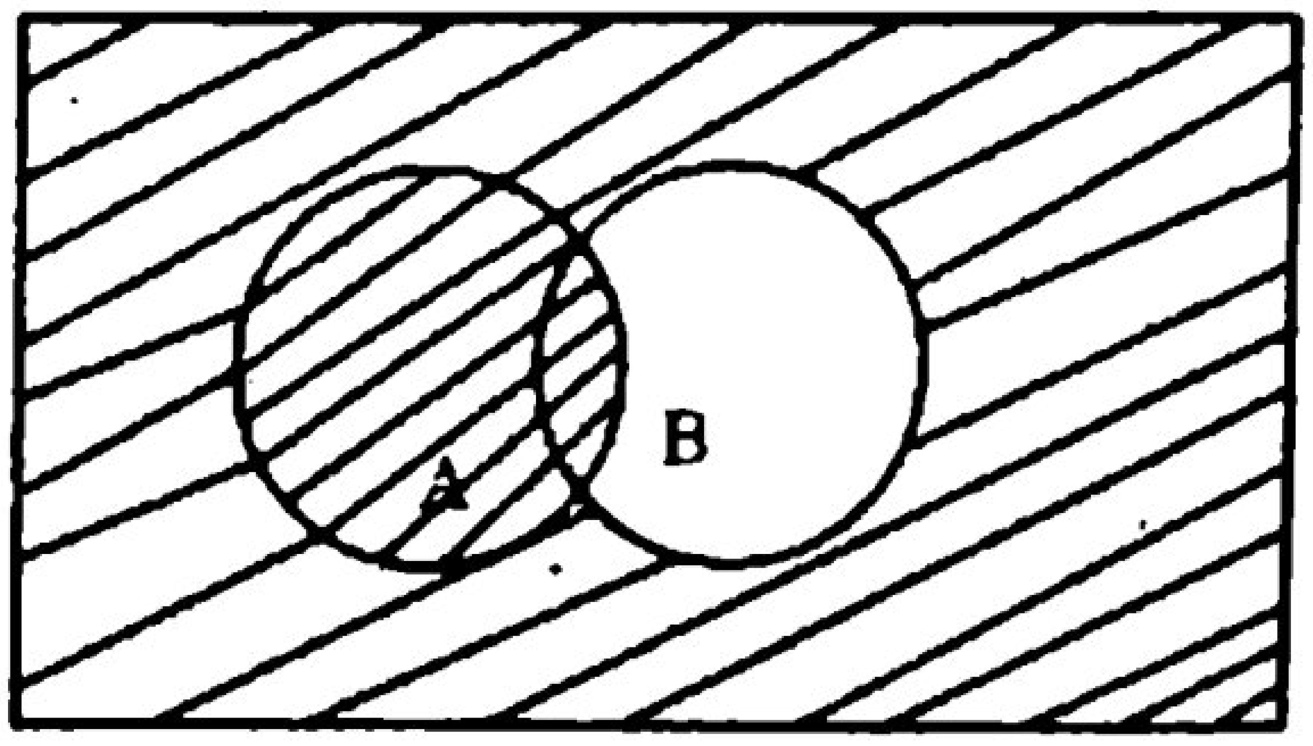
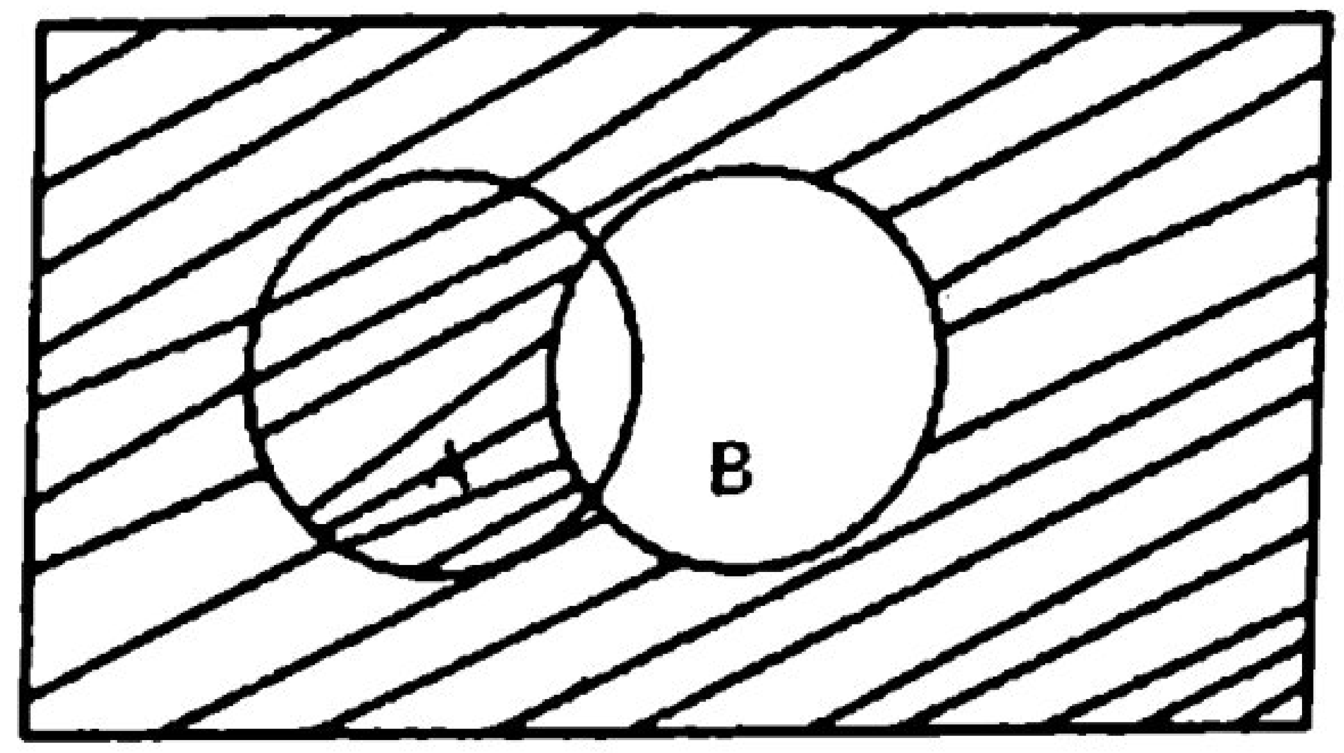
From (i) and (ii), we have

