

We know that $2k\pi + \theta = \theta$ where $k \in \mathbb{Z}$.

Now $\theta = 360^\circ = 0^\circ + (360^\circ) 1 = 0^\circ$ where $k = 1$

$$\begin{aligned} \text{So } \sin 360^\circ &= \sin 0^\circ = 0 & \operatorname{cosec} 360^\circ &= \frac{1}{\sin 360^\circ} = \frac{1}{\sin 0^\circ} = \frac{1}{0} = \infty \text{ (undefined)} \\ \cos 360^\circ &= \cos 0^\circ = 1 & \sec 360^\circ &= \frac{1}{\cos 0^\circ} = \frac{1}{1} = 1, \\ \tan 360^\circ &= \tan 0^\circ = 0 & \cot 360^\circ &= \frac{1}{\tan 0^\circ} = \frac{1}{0} = \infty \text{ (undefined)} \end{aligned}$$

SOLVED EXERCISE 7.3

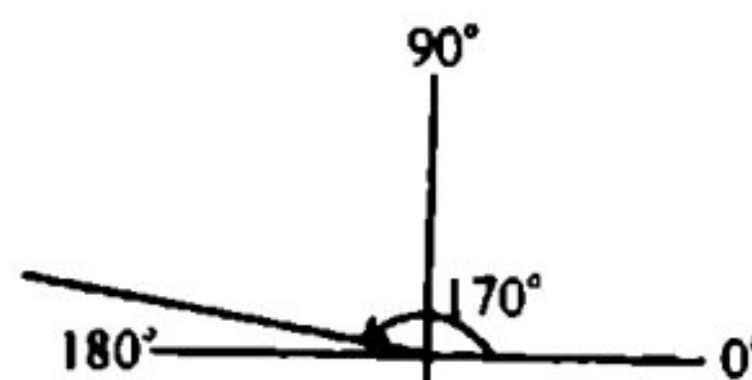
1. Locate each of the following angles in standard position using a protractor or fair free hand guess. Also find a positive and a negative angle conterminal with each given angle.

(i) 170°

Solution

$$\begin{aligned} \text{Positive conterminal angle} \\ &= 360^\circ + 170^\circ \\ &= 530^\circ \end{aligned}$$

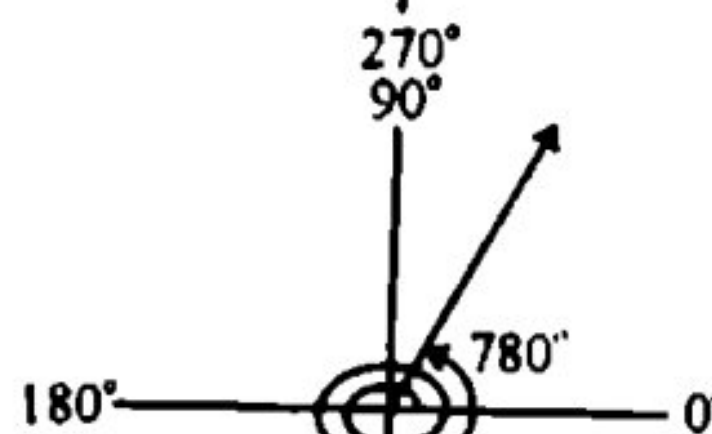
$$\begin{aligned} \text{Negative Coterminal angle} \\ &= 360 - 170^\circ \\ &= 190^\circ \end{aligned}$$



(ii) 780°

Solution

$$\begin{aligned} \text{Positive conterminal angle} \\ &= 780^\circ - 360^\circ - 360^\circ \\ &= 780^\circ - 720^\circ \\ &= 60^\circ \end{aligned}$$

