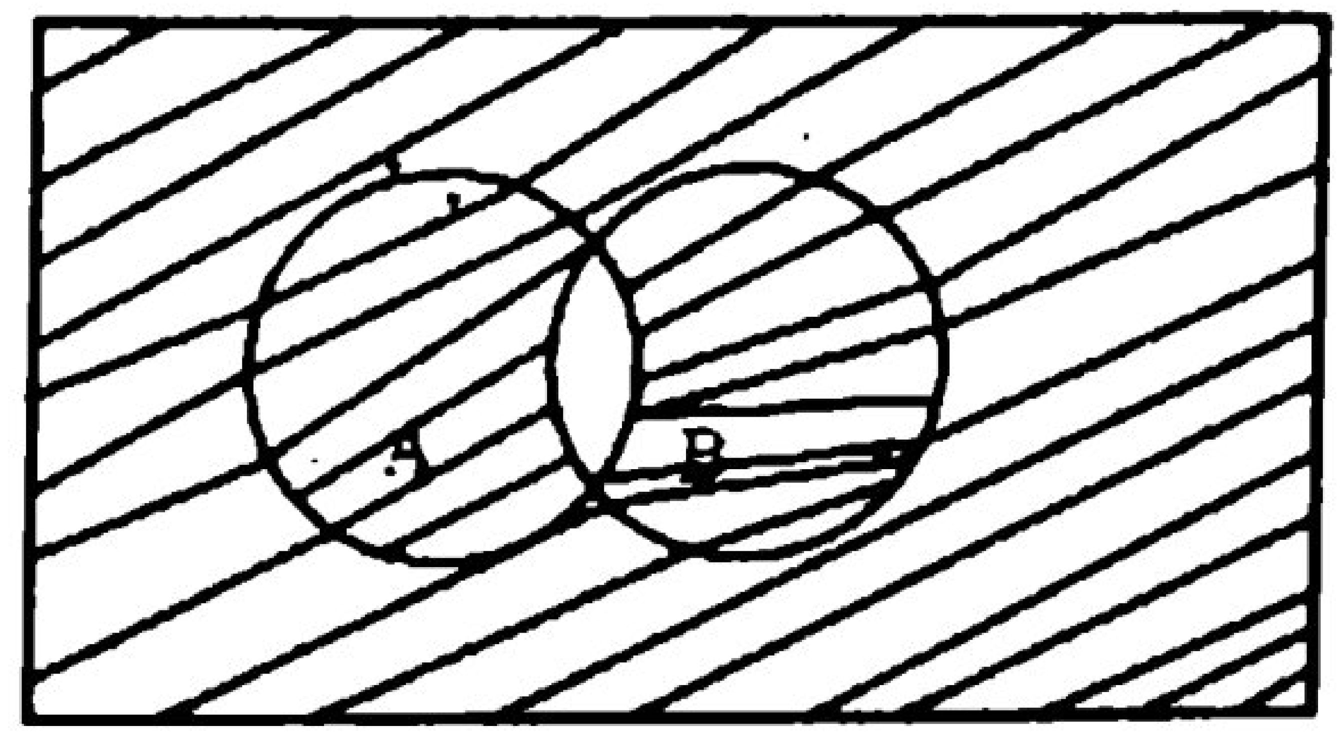
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved



$(A' \cup B)'$



$A' \cup B'$

(v) $(A - B)' = A' \cup B$

Solution:

L.H.S. $= (A - B)'$

$= \cup - (A - B)$

$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} - \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 7, 9\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ _______ (i)