L. H.S. = R.H.S Hence Proved



$(A \cup B)'$



$B' \cup A$

5.1.4 (viii) Ordered pairs and Cartesian product:

5.1.4(a) Ordered pairs:

Any two numbers x and y, written in the form (x, y] is called an ordered pair. In an ordered pair (x, y), the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, iff $x = s$ and $y = t$

5.1.4 (b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by A x B consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

Example: If A = {1, 2, 3} and B = {2, 5}, then find A x B and B x A.

Solution: $A \times B = \{(1,2), (1, 5), (2, 2), (2, 5), (3, 2), (3, 5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A \times B$ has $3 \times 2 = 6$ ordered pairs.

We note that $B \times A - \{(2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3)\}$

Evidently $A \times B \neq B \times A$.

SOLVED EXERCISE 5.4

1. If A = {a, b} and 5 = {c, d}, then find A × B and B × A.

Solution:

$A = \{a, b\}$ and $B = \{c, d\}$

$A \times B = \{a, b\} \times \{c, d\}$

$= \{(a, c), (a, d), (b, c), (b, d)\}$

$B \times A = \{c, d\} \times \{a, b\}$

$= \{(c, a), (c, b), (d, a), (d, b)\}$

2. If A = {0,2,4), B = {–1, 3}, then find A × B, B × A, A × A, B × B.

Solution:

$A = \{0, 2, 4\}$ and $B = \{-1, 3\}$

$A \times B = \{0, 2, 4\} \times \{-1, 3\}$

$= \{(0, -1), (0, 3), (2, -1), (4, -1), (4, 3)\}$

$B \times A = \{-1, 3\} \times \{0, 2, 4\}$

$= \{(-1, 0), (-1, 2), (-1, 4), (3, 0), (3, 2), (3, 4)\}$

$A \times A = \{0, 2, 4\} \times \{0, 2, 4\}$

$= \{(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)\}$

$B \times B = \{-1, 3\} \times \{-1, 3\}$

$= \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$

3. Find a and b, if

(i) $(a - 4, 6 - 2) = (2, 1)$

Solution:

$\Rightarrow$ $a - 4 = 2$ and $b - 2 = 1$

$a = 2 + 4$ $\quad b = 1 + 2$

$a = 6$ $\quad b = 3$

(ii) $(2a + 5, 3) = (7, b - 4)$

Solution:

$\Rightarrow$ $2a + 5 = 7$ and $3 = b - 4$

$2a = 7 - 5$ $\quad b = 4 + 3$

$2a = 12$ $\quad b = 7$

$$a = \frac{12}{2}$$

$a = 6$

(iii) $(3 - 2a. b - 1) - (a - 7, 2b + 5)$

Solution:

$\Rightarrow$ $3 - 2a = a - 7$ and $b - 1 = 2b + 5$

$- 2a - a - 7 - 3$ $\quad b - 2b = 1 + 5$

$- 3a = - 10$ $\quad - b = 6$

$\Rightarrow$ $3a = 10$ $\quad b = - 6$

$$a = \frac{10}{3}$$

4. Find the sets – X and Y, if $X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

Solution:

$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

$\Rightarrow$ $X = \{a, b, c, d\}$ and $Y = \{a\}$

5. If $X = \{a, b, c\}$ and $7 = \{d, e\}$, then find the number of elements in

(i) $X \times Y$

Solution:

Since set X has 3 elements and set y has 2 elements.

Hence, product $X \times Y$ has $3 \times 2 = 6$ elements.

(ii) Y × X

Solution:

Since set y has 2 elements and set X has 3 elements.

(iii) X ×X

Solution:

Since set X has 3 elements.

Hence, product X × X has 9 elements.

Binary relation:

If A and B are any two non-empty sets, then a subset $R \subseteq A \times B$ is called binary relation from set A into set B, because there exists some relationship between first and second element of each ordered pair in R.

Domain of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

Range of relation denoted by Rang R is the set consisting of all the second elements of each ordered pair in the relation.

Function or Mapping:

Suppose A and B are two non-empty sets, then relation $f: A \rightarrow B$ is celled a function.

If (i) Dom f = A (ii) every $x \in A$ appears in one and only one ordered pair in f.

Alternate Definition:

Suppose A and B are two non-empty sets, then relation $f: A \rightarrow B$ is celled a function if (i) Dom f = A (ii) $\forall x \in A$ we can associate some uniqe image element $y = f(x) \in B$.

Domain, Co-domain and Range of Function;

If $f: A \rightarrow B$ is a function, then A is called the domain of f and B is called the co-domain of f.

