$$\frac{m+5n}{m-5n}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5n}{m-5n}+\frac{3n+p}{n-p} \quad \text{\_\_\_\_\_\_\_\_(ii)}$$

Adding eq. (i) and eq. (ii), we get.

$$\frac{m+5n}{m-5n}=\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{2n+p}{n-p}$$

$$=\frac{3p+n}{p-n}+\frac{2n+p}{-(p-n)}$$

$$=\frac{3p+n}{p-n}=\frac{2n+p}{p-n}$$

$$=\frac{(3p+n)-(3n+p)}{p-n}$$

$$=\frac{3p+n-3n-p}{p-n}$$

$$=\frac{2p-2n}{p-n}$$

$$=\frac{2(p-n)}{p-n}=2$$

(iii) Find the value of $\frac{x-6a}{x+6a}-\frac{x-6b}{x+6b}$, if $x=\frac{12ab}{a-b}$

Solution

Given $x=\frac{12ab}{a-b}$ or $x=\frac{(6a)(2b)}{a-b}$

or $\frac{x}{6a}=\frac{2b}{a-b}$

Applying componendo-dividendo theorem, we get.

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$

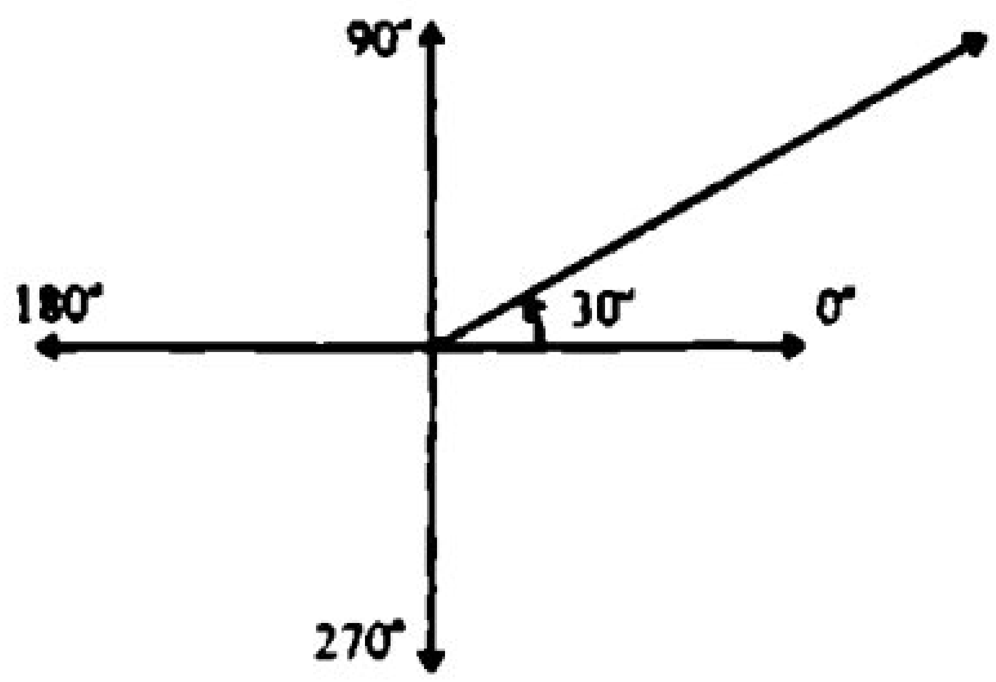
$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$