R.H.S. $= A' \cup B$

$= (\cup - A) \cup B$

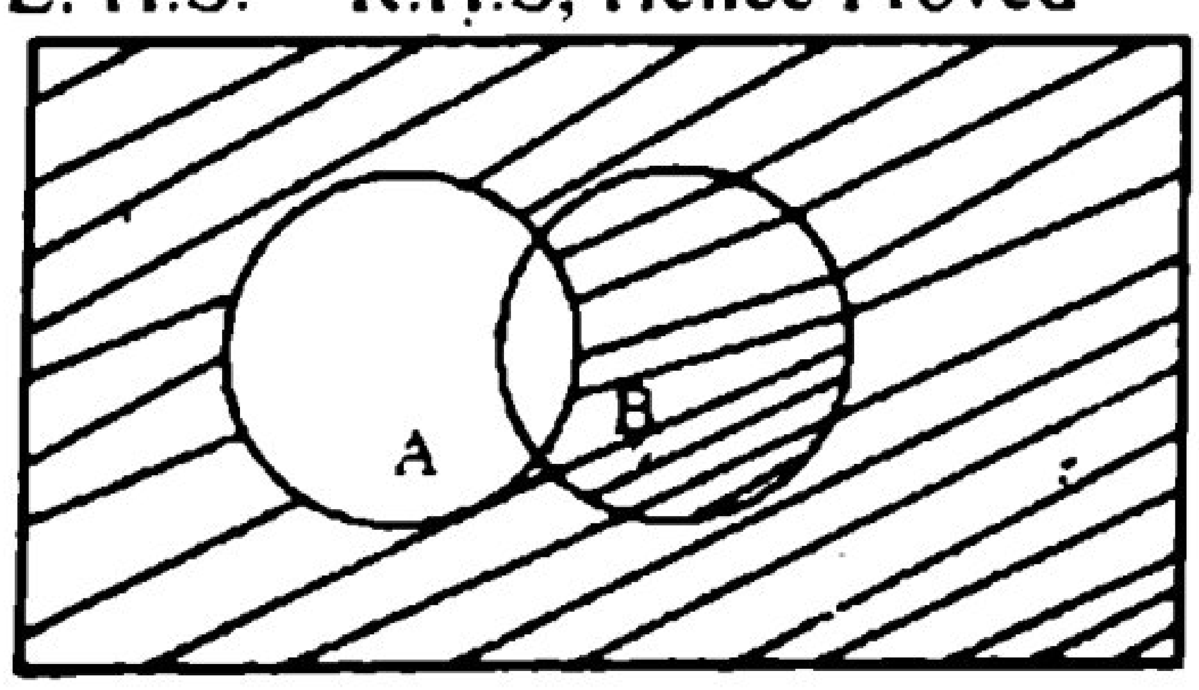
$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\}) \cup \{2, 3, 4, 5, 8\}$

$= \{2, 4, 6, 8, 10\} \cup \{2, 3, 4, 5, 8\}$

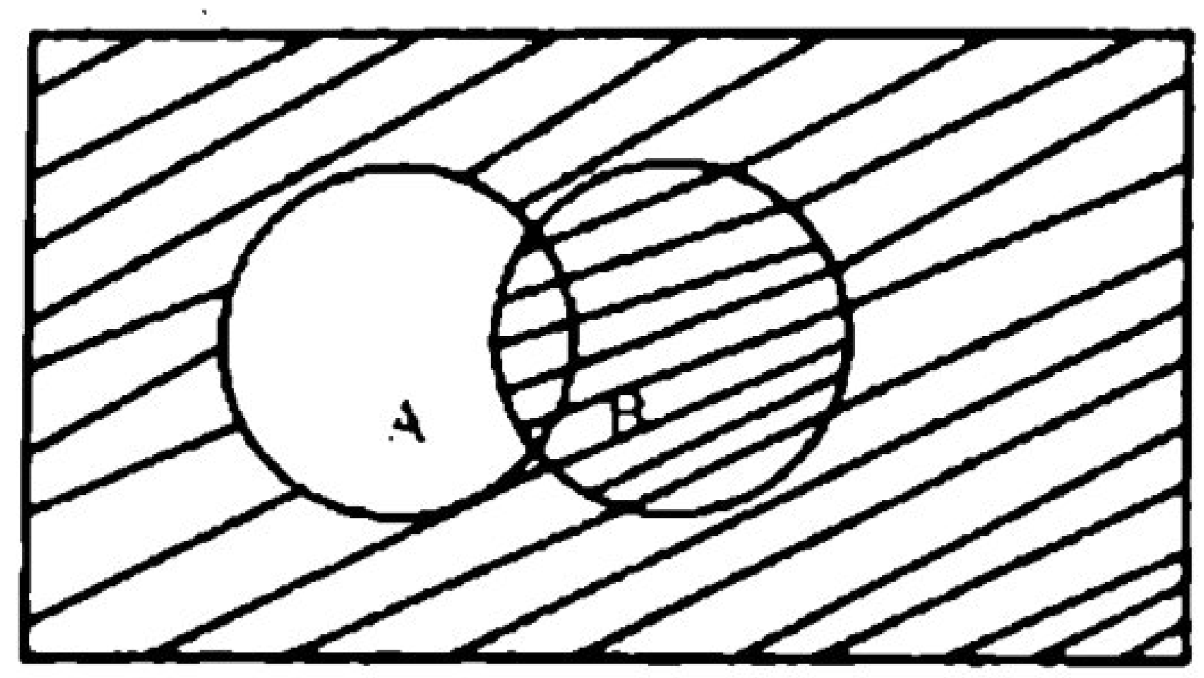
$= \{2, 3, 4, 5, 6, 8, 10\}$ _______ (ii)