Domain f is the set consisting of all first elements of each ordered pair in f and range f is the set consisting of all second elements of each ordered pair in f Example:

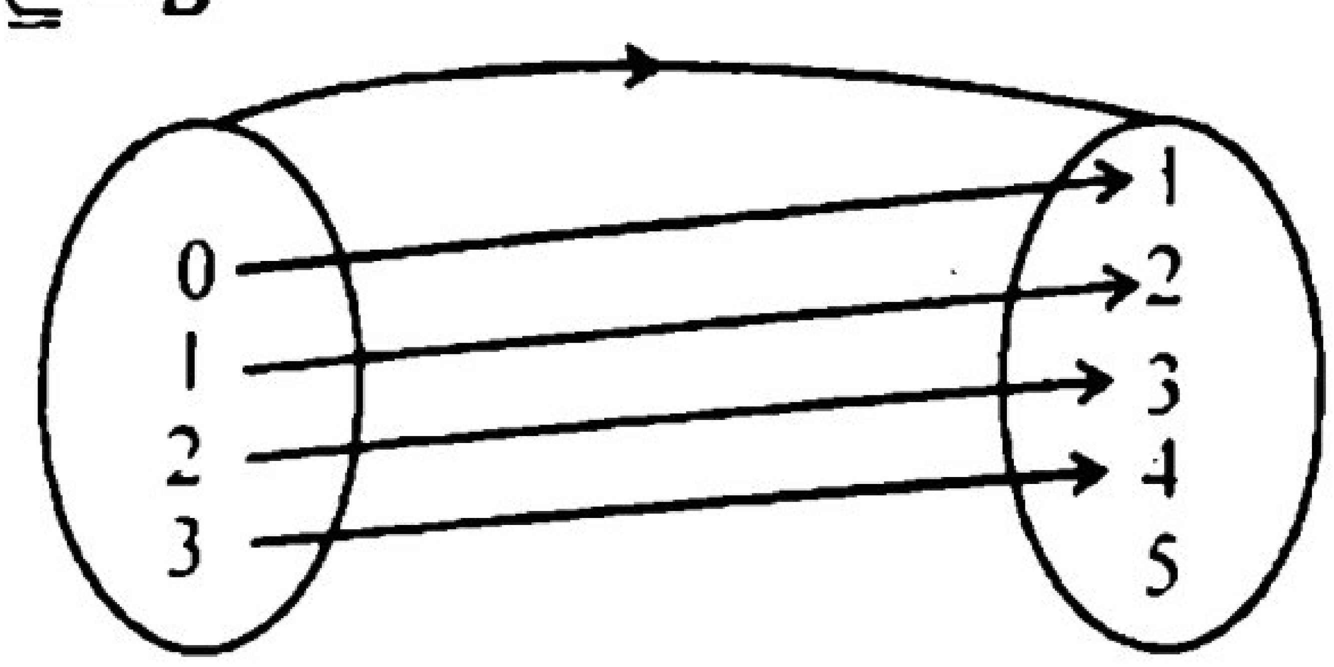
Suppose A = [0, 1, 2, 3} and B = {-1, 2, 3, 4, 5}

Define a function $f: A \rightarrow B$

$f = \{(x, y) \mid y = x + 1 \ \forall x \in A, y \in B)$ $f = \{(0, 1), (1,2), (2, 3), (3, 4)\}$