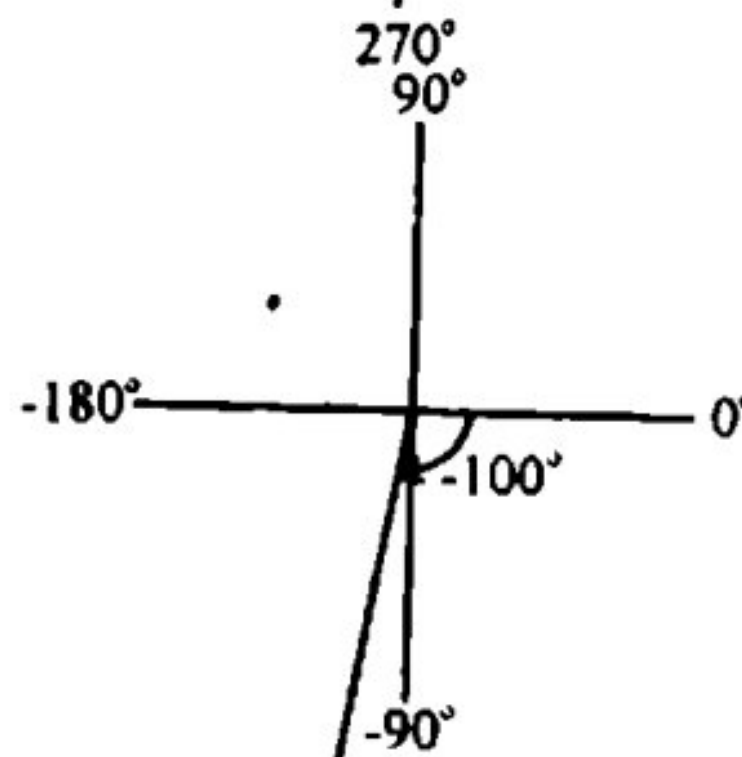


(iii) -100°

Solution

$$\begin{aligned} \text{Positive conterminal angle} \\ &= 360^\circ - 100^\circ \\ &= 260^\circ \end{aligned}$$

$$\begin{aligned} \text{Negative Coterminal angle} \\ &= -360^\circ - 100^\circ \\ &= -460^\circ \end{aligned}$$



(iv) -500°

Solution

Positive conterminal angle

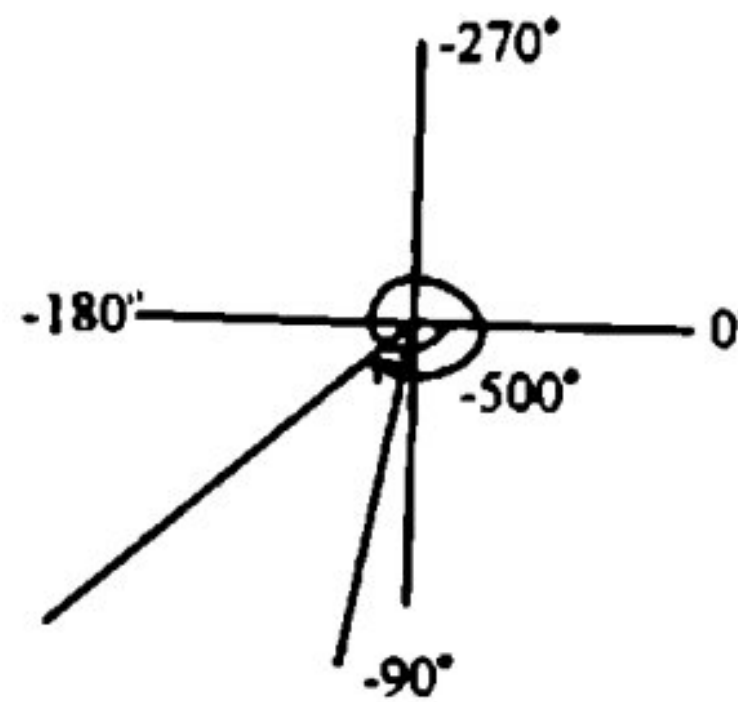
$$= 720^\circ - 500^\circ$$

$$= 220^\circ$$

Negative Conterminal angle

$$= 360^\circ - 500^\circ$$

$$= -140^\circ$$



2. Identify the closest quadrantal angles between which the following angles lies.

(i) 156°

Solution:

The closest quadrantal angles between which 156° lies are 90° and 180° .

(ii) 318°

Solution:

The closest quadrantal angles between which 318° lies are 270° and 360° .

(iii) 572°

Solution

The closest quadrantal angles between which 572° lies are 540° and 630° .

(iv) -330°

Solution

The closest quadrantal angles between which -33° lies are 0° and 90° .

3. Write the closest quadrantal angles between which the angle lies. Write your answer in radian measure.

(i) $\frac{\pi}{3}$

Solution

The closest quadrantal angles between which $\frac{\pi}{3}$ lies are 0 and $\frac{\pi}{2}$.