From (i) and (ii), we have

L. H.S. = R.H.S, Hence Proved



$(A \cup B)'$



$A' \cup B$

(vi) $(B - A)' = B' \cup A$

Solution:

L.H.S. $= (B - A)'$

$= \cup - (B - A)$

$= \{1, 2, 3, 4, \ldots, 10\} - (\{2, 3, 4, 5, 8\} - \{1, 3, 5, 7, 9\})$

$= \{1, 2, 3, 4, \ldots, 10\} - \{2, 4, 8\}$

$= \{1, 3, 4, 5, 6, 7, 9, 10\}$ _______ (i)

R.H.S. $= B' \cap A$

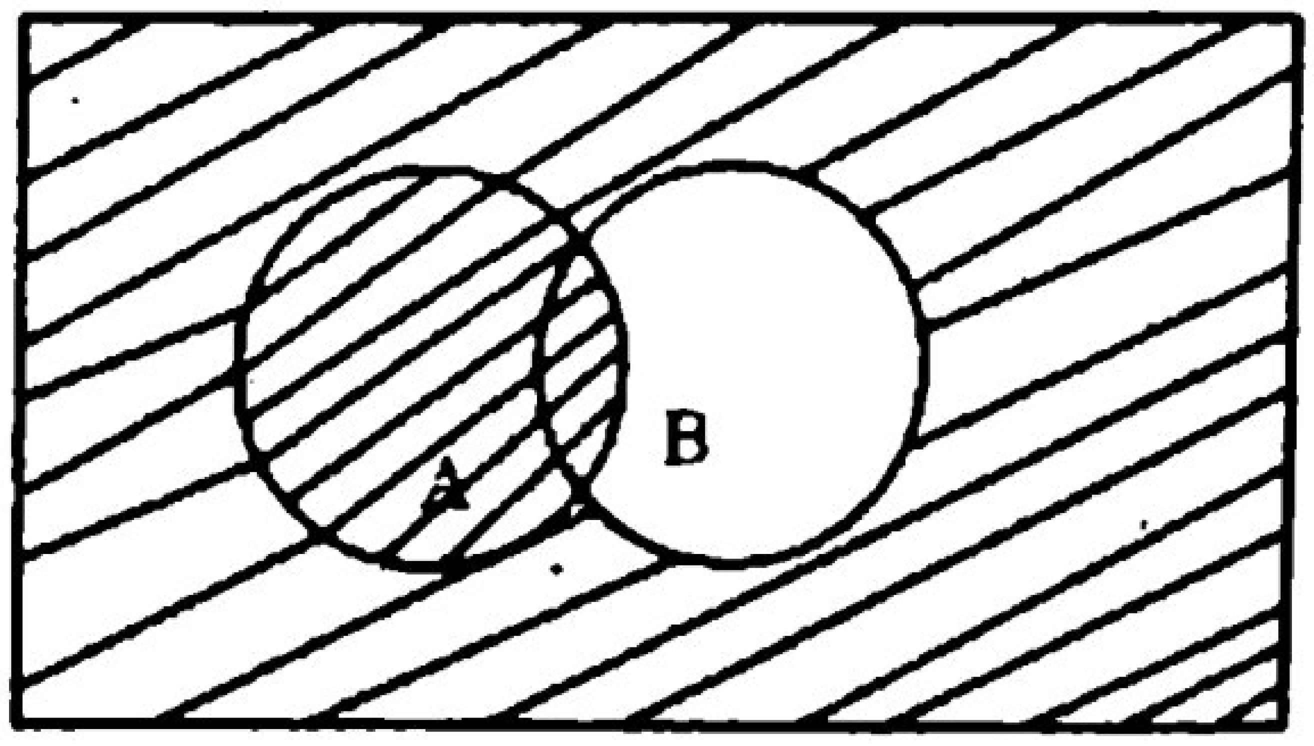
$= (\cup - B) \cup A$

$= (\{1, 2, 3, 4, \ldots, 10\} - \{2, 3, 4, 5, 8\}) \cup \{1, 3, 5, 7, 9\}$

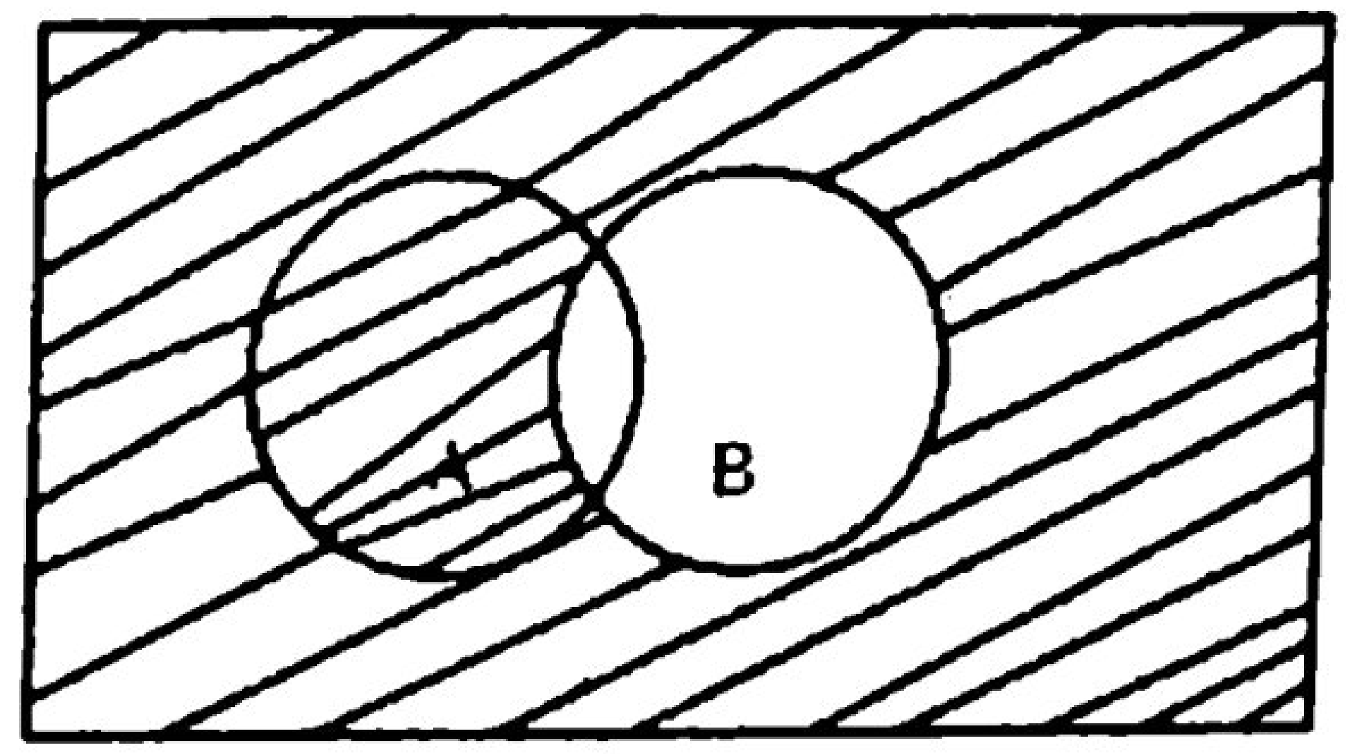
$= \{1, 3, 5, 7, 9, 10\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 3, 5, 7, 9, 10\}$ _______ (ii)

From (i) and (ii), we have
L. H.S. = R.H.S Hence Proved



$(A \cup B)'$



$B' \cup A$

5.1.4 (viii) Ordered pairs and Cartesian product:

5.1.4(a) Ordered pairs:

Any two numbers x and y, written in the form (x, y] is called an ordered pair. In an ordered pair (x, y), the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, iff $x = s$ and $y = t$