Dom f ={0, 1,2,3} = A

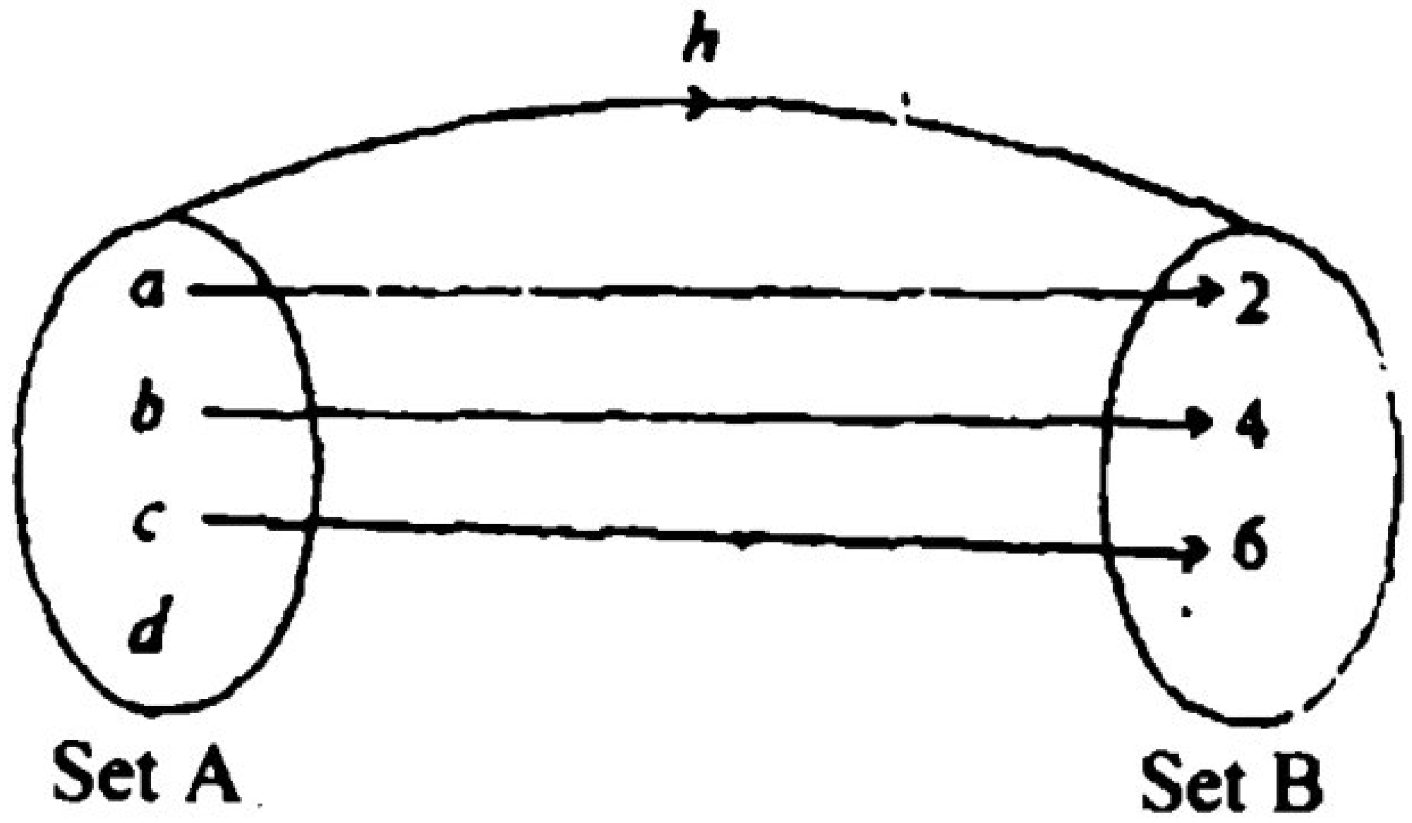
Rang f = $\{1,2,3,4\} \subseteq = B$

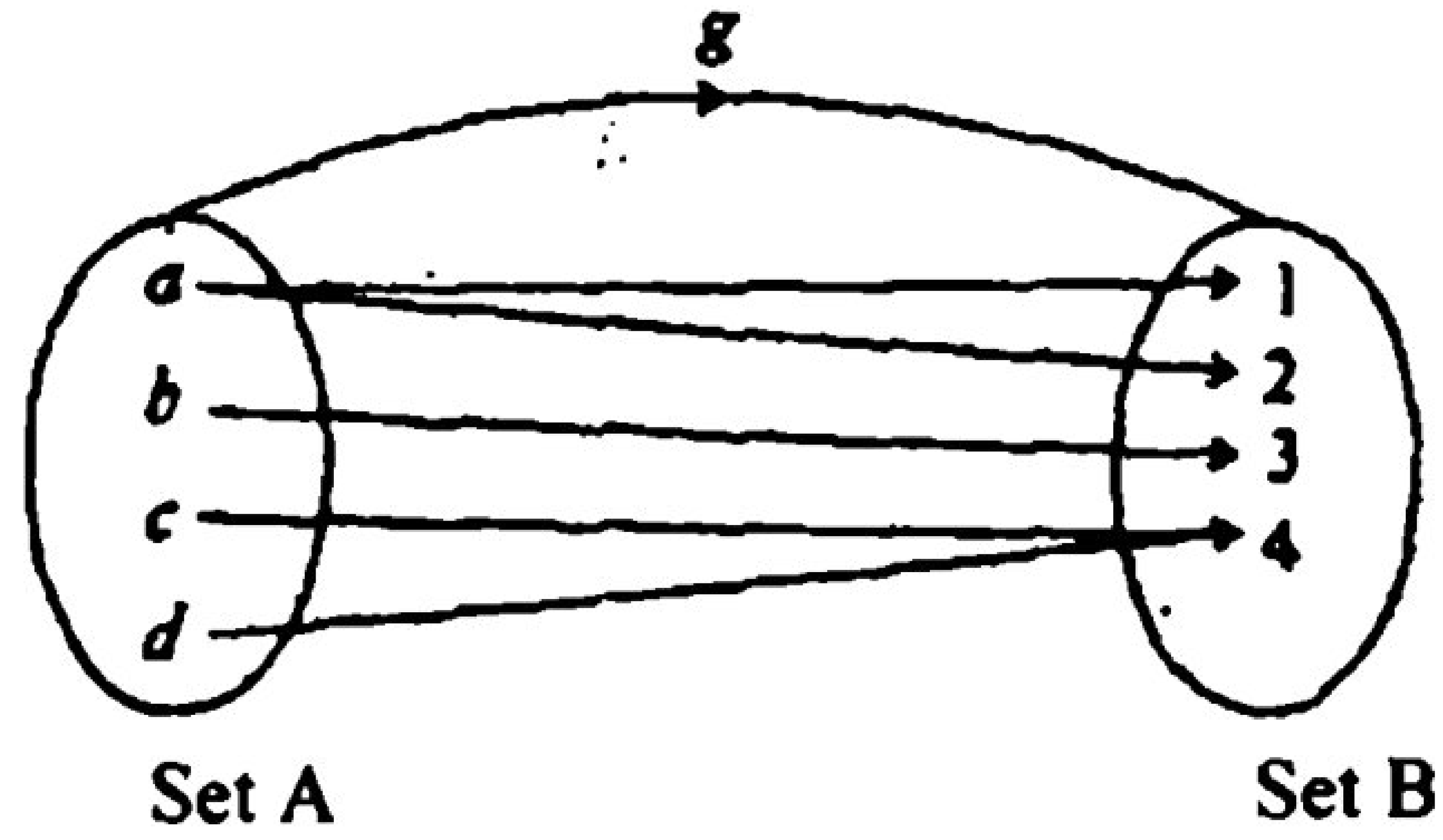


Set A Set B

The following are the examples of relations but not functions.

g is not a function, because an element a e A has two images in set B and A is not a function because an element d G A has no image in set B.



