$$\bar{x}_w = \frac{w_1x_1 + w_2x_2 + \ldots + x_nx_n}{w_1 + w_2 + \ldots + w_n} = \frac{\sum wx}{\sum w}$$

is called the weighted arithmetic mean.

Moving Averages:

Moving averages are defined as the successive averages (arithmetic means) which are computed for a sequence of days/months/years at a time. If we want to find 3-days moving average, we find the average of first 3-days, then dropping the first day and add the succeeding day to this group. Place the average of each 3-days against the mid of days.

SOLVED EXERCISE 6.2

1. What do you understand by measures of central tendency?

Solution:

The specific value of the variable around which the majority of the observations tend to concentrate is called the central tendency.

2. Define Arithmetic mean, Geometric mean, Harmonic mean, mode and median.

Solution:

(i) Arithmetic Means:

Mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number of observations.

$\bar{X} = \frac{\sum X}{n}$ (for ungrouped data) and $\bar{X} = \frac{\sum fX}{\sum f}$ (for grouped data)

(ii) Geometric Means:

Geometric mean of a variable x is the nth positive root of the product of the x_1, x_2, x_3, x_n observation. $G.M = (x_1 \times x_2 \times x_3 \ldots . x_n)^{1/n}$

(iii) Harmonic Means:

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of x_1, x_2, x_3, ...x_n observations.

$H.M = \frac{n}{\sum \frac{1}{x}}$ (for ungrouped data) and $H.M = \frac{n}{\sum \frac{f}{x}}$ (for grouped data)

(iv) Mode:

The most repeated value in an observation is called its mode.

(v) Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.

3. Find arithmetic mean by direct method for the following set of data:

(i) 12, 14, 17, 20, 24, 29, 35, 45.

(ii) 200, 225, 350, 375, 270, 320, 290.

Solution:

(i) $$A.M = \overline{X} = \frac{\sum x}{n} = \frac{12+14+17+20+24+29+35+45}{8}$$

$$= \frac{196}{8} = 24.5$$

(ii) $$A.M = \overline{X} = \frac{\sum x}{n} = \frac{200+225+350+375+270+320+290}{7}$$

$$= \frac{2030}{7} = 290$$

4. For each of the data in Q. No 3, compute arithmetic mean using indirect method.

Solution:

(i) Take any constant say 24 and take deviations from it (24).

A = 24

X	D = X − A
12	12 − 24 = − 12
14	14 − 24 = − 10
17	17 − 24 = − 7
20	20 − 24 = − 4
24	24 − 24 = 0
29	29 − 24 = 5
35	35 − 24 = 11
45	45 − 24 = 21
n = 8	ΣD = 4

$$\overline{X} = A + \frac{\sum D}{n}$$

$$= 24 + \frac{4}{8} = 24 + \frac{1}{2} = 24\frac{1}{2} = 24.5$$

(ii) Take any constant 270 and take deviations from it (270).

A = 270

X	D = X − A
200	200 − 270 = − 70
225	225 − 270 = − 45
350	350 − 270 = 80
375	375 − 270 = 105

270 320 290	270 – 270 = 0 320 – 270 = 50 290 – 270 = 20
n = 7	$\sum D = 140$

$$\overline{X} = A + \frac{\sum D}{n}$$

$$= 270 + \frac{140}{7} = 270 + 20 = 290$$

5. The marks obtained by students of class XI in mathematics are given below. Compute arithmetic mean by direct and indirect methods.

Classes / Groups	Frequency
0 — 9	2
10 — 19	10
20 — 29	5
30 — 39	9
40 — 49	6
50 — 59	7
60 — 69	1

Solution:

Direct Method:

Classes/groups	Mid-points (x)	f	fx
0 – 9	4.5	2	4.5 × 2 = 9.0
10 – 19	14.5	10	14.5 × 10 = 145.0
20 – 29	24.5	5	24.5 × 5 = 122.5
30 – 39	34.5	9	34.5 × 9 = 310.5
40 – 49	44.5	6	44.5 × 6 = 267.0
50 – 59	54.5	7	54.5 × 7 = 381.5
60 – 69	64.5	1	64.5 × 1 = 64.5
		$n = \sum f = 40$	1300

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{1300}{40} = 32.5$$

Indirect, short cut method:

Let A = 34.5

Classes/ groups	f	Mid-point (x)	D = X – A	$U = \frac{D}{10}$	fD	$f(U) = \frac{f(D)}{3}$
0 – 9	2	4.5	4.5 – 34.5 = – 30	–3	–60	–6
10 – 19	10	14.5	14.5 – 34.5 = – 20	–2	–200	–20
20 – 29	5	24.5	24.5 – 34.5 = – 10	–1	–50	–5

30 – 39	9	34.5	34.5 – 34.5 = 0	0	0	0
40 – 49	6	44.5	44.5 – 34.5 = 10	1	60	6
50 – 59	7	54.5	54.5 – 34.5 = 20	2	140	14
60 – 69	1	64.5	64.5 – 34.5 = 30	3	30	3
Total	40				–80	–8

$$\bar{X} = A + \frac{\sum fD}{\sum f} \quad \text{or} \quad \bar{X} = A + \frac{\sum f(U)}{\sum f} \times h$$

$$= 34.5 + \frac{(-80)}{40} \qquad = 34.5 + \frac{-8}{40} \times h$$

$$= 34.5 - 2 \qquad = 34.5 + \frac{-8}{40} \times 10$$

$$= 32.5 \qquad = 34.5 - 2 = 32.5$$

6. **The following data relates to the ages of children in a school. Compute the mean age by direct and short-cut method taking any provisional mean.**
(Hint. Take A = 8)

Class limits	Frequency
4—6	10
7—9	20
10—12	13
13—15	7
Total	50

Also Compute Geometric mean and Harmonic mean.

Solution:

Class Limits	Mid points (x)	f	f x
4 – 6	5	10	5 × 10 = 50
7 – 9	8	20	8 × 20 = 160
10 – 12	11	13	11 × 13 = 143
13 – 15	14	7	14 × 7 = 98
Total	$\sum f = 50$		$\sum fx = 451$

$$A.M = \frac{\sum fx}{\sum f} = \frac{451}{50} = 9.02$$

Indirect, short cut method:

Let A = 11

Classes/ groups	f	Mid-point (x)	D = X – A	$U = \frac{D}{3}$	fD	$fU = \frac{fD}{3}$
4 – 6	10	5	5 – 11 = – 6	– 2	–60	–20
7 – 9	20	8	8 – 11 = – 3	–1	–60	–20

10 – 12	13	11	11 – 11 = 0	0	0	0
13 – 15	7	14	14 – 11 = 3	1	21	7
	50				–99	–33

$$\bar{X} = A + \frac{\sum fD}{\sum f} \quad \text{or} \quad \bar{X} = A + \frac{\sum fU}{\sum f} \times h$$

$$= 11 - \frac{99}{50} \qquad = 11 + \frac{33}{50} \times 3$$

$$= 11 - 1.98 \qquad = 11 - \frac{99}{50}$$

$$= 9.02 \qquad = 11 - 1.98 = 9.02$$

Geometric Mean:

We Proceed as follows:

Class limits	f	Mid points x	Log x	f log x
4 – 6	10	5	0.69897	6.9897
7 – 9	20	8	0.90309	18.0618
10 – 12	13	11	1.04139	13.53807
13 – 15	7	14	1.14613	8.02291
$\sum f = 50$			$\sum f \log x$	= 46.61248

$$G.M = \text{Anti log}\left(\frac{\sum f \log x}{\sum f}\right) = \text{Anti log}\left(\frac{46.61248}{50}\right)$$

$$= \text{Anti log}(0.9322496) = 8.553$$

Harmonic Means

Class limits	f	Mid points x	$\frac{f}{x}$
4 – 6	10	5	$\frac{10}{5} = 2.0$
7 – 9	20	8	$\frac{20}{8} = 2.5$
10 – 12	13	11	$\frac{13}{11} = 1.18$
13 – 15	7	14	$\frac{7}{14} = 0.50$
	$\sum f = 50$		$\sum \frac{f}{x} = 6.18$

$$H.M = \frac{\sum f}{\sum \frac{f}{x}} = \frac{50}{6.18} = 8.09$$

7. The following data shows the number of children in various families. Find mode and median.

9, 11, 4, 5, 6, 8. 4, 3, 7, 8, 5, 5, 8, 3. 4, 9, 12, 8, 9, 10, 6,1, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10,9,7,6,9,5.