(ii) $\frac{3\pi}{4}$

Solution

The closest quadrantal angles between which $\frac{3\pi}{4}$ lies are $\frac{\pi}{2}$ and π .

(iii) $-\frac{\pi}{4}$

Solution

The closest quadrantal angles between which $-\frac{\pi}{4}$ lies are 0 and $-\frac{\pi}{4}$.

(iv) $-\frac{3\pi}{4}$

Solution

The closest quadrantal angles between which $-\frac{3\pi}{4}$ lies are $-\frac{\pi}{2}$ and $-\pi$.

4. In which quadrant (9) lie when

(i) $\sin\theta > 0$, $\tan\theta < 0$

Solution

II

(ii) $\cos\theta < 0$, $\sin\theta < 0$

Solution

III

(iii) $\sec\theta > 0$, $\sin\theta < 0$

Solution

IV

(iv) $\cos\theta < 0$, $\tan\theta < 0$

Solution

II

(v) $\operatorname{cosec}\theta > 0$, $\cos\theta > 0$

Solution

I

(vi) $\sin\theta < 0$, $\sec\theta < 0$

Solution

III

5. Fill in the blanks.

(i) $\cos(-150^\circ) = \dots\dots\dots \cos 150^\circ$

Solution

(ii) $\sin(-310^\circ) = \dots\dots\dots \sin 310^\circ$

Solution

- ve

(iii) $\tan(-210^\circ) = \dots\dots\dots \tan 210^\circ$

Solution

- ve

(iv) $\cot(-45^\circ) = \dots\dots\dots \cot 45^\circ$

Solution

- ve

(v) $\sec(-60^\circ) = \dots\dots\dots \sec 60^\circ$

Solution:

+ ve

(vi) $\operatorname{cosec}(-137^\circ) = \dots\dots\dots \operatorname{cosec} 137^\circ$

Solution:

- ve

6. The given point P lies on the-terminal side of θ . Find quadrant of θ and all six trigonometric ratios.

(I) $(-2, 3)$

Solution

$(-2, 3)$

Here $x = -2$ and $y = 3$

So, the quadrant of θ is II.

Now, by Pythagorus theome,

We have

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-2)^2 + (3)^2}$$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13}$$

The six trigonometric ratios are

$$\sin \theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$$

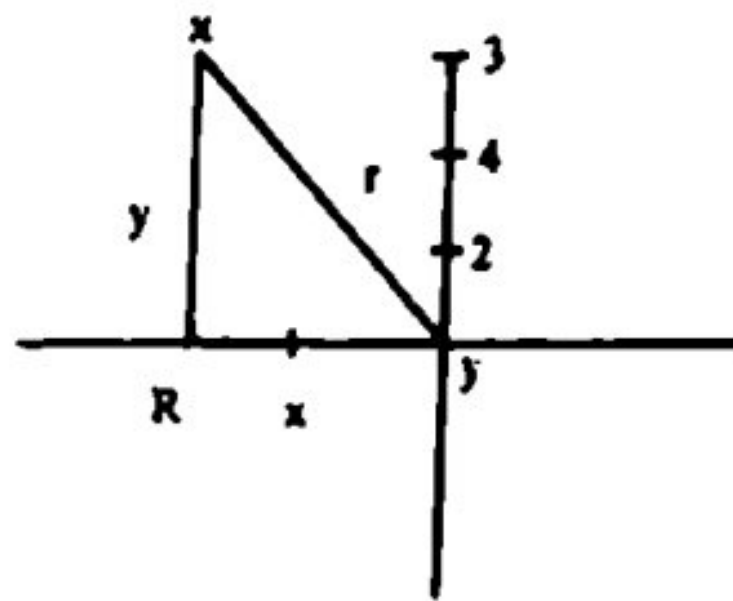
$$\cos \theta = \frac{x}{r} = \frac{-2}{\sqrt{13}}$$

$$\tan \theta = \frac{y}{x} = \frac{3}{-2}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\sec \theta = \frac{r}{x} = \frac{\sqrt{13}}{-2}$$