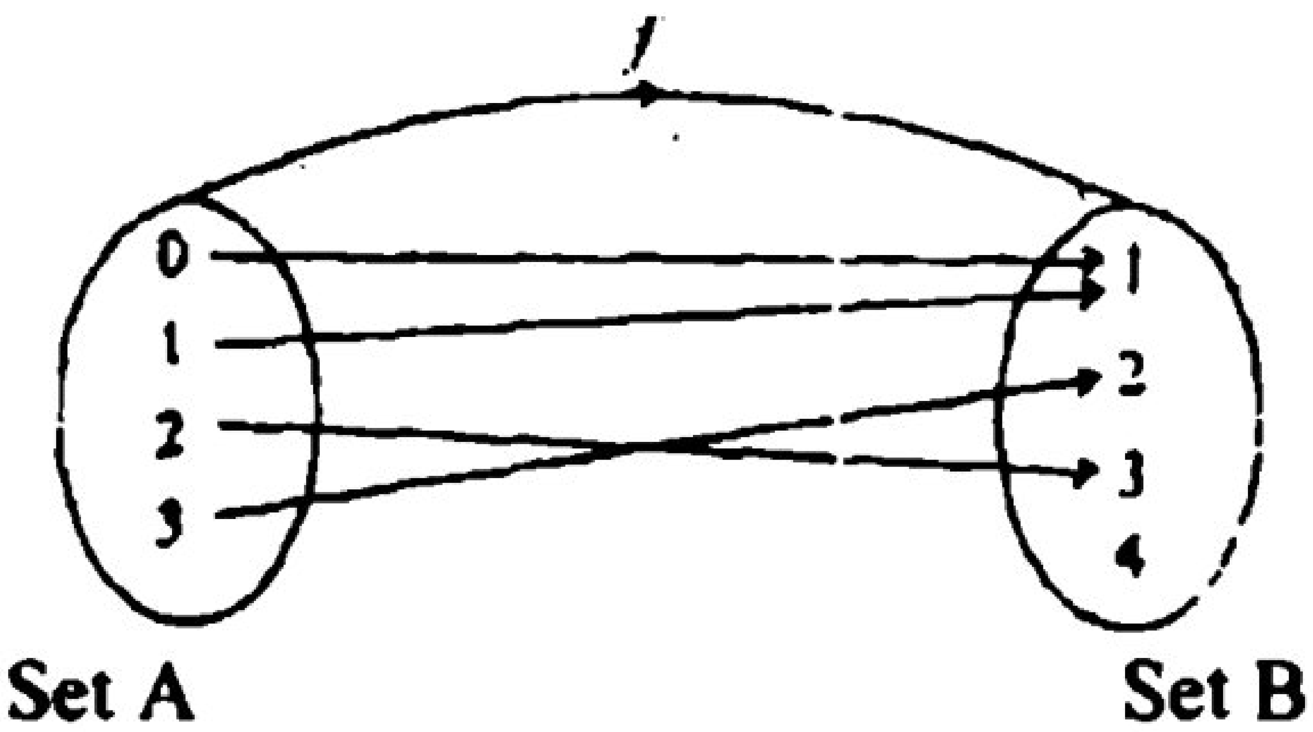


Demonstrate the following:

(a) Into function:

A function f: $A \rightarrow B$ is called an into function, if at least one element in B is not an image of some element of set A i.e.,

Range of $f \subset$ set R.



For example, we define a function $f : A \rightarrow S$ such that

$f = \{(0,1), (1,1), (2,3), (3, 2)\}$

where $A = \{0, 1, 2, 3\}$ and $B = \{1,2, 3, 4\}$ f is an into function.

(b) One-one function:

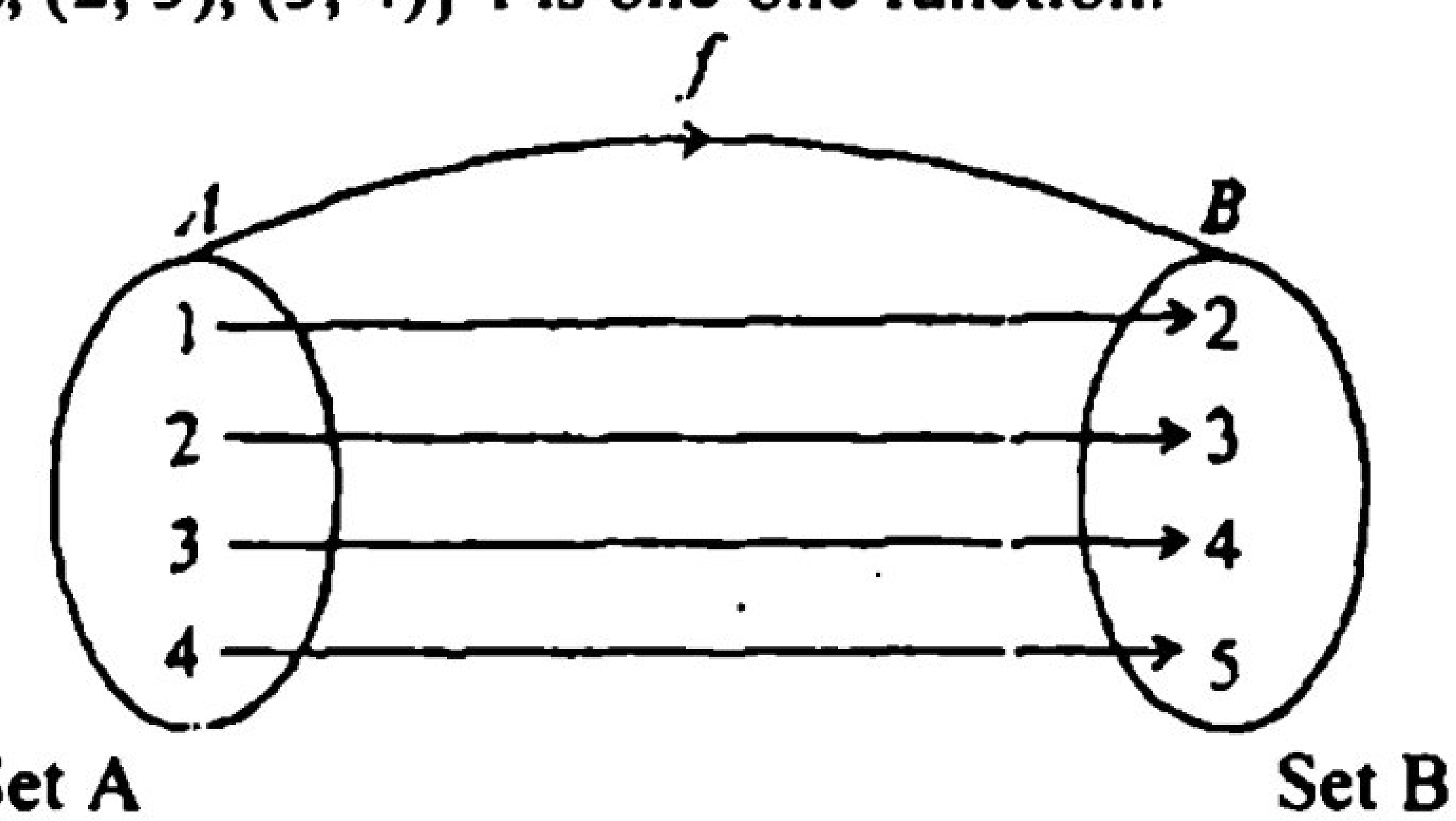
A function f: $A \rightarrow B$ is called one-one function, if all distinct elements of A. have distinct images in B, i. e., $f(x_1) = f(x_2) = \Rightarrow x_1 = x_2 \cong A$ or $\forall\, x_1 \neq x_2 \in A \Rightarrow f\{x_1\} \neq f\{x_2\}$