Solution:

Writing the observations in ascending order

2, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 11, 11, 12.

Mode:

The most frequent observation = 9, 4

Number of observation = 38

Therefore, median is the mean of 19^{th} and 20^{th} observation = $\frac{7+7}{2} = 7$

8. Find Modal number of heads for the following distribution showing the number of heads when 5 coins are tossed. Also determine median.

X (number of heads)	Frequency (number of times)
1	3
2	8
3	5
4	3
5	1

Solution:

Mode:

The most frequent observation = 2

For median, we make cumulative frequency column.

x	Frequency	Cumulative frequency
1	3	3
2	8	3 + 8 = 11
3	5	11 + 5 = 16
4	3	16 + 3 = 19
5	1	19 + 1 = 20

Median = the class containing $\left(\frac{n}{2}\right)^{th}$ observation.

= the class containing $\left(\frac{20}{2}\right)^{th}$ observation.

= the class containing $(10)^{th}$ observation.

Median = 2

9. The following frequency distribution the weights of boys in kilogram. Compute mean, median, mode.

Class Intervals	Frequency
1—3	2
4—6	3
7—9	5
10—12	4
13—15	6
16—18	2
19—21	1

Solution:

C.I	f	Mid Points (x)	fx	Class Boundaries	Cumulative frequency
1 – 3	2	2	4	0.5 – 3.5	2
4 – 6	3	5	15	3.5 – 6.5	2 + 3 = 5
7 – 9	5	8	40	6.5 – 9.5	5 + 5 = 10
10 – 12	4	11	44	9.5 – 12.5	10 + 4 = 14
13 – 15	6	14	84	12.5 – 15.5	14 + 6 = 20
16 – 18	2	17	34	15.5 – 18.5	20 + 2 = 22
19 – 21	1	20	20	18.5 – 21.5	22 + 1 = 23
	23		241		

$$\text{Mean} = \overline{X} = \frac{\sum fx}{\sum f} = \frac{241}{23} = 10.478$$

Median:

Median class = class containing $\left(\frac{n}{2}\right)^{th}$ observation.

$$= \left(\frac{23}{2}\right)^{th} = (11.5)^{th} \text{ observation.}$$

Median class is 9.5 – 12.5

Here $l = 9.5$, $c = 10$, $f = 4$, $h = 3$

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 9.5 + \frac{3}{4}\left(\frac{23}{2} - 10\right) = 9.5 + \frac{3}{4}\left(\frac{3}{2}\right) = 9.5 + \frac{9}{8} = 9.5 + 1.125 = 10.625$$

Mode: $\text{Mode} = \ell + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$

Here $l = 12.5$, $f_m = 6$, $f_1 = 4$, $f_2 = 2$, $h = 3$

$$\therefore \text{Mode} = 12.5 + \frac{6 - 4}{2(6) - 4 - 2} \times 3 = 12.5 + \frac{2}{6} \times 3 = 12.5 + 1 = 13.5$$

10. A student obtained the following marks at a certain examination: English 73, Urdu 82, Mathematics 80, History 67 and Science 62.

(i) If the weights accorded these marks are 4, 3, 3, 2 and 2, respectively, what is an appropriate average mark?

(ii) What is the average mark if equal weights are used?

Solution:

Marks (x)	Weight (w)	xw
73	4	$73 \times 4 = 292$
82	3	$82 \times 3 = 246$
80	3	$80 \times 3 = 240$
67	2	$67 \times 2 = 134$
62	2	$62 \times 2 = 124$
$\sum x = 364$	$\sum w = 14$	$\sum xw = 1036$

(i) $\bar{X}_w = \frac{\sum Xw}{\sum w} = \frac{1036}{14} = 74$

(ii) $\bar{X} = \frac{\sum x}{n} = \frac{364}{5} = 72.8$

11. On a vacation trip a family bought 21.3 liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter. Find the mean price paid per liter.

Solution:

X	W	XW
21.3	39.90	(21.3) (39.90) = 849.87
18.7	42.90	(21.3) (39.90) = 849.87
23.5	40.90	(21.3) (39.90) = 849.87
$\sum x = 63.5$		$\sum xW = 2613.25$

Mean price $= \frac{\sum XW}{\sum X} = \frac{2613.25}{63.5} = 41.15$ rupees per liter.

