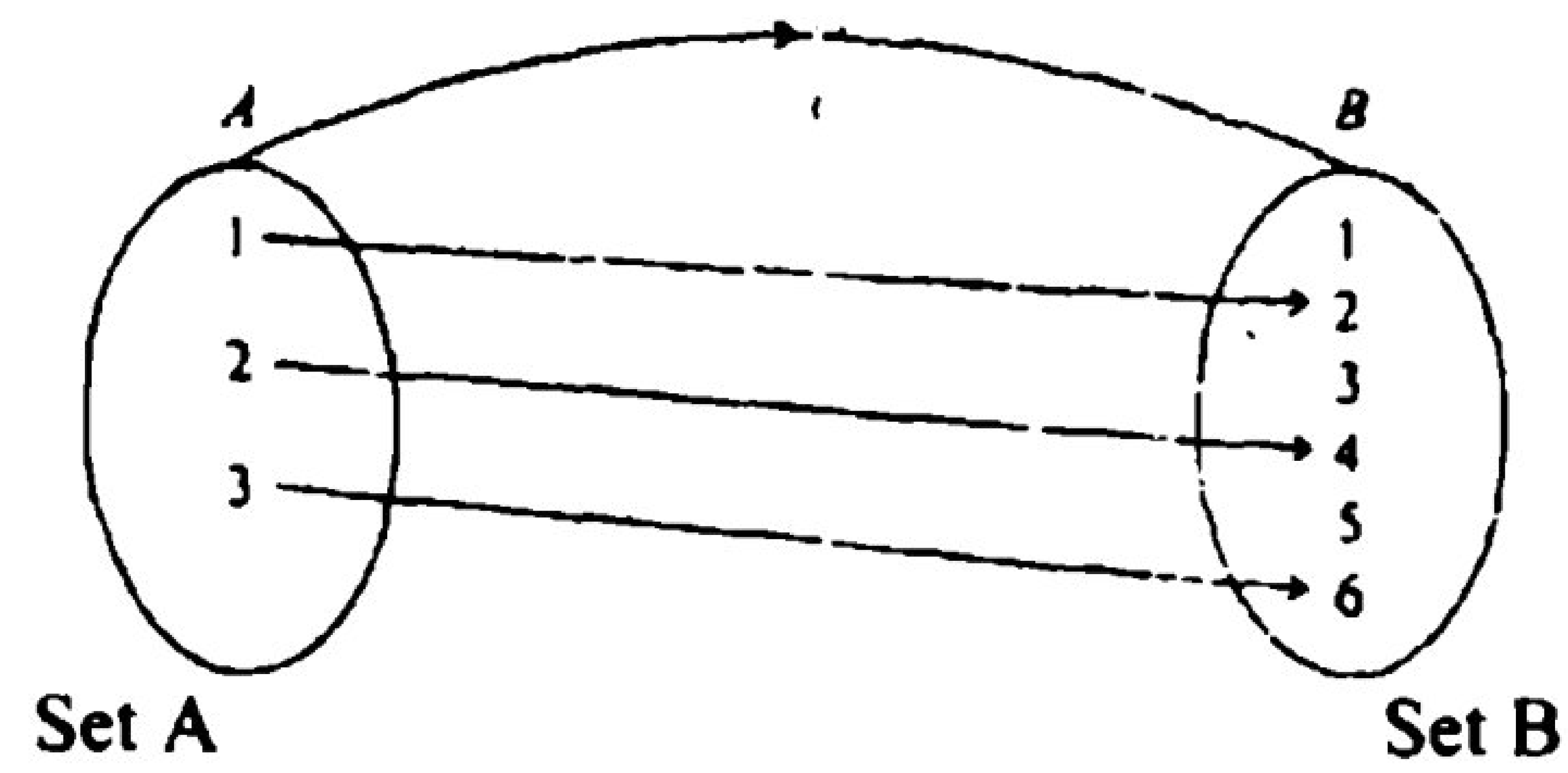

Examine whether a given relation is a function:

A relation in which each x e its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one-one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. • The domain of/is A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n(A) = n(B).