For example,, if $A = \{0, 1, 2, 3\}$

and $B = \{1, 2, 3, 4, 5\}$, then we define a function f: $A \rightarrow B$ such that

$f = \{(x, y) \mid y = x + 1, \forall\, x \in A, y \in B\}$.

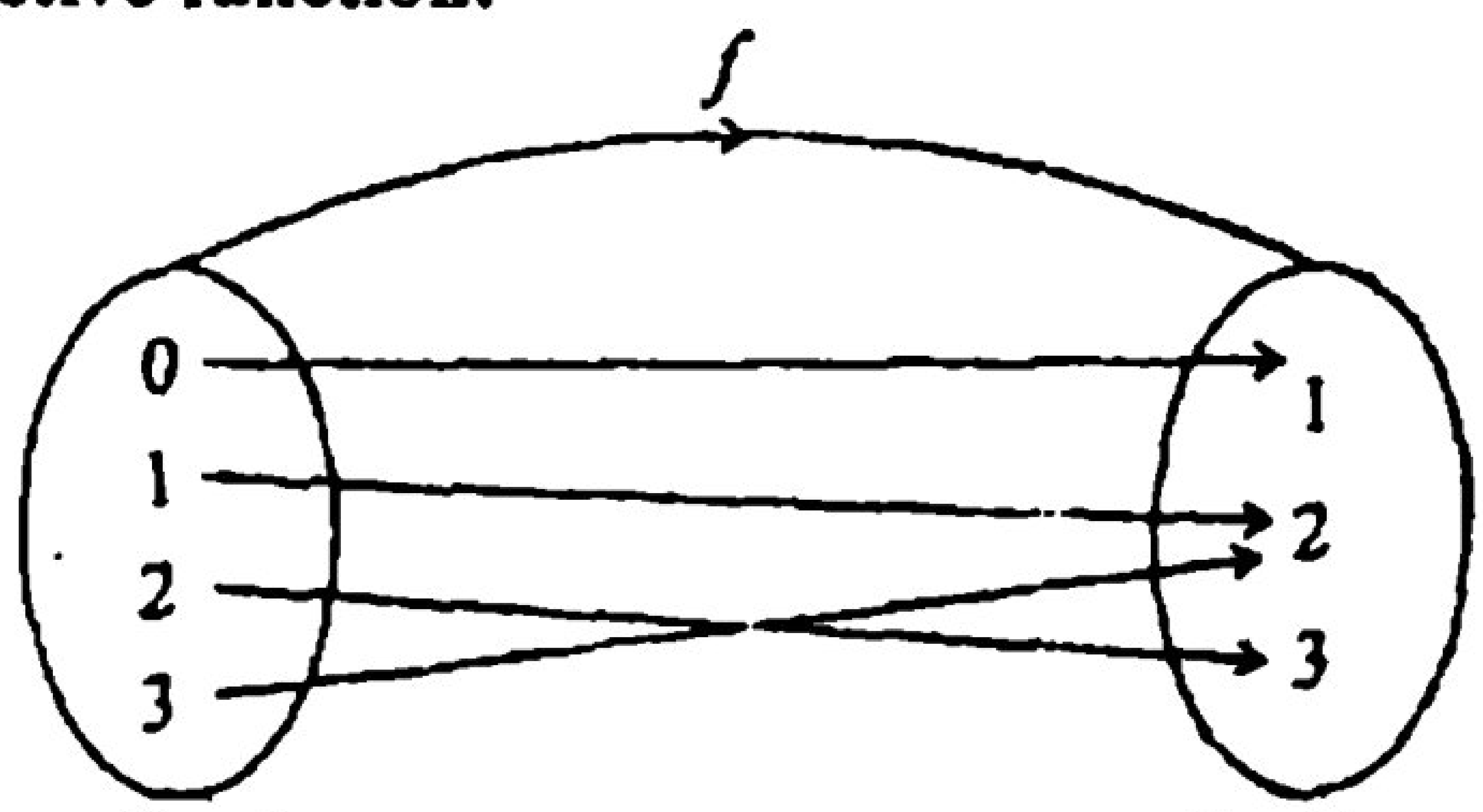
$= \{(0,1,), (1,2), (2, 3), (3, 4)\}$ f is one-one function.