$$\frac{x+6a}{x-6a}=\frac{a+b}{3b-a}$$

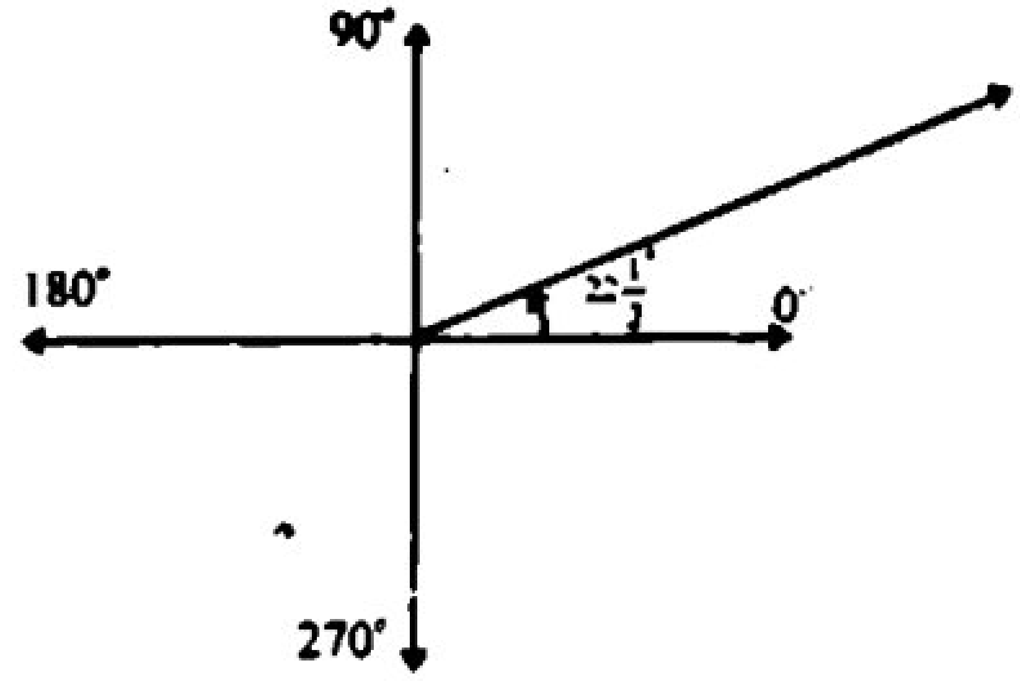
SOLVED EXERCISE 7.1

1. Locate the following angles:

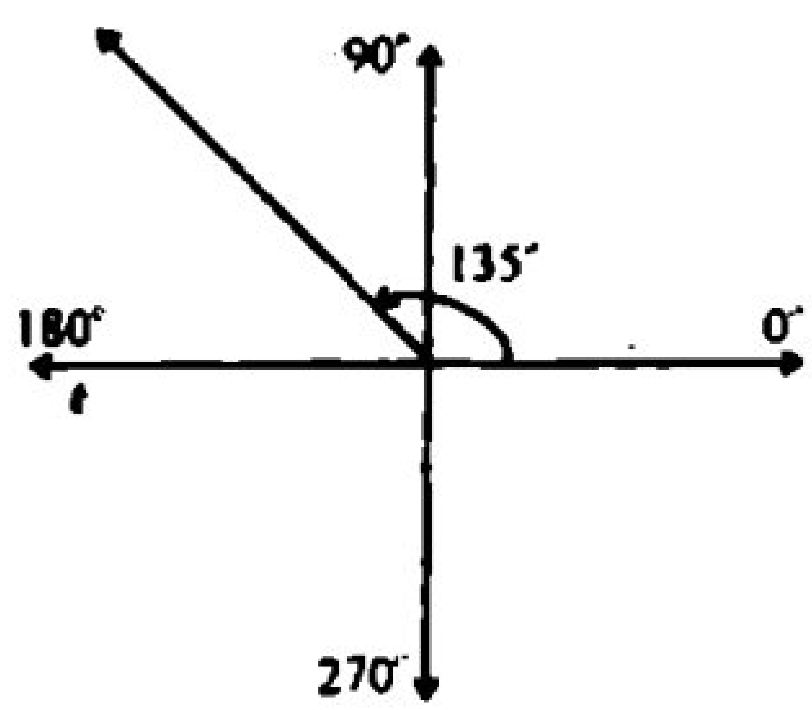
(i) 30°



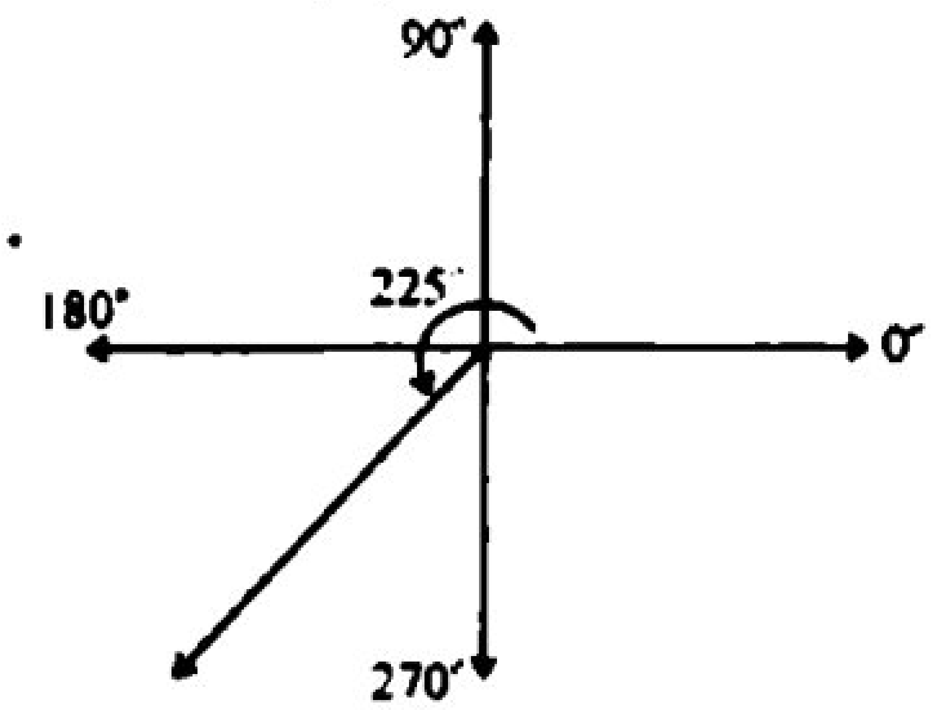
(ii) $22\frac{1}{2}^{\circ}$



(iii) 135°