12. Calculate simple moving average of 3 years from the following data;

Years	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
Values	**102**	**108**	**130**	**140**	**1 158**	**180**	**196**	**210**	**220**	**230**

Solution:

Years	Value	3 – years moving Total	3 – years moving Average
2001	102		–
2002	108	340	340/3 = 113.33

2003	130	378	378/3 = 126.00
2004	140	428	428/3 = 142.67
2005	158	478	478/3 = 159.33
2006	180	534	534/3 = 178.00
2007	196	586	586/3 = 195.33
2008	210	626	626/3 = 208.67
2009	220	660	660/3 = 220.00
2010	230	–	

13. Determine graphically for the following data and check your answer by using formulae.

(i) Median and Quartiles using cumulative frequency polygon.

(ii) Mode using Histogram.

Class Boundaries	Frequency
10—20	2
20—30	5
30—40	9
40—50	6
60—70	1

Solution:

Class Boundaries	f	c. f	
10 – 20	2	2	
20 – 30	5	7	
30 – 40	9	16	Median class.
40 – 50	6	22	Q_3 class
50 – 60	4	26	
60 – 70	1	27	
	$\sum f = 27$		

Median class $= \left(\frac{n}{2}\right)^{th}$ obsrvation $= \left(\frac{27}{2}\right)^{th} = (13.5)^{th}$ obsrvation.

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Here $l = 30$, $h = 10$, $f = 9$, $n = 27$, $c = 7$

Thus median $x = 30 + \frac{10}{9}\left(\frac{27}{2} - 7\right) = 30 + \frac{10}{9}\left(\frac{13}{2}\right) = 30 + 7.22 = 37.22$

To find Q_3

We have to find $3\left(\frac{n}{4}\right)^{th}$ observation.

Q_3 Class $= 3\left(\frac{n}{4}\right)^{th}$ observation $= 3\left(\frac{27}{4}\right)^{th}$ observation

$= 3\ (6.75)^{th}$ observation $= (20.25)^{th}$ observation.