$$\cot \theta = \frac{x}{y} = \frac{-2}{3}$$



(ii) $(-3, -4)$

Solution

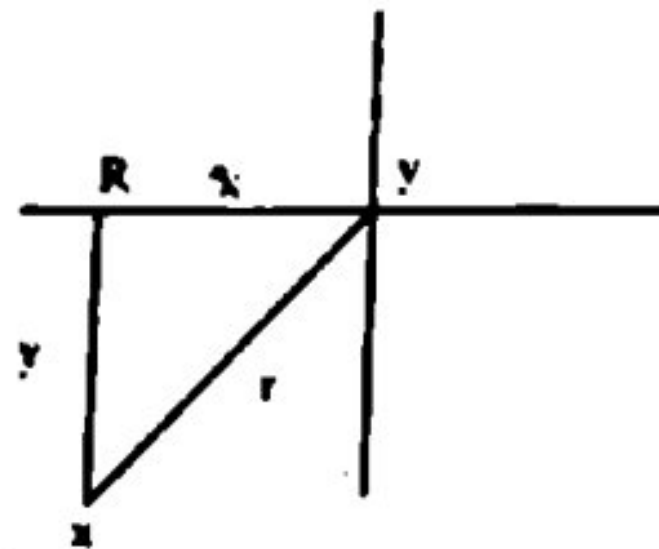
$(-3, -4)$

Here $x = 3$ and $y = -4$

So, the quadrant of θ is III.

Now by Pythagoras theorem, we have

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-3)^2 + (-4)^2} \\ &= \sqrt{9 + 16} = \sqrt{25} = 5 \end{aligned}$$



The six trigonometric ratios are

$$\sin \theta = \frac{y}{r} = \frac{-4}{5}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = \frac{5}{-4}$$

$$\cos \theta = \frac{x}{r} = \frac{-3}{5}$$

$$\sec \theta = \frac{r}{x} = \frac{5}{-3}$$

$$\tan \theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

(iii) $(\sqrt{2}, 1)$

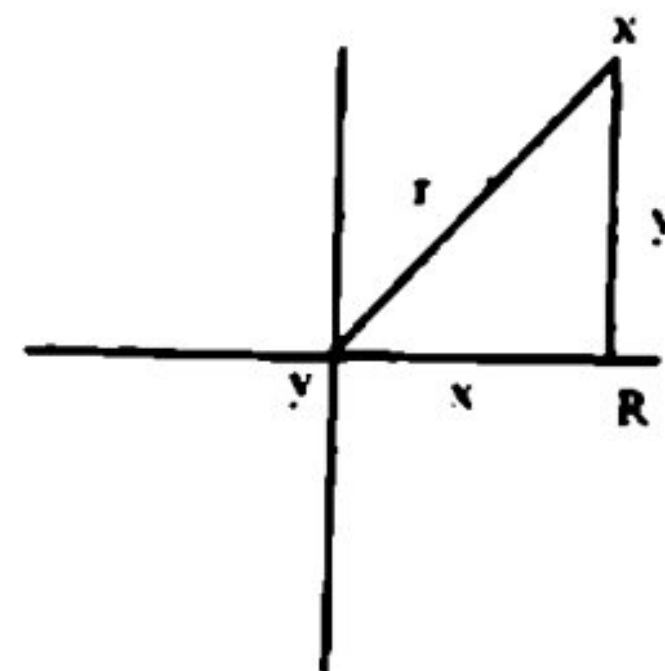
Solution

$(\sqrt{2}, 1)$

Here $x = \sqrt{2}$ and $y = 1$

So, the quadrant of θ is I

Now by Pythagoras theorem, we have



$$\begin{aligned}
 r &= \sqrt{x^2 + y^2} \\
 &= \sqrt{(\sqrt{2})^2 + (1)^2} \\
 &= \sqrt{2+1} \\
 &= \sqrt{3}
 \end{aligned}$$

The six trigonometric ratios are

$$\begin{aligned}
 \sin \theta &= \frac{y}{r} = \frac{1}{\sqrt{3}} & \operatorname{Cosec} \theta &= \frac{r}{y} = \frac{\sqrt{3}}{1} = \sqrt{3} \\
 \cos \theta &= \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}} & \sec \theta &= \frac{r}{x} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}} \\
 \tan \theta &= \frac{y}{x} = \frac{1}{\sqrt{2}} & \cot \theta &= \frac{x}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}
 \end{aligned}$$