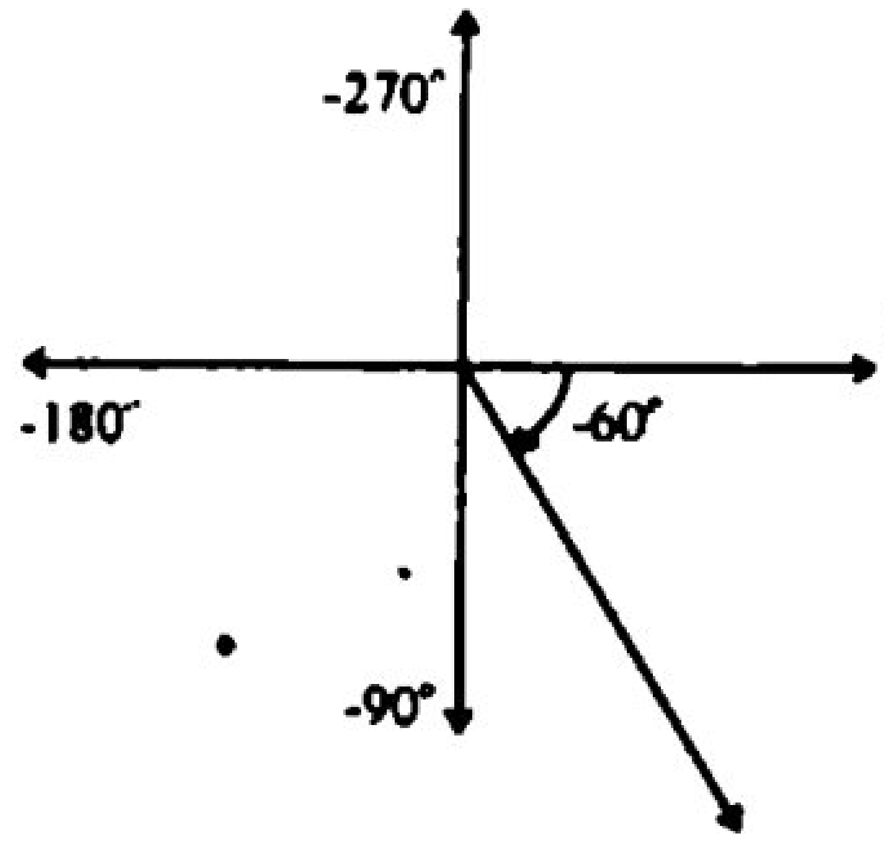


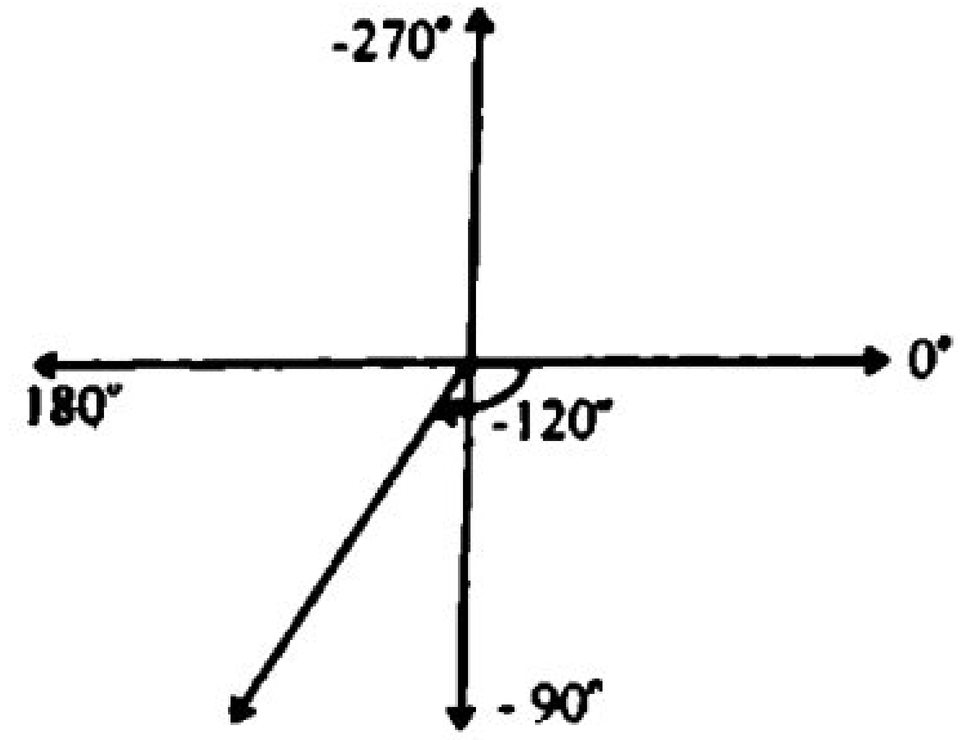
(iv) 225°



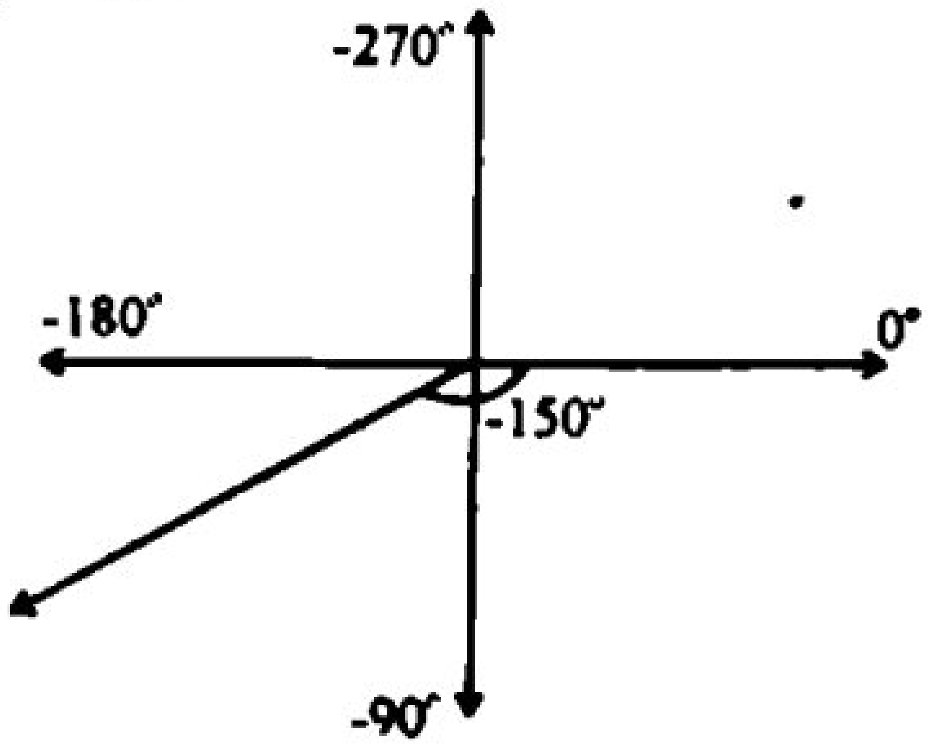
(v) – 60°

(vi) – 120°

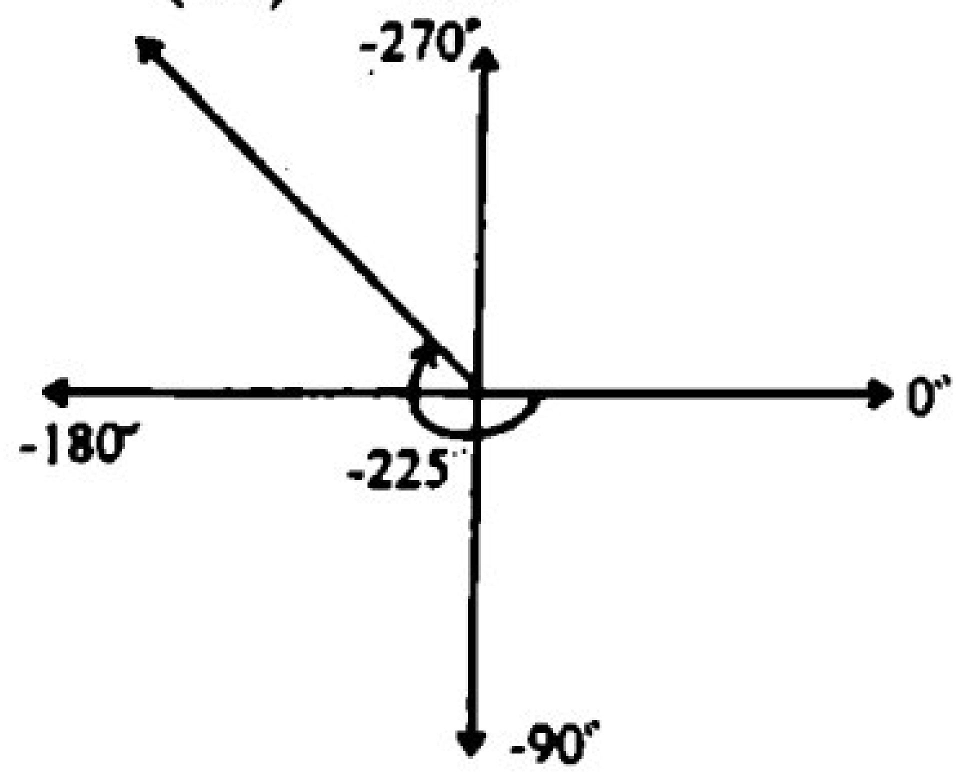




(vii) – 150°



(viii) – 225°



2. Express the following sexagesimal measures of angles in decimal form.

(i) 45° 30'

Solution

$$45^\circ\ 30' = 45^\circ + \left(\frac{30}{60}\right)^\circ = 45^\circ + 0.5^\circ = 45.5^\circ$$

(ii) 60°30'30"

Solution:

$$60^\circ\ 30'\ 30'' = 60^\circ + \left(\frac{30}{60}\right)^\circ + \left(\frac{30}{60\times 60}\right)^\circ$$

$$= 60^\circ + \frac{30}{60}^\circ + \frac{30}{3600}^\circ$$

$$= 60^\circ + 0.5^\circ + 0.0083^\circ$$

$$= 60.5083^\circ$$

(iii) 125° 22'50"

Solution

$$125^\circ\ 22'\ 50'' = 125^\circ + \left(\frac{22}{60}\right)^\circ + \left(\frac{50}{60\times 60}\right)^\circ$$