5.1.4 (b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by A x B consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

Example: If A = {1, 2, 3} and B = {2, 5}, then find A x B and B x A.

Solution: $A \times B = \{(1,2), (1, 5), (2, 2), (2, 5), (3, 2), (3, 5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A \times B$ has $3 \times 2 = 6$ ordered pairs.

We note that $B \times A - \{(2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3)\}$

Evidently $A \times B \neq B \times A$.

SOLVED EXERCISE 5.4

1. If A = {a, b} and 5 = {c, d}, then find A × B and B × A.

Solution:

$A = \{a, b\}$ and $B = \{c, d\}$

$A \times B = \{a, b\} \times \{c, d\}$

$= \{(a, c), (a, d), (b, c), (b, d)\}$

$B \times A = \{c, d\} \times \{a, b\}$

$= \{(c, a), (c, b), (d, a), (d, b)\}$

2. If A = {0,2,4), B = {–l, 3}, then find A × B, B × A, A × A, B × B.

Solution:

$A = \{0, 2, 4\}$ and $B = \{-1, 3\}$

$A \times B = \{0, 2, 4\} \times \{-1, 3\}$

$= \{(0, -1), (0, 3), (2, -1), (4, -1), (4, 3)\}$

$B \times A = \{-1, 3\} \times \{0, 2, 4\}$