(c) Into and one-one function: (injective function)

The function f discussed in (b) is also an into function. Thus f is an into and one-one function.

(d) An onto or surjective function:



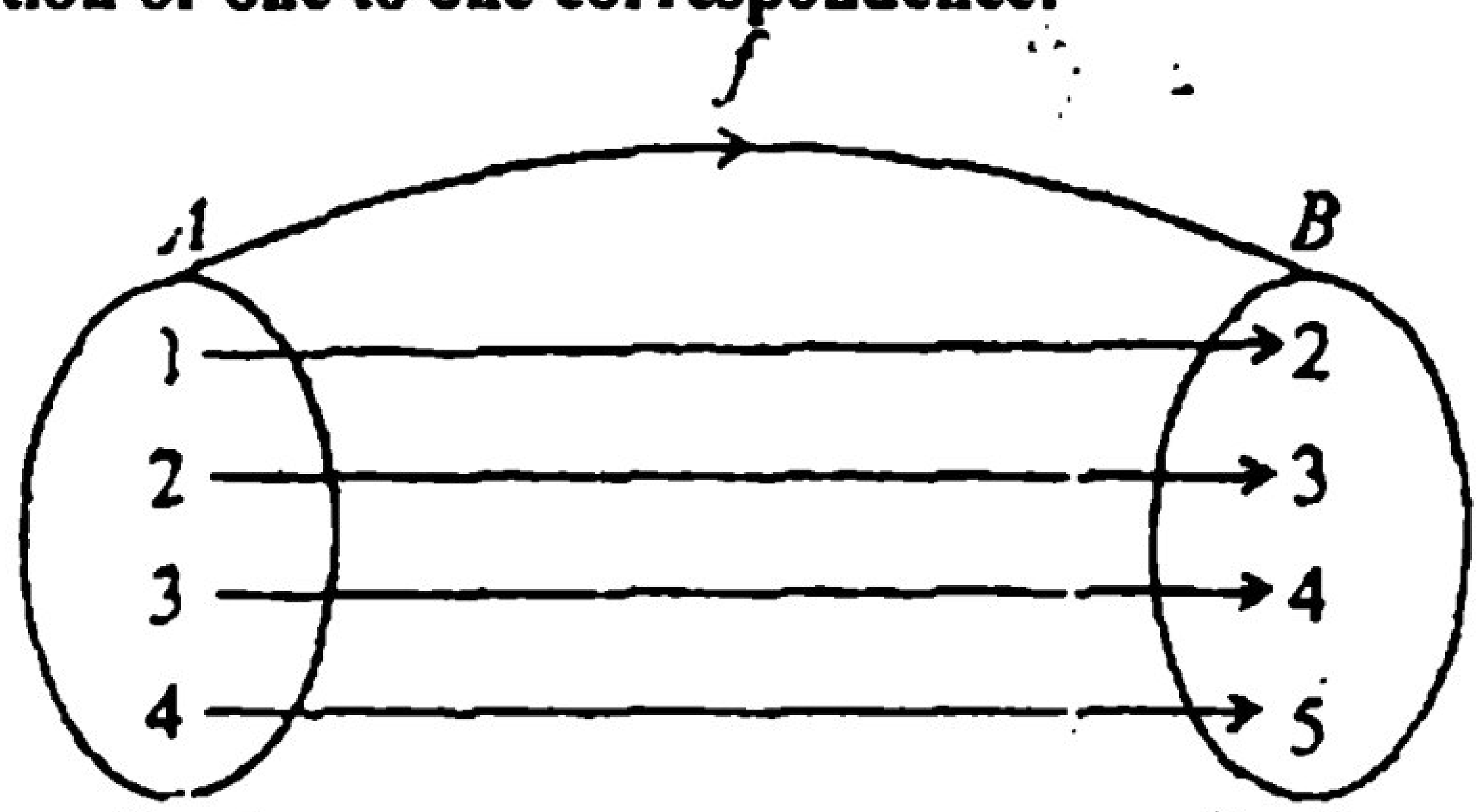
Set A Set B

A function f: A → B is called an onto function, if every element of set B is an image of at least one element of set A i.e. Range of f = B.

For example if A = {0, 1, 2, 3}
and B = {1, 2, 3}, then f: A → B such
that f = {(0, 1), (1, 2), (2, 3), (3, 2)}.

Here Rang f = {1,2, 3}= B.
Thus f so defined is an onto function.

(c) Bijcctive function or one to one correspondence:



Set A Set B

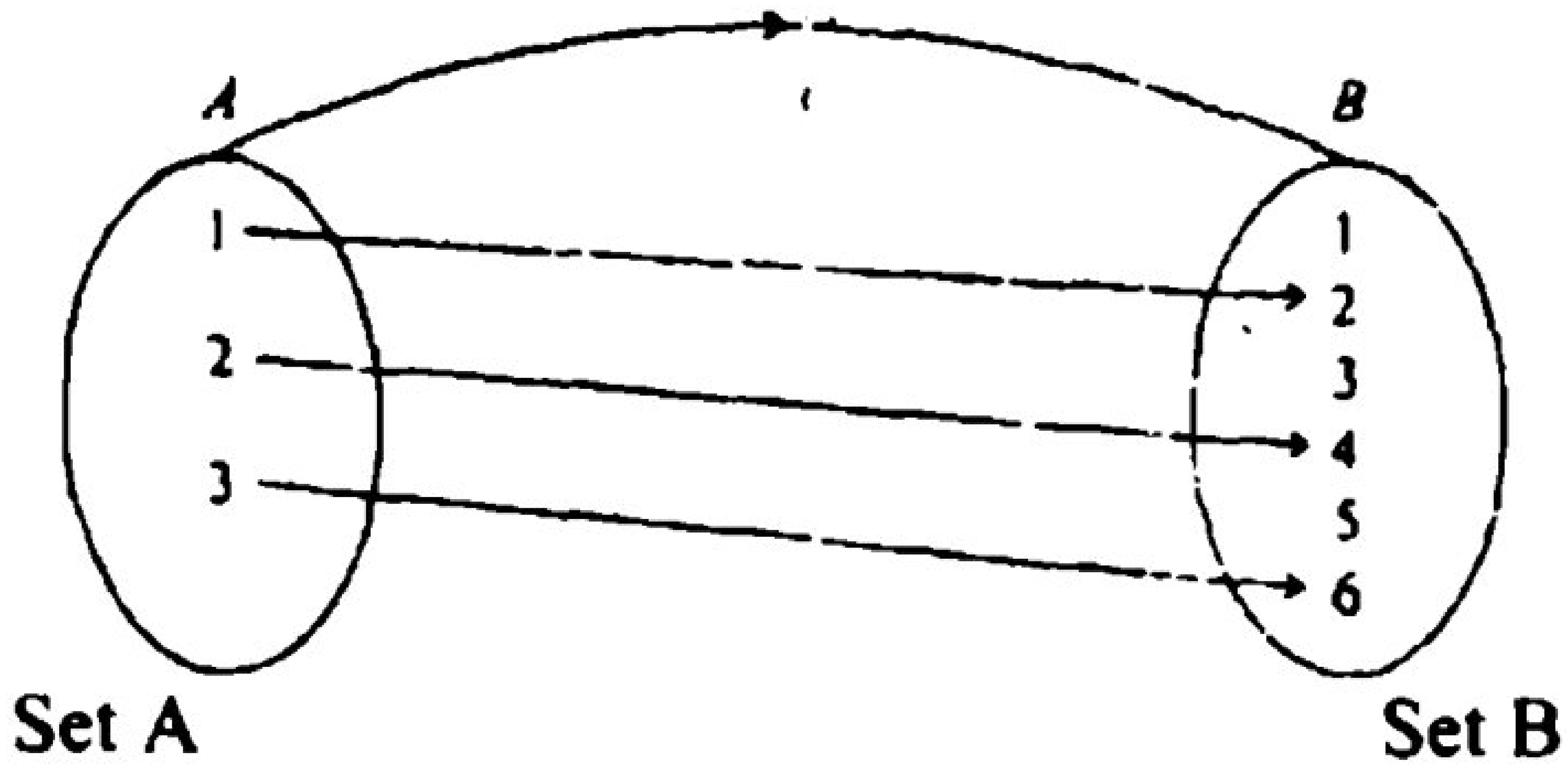
A function f: A, → B is called bijective function if f function f is one-one and onto.
For example, if A = {1, 2, 3, 4} and B ={2, 3, 4, 5}
We define a function f: A → B such that f = {(x, y) | y = x. + 1, ∀ x ∈ A, y ∈ B}
Then f = {(1, 2), (2, 3), (3, 4), (4, 5)}

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of.0 is the image of at least one element of A.

Note: (1) Even function is a relation but converse may not be true.
(2) Even function may not be one-one
(3) Every function may not be onto.

Example: Suppose A = {1,2,3}
B = (1,2,3,4,5,6}

We define a function f: A → B = {(x. y) | y = 2x, ∀ x ∈ A ,y ∈ B}
Then f ={(1, 2), (2, 4), (3, 6)}
Evidently this function is one-one but riot an onto.



Examine whether a given relation is a function:

A relation in which each x e its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one-one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. • The domain of/is A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n(A) = n(B).

SOLVED EXERCISE 5.5

1. If L = {a, b, c}, M = {3, 4}, then Find two binary relations of L × M and M × L.

Solution:

$$L = \{a,b,c\}, \quad \{3,4\}$$
$$L \times M = \{a,b,c\} \times \{3,4\}$$
$$= \{(a,3), (a,4),(b,3),(b,4),(c,3),(c,4)\}$$
$$\text{Then } R_1 = \{(a,3), (b,4),(c,3)\}$$
$$R_2 = \{(a,4), (b,3),(c,4)\}$$
$$= \{(3,a), (3,b),(3,c),(4,a),(4,b),(4,c)\}$$
$$\text{Here} \quad R_1 = \{(3,a), (4,a),(4,c)\}$$
$$R_2 = \{(3,b), (4,c)\}$$

2. If Y = {–2, 1, 2}, then make two binary relations for Y × Y. Also find their domain and range.

Solution:

$$Y = \{-2, 1, 2\}$$
$$Y \times Y = \{-2, 1, 2\} \times \{-2, 1, 2\}$$
$$= \{(-2, -2)\}, (-2,1),(-2,2), (1, -2), (1,1), (1,2), (2, -2), (2,1), (2,2)\}$$
$$R_1 = \{(-2, -2)\}, (-2,1), (1, 2), (2,2)$$
$$\text{Dom } R_1 = \{-2, 1, 2\}$$
$$\text{Dom } R_1 = \{-2, 1, 2\}$$
$$\text{Range } R_1 = \{-2, 1, 2\}$$
$$\text{and} \quad R_2 = \{(-2, 1), (1, 1),(-2, 2)\}$$
$$\text{Dom } R_2 = \{-2, 1\}$$