$$= 125^\circ + \frac{22}{60}^\circ + \frac{50}{3600}^\circ$$

$$= 125^\circ + 0.3667^\circ + 0.0139^\circ$$

$$= 125.3806^\circ$$

3. Express the following into D° M' S". form.

(i) 47.36°

Solution

$$47.36^\circ = 47^\circ + 0.36^\circ$$

$$= 47^\circ + \left(\frac{36}{100}\right)^\circ$$

$$= 47^\circ + \frac{9}{25}^\circ$$

$$= 47^\circ + \left(\frac{9}{25}\times 60\right)'$$

$$= 47^\circ + 21.6'$$

$$= 47^\circ + 21' + (0.6\times 60)''$$

$$= 47^\circ + 21' + 36''$$

$$= 47^\circ 21' 36''$$

(ii) 125.45°

Solution

$$125.45^\circ = 125^\circ + 0.45^\circ$$

$$= 125^\circ + \left(\frac{45}{100}\right)^\circ$$

$$= 125^\circ + \frac{9}{20}^\circ$$

$$= 125^\circ + \left(\frac{9}{20}\times 60\right)'$$

$$= 125^\circ + 27'$$

$$= 125^\circ 27'$$

(iii) 225.75°

Solution

$$225.75^\circ = 225^\circ + 0.75^\circ$$

$$= 225^\circ + \left(\frac{75}{100}\right)^\circ$$

$$= 225^\circ + \frac{3}{4}^\circ$$

$$= 225^\circ + \left(\frac{3}{4}\times 60\right)'$$

$$= 225^\circ + 45'$$

$$= 225^\circ 45'$$

(iv) − 22.5°

Solution

$$-22.50^\circ = -22^\circ - 0.5^\circ$$

$$= -22^\circ - \left(\frac{5}{10}\right)^\circ$$

$$= -22^\circ - \left(\frac{5}{10}\times 60\right)'$$

$$= -22^\circ - 30'$$

$$= -22^\circ 30'$$

(v) − 67.58°

Solution

$$-67.58^\circ = -67^\circ - 0.58^\circ$$

$$= -67^\circ - \left(\frac{58}{100}\right)^\circ$$

$$= -67^\circ - \left(\frac{58}{100}\times 60\right)'$$

$$= -67^\circ - 34.8'$$

$$= -67^\circ - 34' + (0.8\times 60)''$$

$$= -67^\circ - 34' + 48''$$

$$= -67^\circ 34' 48''$$

(vi) 315.18°

Solution

$$315^\circ.18^\circ = 315^\circ + 0.18^\circ$$

$$= 315^\circ + \left(\frac{18}{100}\right)^\circ$$

$$= 315^\circ + \left(\frac{18}{100} \times 60\right)'$$

$$= 315^\circ + 10.8'$$

$$= 315^\circ + 10' + (0.8 \times 60)''$$

$$= 315^\circ + 10' + 48''$$

$$= 315^\circ 10' 48''$$

4. Express the following angles into radians.

(i) 30°

Solution

$$30^\circ = 30 \times 1^\circ = 30 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= \frac{\pi}{6} \text{ radians}$$

(ii) (60)°

Solution

$$60^\circ = 60 \times 1^\circ = 60 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= \frac{\pi}{3} \text{ radians}$$

(iii) 135°

Solution

$$135^\circ = 135 \times 1^\circ = 135 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= \frac{3\pi}{4} \text{ radians}$$

(iv) 225°

Solution

$$225^\circ = 225 \times 1^\circ = 225 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= \frac{5\pi}{4} \text{ radians}$$

(v) -150°

Solution

$$-150^\circ = -150 \times 1^\circ = -150 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= -\frac{5\pi}{6} \text{ radians}$$

(vi) -225°

Solution

$$-225^\circ = -225 \times 1^\circ = -225 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= -\frac{5\pi}{4} \text{ radians}$$

(vii) 300°

Solution:

$$300^\circ = 300 \times 1^\circ = 300 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= \frac{5\pi}{3} \text{ radians}$$

(viii) 315°

Solution:

$$315^\circ = 315 \times 1^\circ = 315 \times \left(\frac{\pi}{180} \text{ radians}\right)$$

$$= \frac{7\pi}{4} \text{ radians}$$

5. Convert each of following to degrees.

(i) $\frac{3\pi}{4}$

Solution

$$\frac{3\pi}{4}=\frac{3\pi}{4}\text{ radian}=\frac{3\pi}{4}\times 1\text{ radians}$$

$$=\frac{3\pi}{4}\times\frac{180^\circ}{\pi}=135^\circ$$

(ii) $\frac{5\pi}{6}$

Solution

$$\frac{5\pi}{6}=\frac{5\pi}{6}\text{ radian}=\frac{5\pi}{6}\times 1\text{ radians}$$

$$=\frac{5\pi}{4}\times\frac{180^\circ}{\pi}=150^\circ$$

(iii) $\frac{7\pi}{8}$

Solution

$$\frac{7\pi}{8}=\frac{7\pi}{8}\text{ radian}=\frac{7\pi}{8}\times 1\text{ radians}$$

$$=\frac{7\pi}{4}\times\frac{180^\circ}{\pi}=157.5^\circ$$

$$=157^\circ+0.5^\circ=157^\circ+30'$$

$$=157^\circ 30'$$

(iv) $\frac{13\pi}{16}$

Solution

$$\frac{13\pi}{16}=\frac{13\pi}{16}\text{ radian}=\frac{13\pi}{16}\times 1\text{ radians}$$

$$=\frac{13\pi}{16}\times\frac{180^\circ}{\pi}=146.25^\circ$$

$$=146^\circ+0.25^\circ=146^\circ+\left(\frac{25}{100}\right)^\circ$$

$$=146^\circ+\left(\frac{25}{100}\times 60\right)'=146^\circ+15'$$

$$=146^\circ 15'$$

(v) 3

Solution

$3 = 3 \text{ radian} = 3 \times 1 \text{ radians}$

$$= 3 \times \frac{180^\circ}{\pi} \qquad = 3 \times 57.29.295779$$

$$= 171^\circ + 89^\circ \qquad = 171^\circ + 0.89^\circ$$

$$= 171^\circ + \left(\frac{89}{100}\right)^\circ = 171^\circ + \left(\frac{89}{100} \times 60\right)'$$