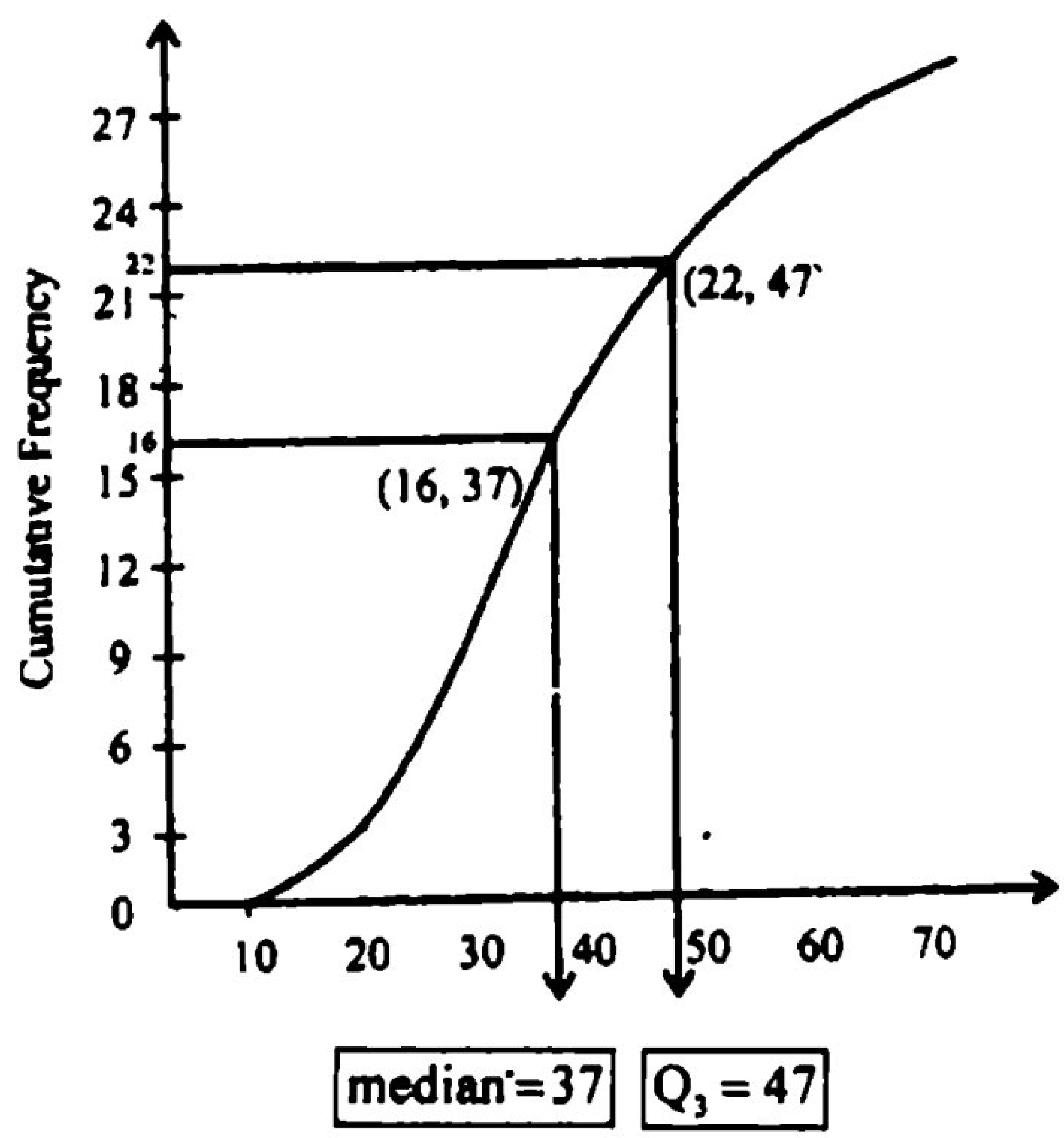
Q_3 Class is 40 – 50

Now $\quad Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - c\right)$

Here, $\quad l = 40, h = 10, f = 6, n = 27, c = 16$

$$Q_3 = 40 + \frac{10}{6}\left(\frac{3 \times 27}{4} - 16\right) = 40 + \frac{10}{6}(20.25 - 16)$$

$$= 40 + \frac{10}{6}(4.25) = 40 + 7.08 = 47.08$$



Measures of Dispersion:

Statistically, Dispersion means the spread or scatterness of observations in a data set. The spread or scatterness in a data set can be seen in two ways:

(i) The spread between two extreme observations in a data set.

(ii) The spread of observations around an average say their arithmetic mean.

The purpose of finding Dispersion is to study the behavior of each unit of population around the average value. This also helps in comparing two sets of data in more detail.

The measures that are used to determine the degree or extent of variation in a data set are called Measures of Dispersion.

We shall discuss only some important absolute measures of dispersion now.

(i) Range:

Range measures the extent of variation between two extreme observations of a data set.

It is given by the formula:
Range = $X_{max} - X_{min} = X_m - X_0$
where $X_{max} = X_m$ = the maximum, highest or largest observation.
$X_{min} = X_o$ = the minimum lowest or smallest observation.
The formula to find range for grouped continuous data is given below:
Range = (Upper class boundary of last group) – (lower class boundary of first group).

(ii) Variance:
Variance is defined as the mean of the squared deviations of x_i (i = , 2, ... n) observations from their arithmetic mean. In symbols,

$$\text{Variance of } X = Var(X) = S^2 = \frac{\Sigma(X-\bar{X})^2}{n}$$

(iii) Standard Deviation:
Standard deviation is defined as the positive square root of mean of the squared deviations of X_i (i = 1, 2, ... n) observations from their arithmetic mean. In symbols we write.

$$\text{Standard Deviation of } X = S.D(X) = S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$$

Computation of Variance and Standard Deviation:
We use the following formulae to compute Variance and Standard Deviation for Ungrouped and Grouped Data.

Ungrouped Data
The formula of Variance is given by:

$$Var(X) = S^2 = \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2$$

And Standard deviation is given by:

$$S.D(X) = S = \sqrt{\left[\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2\right]}$$

SOLVED EXERCISE 6.3

1. What do you understand by Dispersion?

Solution:

Dispersion:
Dispersion means the spread or scatterness of observations in a data set. By dispersion we mean the extent to which the observations in a sample or in a population are spread out.
The main measures of dispersion are range, variance and standard deviation.

2. How do you define measure of dispersion?