SOLVED EXERCISE 5.5

1. If L = {a, b, c}, M = {3, 4}, then Find two binary relations of L × M and M × L.

Solution:

$$
\begin{aligned}
L &= \{a,b,c\}, \quad \{3,4\} \\
L \times M &= \{a,b,c\} \times \quad \{3,4\} \\
&= \{(a,3), (a,4),(b,3),(b,4),(c,3),(c,4)\} \\
\text{Then } R_1 &= \{(a,3), (b,4),(c,3)\} \\
R_2 &= \{(a,4), (b,3),(c,4)\} \\
. &= \{(3,a), (3,b),(3,c),(4,a),(4,b),(4,c)\} \\
\text{Here} \quad R_1 &= \{(3,a), (4,a),(4,c)\} \\
R_2 &= \{(3,b), (4,c)\}
\end{aligned}
$$

2. If Y = {–2, 1, 2}, then make two binary relations for Y × Y. Also find their domain and range.

Solution:

$$
\begin{aligned}
Y &= \{-2, 1, 2\} \\
Y \times Y &= \{-2, 1, 2\} \times \{-2, 1, 2\} \\
&= \{(-2, -2)\}, (-2,1),(-2,2), (1, -2), (1,1), (1,2), (2, -2), \\
&\quad (2,1), (2,2)\} \\
R_1 &= \{(-2, -2)\}, (-2,1), (1, 2), (2,2) \\
\text{Dom } R_1 &= \{-2, 1, 2\} \\
\text{Dom } R_1 &= \{-2, 1, 2\} \\
\text{Range } R_1 &= \{-2, 1, 2\} \\
\text{and} \quad R_2 &= \{(-2, 1), (1, 1),(-2, 2)\} \\
\text{Dom } R_2 &= \{-2, 1\}
\end{aligned}
$$

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Range $R_2 = \{1,2\}$

3. If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each:
(i) L × L (ii) L × M (iii) M × M

Solution

(i) L × L

$$L = \{a,b,c\}$$
$$\text{Now } L \times L = \{a,b,c\} \times \{a,b,c\}$$
$$= \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$$
$$R_1 = \{(a, a)\}$$
$$R_2 = \{(a,b), (b,b), (c,b)\}$$

(ii) L × M

$$L = \{a,b,c\}$$
$$M = \{d,e,f,g\} \times \{a,b,c\}$$
$$\text{Now } L \times M = \{a,b,c\}, \{d,e,f,g\}$$
$$= \{(a,d), (a,e), (a,f), (a,g), (b,d), (b,e), (b,f), (b,g), (c,d), (c,e), (c,f), (c,g)\}$$
$$\text{Here } R_1 = \{(a,d), (b,f)\}$$
$$R_2 = \{(b,e), (b,g), (c,d), (c,e)\}$$

(iii) M × M

$$M = \{d,e,f,g\}$$
$$\text{Now } M \times M = \{d,e,f,g\} \times \{d,e,f,g\}$$
$$= \{(d,d), (d,e), (d,f), (d,g), (e,d), (e,e), (e,f), (e,g), (f,d), (f,e), (f,f), (f,g), (gd), (ge), (g,f), (g,g)\}$$
$$\text{Here } R_1 = \{(d,e), (d,f), (f,f)\}$$
$$R_2 = \{(d,f), (e,d), (e,e), (g,g)\}$$

4. If set M has 5 elements, then find the number of binary relations in M.

Solution:

Number of elements in M = 5
Number of elements in M = 5
Number of elements in M × M = $2^{5\times5} = 2^{25}$

5. If L = {x | x ∈ N ∧ x ≤ 5}, M = {y | y ∈ P ∧ y < 10}, then make the following relations from L to M.

(i) $R_1 = \{(x, y) \mid y < x\}$ (ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$ (iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Also write the domain and range of each relation.

(i) $R_1 = \{(x, y) \mid y < x\}$

Solution

$$R_1 = \{x \mid x \in N \wedge x \leq 5\}$$
$$\text{Thus, } L = \{1,2,3,4,5\}$$
and
$$M = \{y \mid y \in P \wedge y < 10\}$$
$$\text{Thus, } M = \{2,3,5,7\}$$

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Now $L \times M = \{1,2,3,4,5\}, \{2,3,5,7\}$

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5.5), (5,7)$

$R_1 = \{(x,y)|y < x\}$

$= \{(3,2) (4,2), (4,3),),(5,2), (5,3)\}$

$\text{Dom } R_1 = \{3, 4, 5\}$

$\text{Range } R_1 = \{2, 3\}$

$R_2 = \{(x,y)| y = x\}$

(ii) $\mathbf{R_2 = \{(x, y) \mid y = x\}}$

$R_2 = \{(2,2), (3,3), (5,5)\}$

$\text{Dom } R_2 = \{2,3,5\}$

$\text{Range } R_2 = \{2,3,5\}$

iii) $\mathbf{R_3 = \{(x, y\}| x + y = 6\}}$

$R_3 = \{(1,5), (3,3),(4,2)\}$

$\text{Dom } R_3 = \{1,3,4\}$

$\text{Range } R_3 = \{5,3,2\}$

iv) $\mathbf{R_4 = \{(x, y) \mid y - x = 2\}}$

$R_3 = \{(1,3), (3,5),(5,7)\}$

$\text{Dom } R_3 = \{1,3,5\}$

$\text{Range } R_3 = \{3,5,7\}$

Also write the domain and range of each relation.

6. Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and the range.

Solution:

(i) $\mathbf{R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}}$

Bijective

$\text{Dom } R_1 = \{1,2,3,4\}$

$\text{Range } R_1 = \{1,2,3,4\}$

(ii) $\mathbf{R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}}$

Relation

$\text{Dom } R_2 = \{1,2,3\}$

$\text{Range } R_2 = \{1,,2, 4, 5\}$

(iii) $\mathbf{R_3 = \{(b, a), (c, a), (d, a)\}}$

Function

$\text{Dom } R_3 = \{b,c,d\}$

$\text{Range } R_3 = \{a\}$

(iv) $\mathbf{R_4 = \{(1, 1), (2, 3), (3, 4), (5, 4)\}}$

On to function

$\text{Dom } R_4 = \{1,2,3,4,5\}$

$\text{Range } R_4 = \{1,3,4\}$

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(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

One-one function

Dom R_5 = {a,b,c,d}

Dom R_5 = {a,b,d,e}

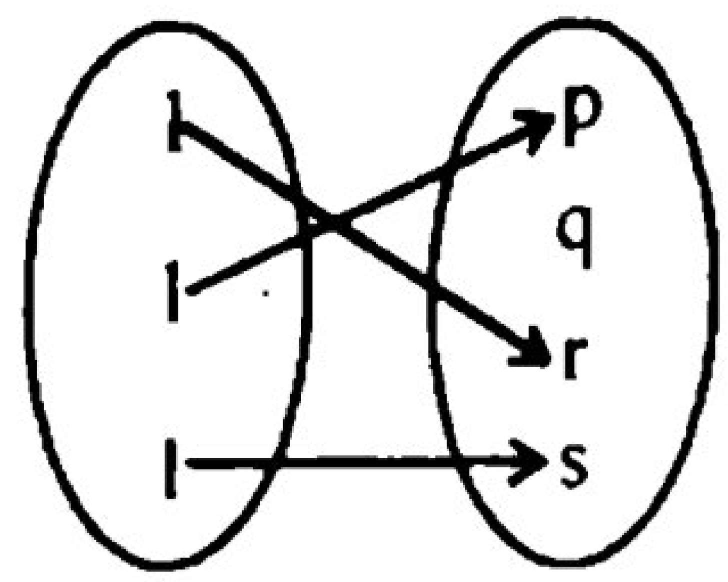
(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

Relation

Dom R_6 = {1,2,3}

Dom R_6 = {2,3,4}

(vii) R_7 =

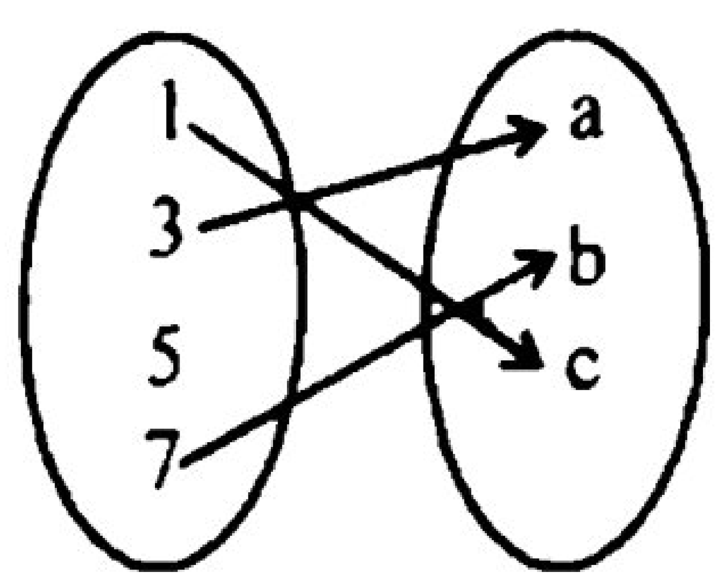


one-one function

Dom R_7 = {1,2,3}

Dom R_7 = {r,p,s}

(viii) R_8 =



Relation

Dom R_8 = {1,3,7}

Dom R_8 = {c,a,b}

MISCELLANEOUS EXERCISE - 5

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick mark (✓) the correct answer.

(i) A collection ^f well-defined distinct objects is called

(a) subset (b) power set

(c) set (d) none of these

(ii) A set $Q = \left\{\frac{a}{b} \mid a, b \in Z \wedge b \neq 0\right\}$ **is called a set of**

(a) Whole numbers (b) Natural numbers

(c) Irrational numbers (d) Rational numbers

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