$$= 171^\circ + 53.4' \qquad = 171^\circ + 53' + (0.4 \times 60)''$$

$$171^\circ + 53' + 24'' = 171^\circ + 53'24''$$

(vi) 4.5

Solution

$$4.5 = 4.5 \text{ radians} \qquad = 4.5 \times 1 \text{ radians}$$

$$= 4.5 \times \frac{180^\circ}{\pi} \qquad = 4.5 \times 57.295779$$

$$= 257.83^\circ \qquad = 257^\circ + 0.83^\circ$$

$$= 257^\circ + 49.8' \qquad = 257^\circ + 49' + (0.8 \times 60)''$$

$$= 257^\circ + 49' + 48'' = 257^\circ 49' 48''$$

(vii) $\frac{-7\pi}{8}$

Solution

$$-\frac{7\pi}{8} \qquad = -\frac{7\pi}{8} \text{ radians} = -\frac{7\pi}{8} \times 1 \text{ radians}$$

$$= -\frac{7\pi}{8} \times \frac{180^\circ}{\pi} = -157.5^\circ$$

$$= -157^\circ + 0.5^\circ = -157^\circ + \left(\frac{5}{10}\right)^\circ$$

$$= -157^\circ + \left(\frac{1}{2} \times 60\right)' = -157^\circ + 30'$$

$$= -157^\circ 30'$$

(viii) $\frac{13}{16}\pi$

Solution

$$-\frac{13\pi}{8} = -\frac{13\pi}{8}\text{ radians} = -\frac{13\pi}{8}\times 1\text{ radians}$$

$$= -\frac{13\pi}{8}\times\frac{180^\circ}{\pi} = -146.25^\circ$$

$$= -146^\circ + 0.25^\circ = -146^\circ + \left(\frac{25}{100}\right)^\circ$$

$$= -146^\circ + \left(\frac{1}{4}\times 60\right)' = -146^\circ + 15'$$

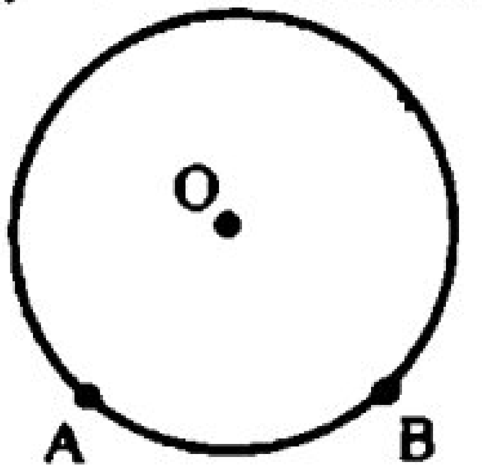
$$= -146^\circ 15'$$

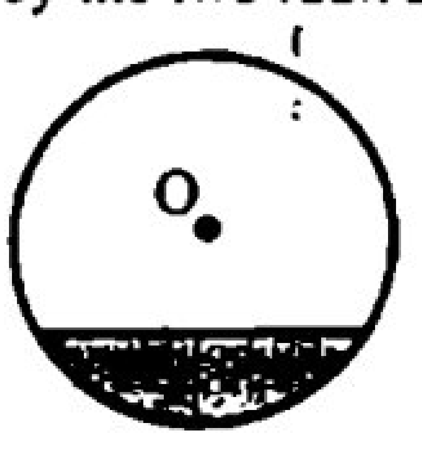
Sector of a Circle

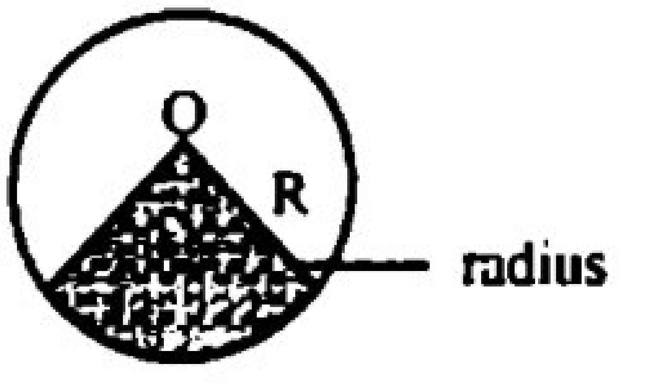
(i) A part of the circumference of a circle is called an arc.

(ii) A part of the circle bounded by an arc and a chord is called segment of a circle.

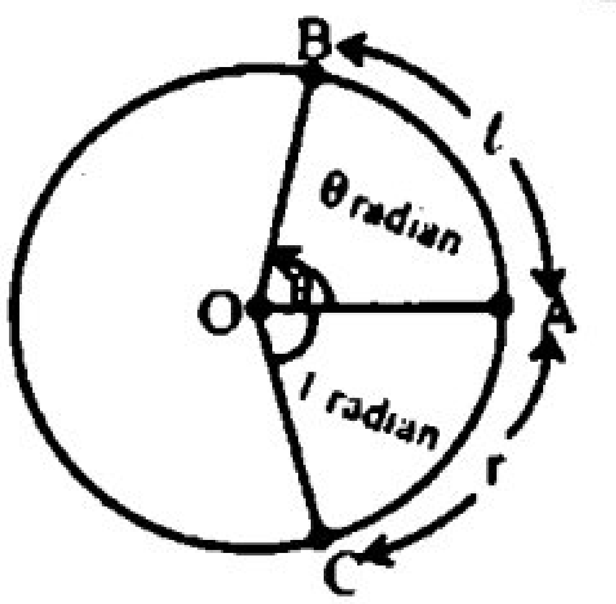
(iii) A part of the circle bounded by the two radii and an arc is called sector of the circle.







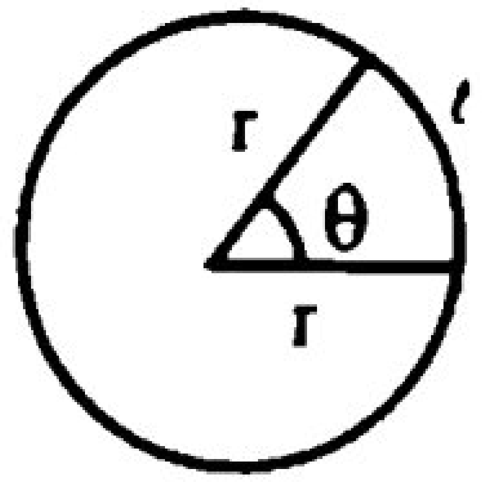
To establish the rule $\ell = r\theta$, where r is the radius of the circle, ℓ the length of circular arc and θ the central angle measured in radians:



Let an arc AB denoted by ℓ subtends an angle G radian at the centre of the circle. It is a fact of plane geometry that measure of central angles of the arcs of a circle are proportional to the lengths of their arcs.

$$\frac{m\angle AOB}{m\angle AOB} = \frac{m\overline{AB}}{m\overline{AC}}$$

$$\Rightarrow \quad \frac{\theta\text{ radian}}{1\text{ radian}} = \frac{\ell}{r} \Rightarrow \frac{\ell}{r} = \theta \text{ or } \ell = r\theta$$



Area of a circular sector

Consider a circle of radius r units and an arc of length units, subtending an angle θ at O.

Area of the circle = πr^2

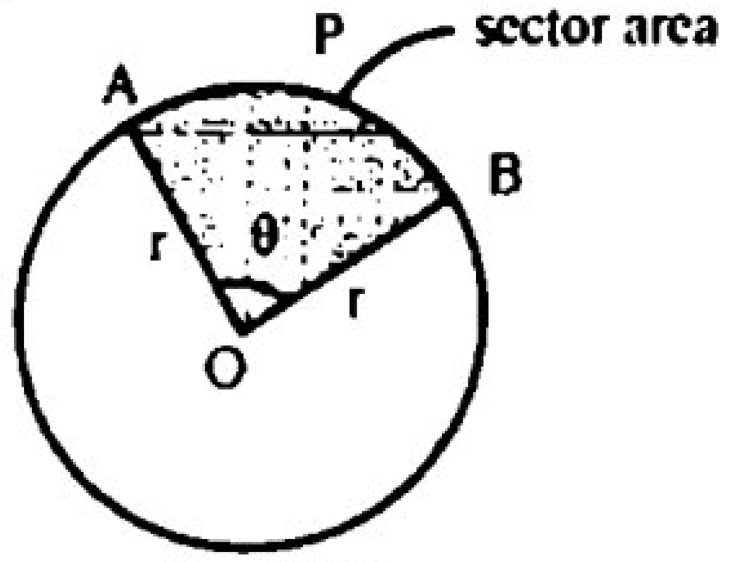


Fig. 7.2.2

Angle of the circle = πr^2

Angle of the circle = πr^2

Angle of the sector = 0 radian

Then by elementary geometry we can use the proportion,

$$\frac{\text{area of sector AOBP}}{\text{area of circle}} = \frac{\text{angle of the sector}}{\text{angle of the circle}}$$

or $$\frac{\text{area of sector AOBP}}{\pi r^2} = \frac{\theta}{2\pi}$$

$\Rightarrow$ area of sector AOBP $= \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2\theta$ angle of the sector AOBP $= \frac{1}{2} r^2\theta$

SOLVED EXERCISE 7.2

1. Find θ, when:

(i) l = 4.5m, r = 3.5m

Solution

We know that

$l = r\theta \qquad \Rightarrow 0 = \frac{l}{r}$

$0 = \frac{2}{3.5} \qquad \Rightarrow 0 = 0.57 \text{ radians}$