7. If $\cos \theta = -\frac{2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.

Solution

As $\cos \theta = -\frac{2}{3}$ and terminal arm of the angle θ is in quadrant II, so

$$x = -2 \text{ and } r = 3$$

By Phthagoras theorem, we have

$$r^2 = x^2 + y^2$$

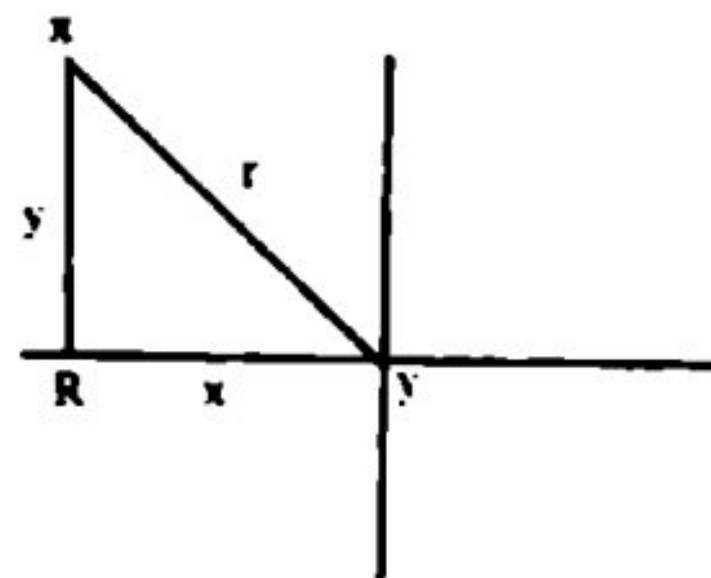
$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{(3)^2 - (-2)^2}$$

$$y = \sqrt{9-4}$$

$$y = \sqrt{5}$$



The six trigonometric ratios are

$$\begin{aligned}
 \sin \theta &= \frac{y}{r} = \frac{\sqrt{5}}{3} & \operatorname{Cosec} \theta &= \frac{r}{y} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5} \\
 \cos \theta &= \frac{x}{r} = \frac{-2}{3} = -\frac{2}{3} & \sec \theta &= \frac{r}{x} = \frac{3}{-2} = -\frac{3}{2} \\
 \tan \theta &= \frac{y}{x} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2} & \cot \theta &= \frac{x}{y} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}
 \end{aligned}$$

8. If $\tan \theta = \frac{4}{3}$ and $\sin \theta < 0$, find the values of other trigonometric functions at θ .

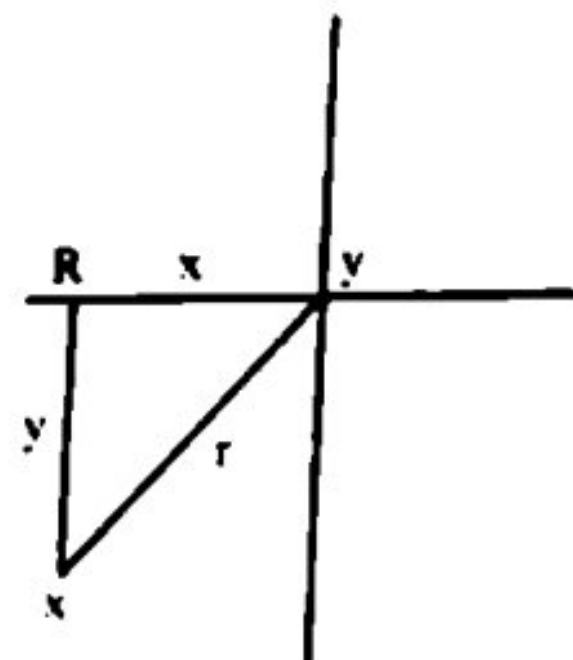
Solution

As $\tan \theta = \frac{4}{3}$ and $\sin \theta < 0$ (terminal arm of the angle θ is in quadrant III), so.

$$x = 3 \text{ and } y = 4$$

By Phthagoras theorem, we have

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(3)^2 + (4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$



The six trigonometric ratios are

$$\sin \theta = -\frac{y}{r} = -\frac{4}{5}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = -\frac{5}{4}$$

$$\cos \theta = -\frac{x}{r} = -\frac{3}{5}$$

$$\sec \theta = -\frac{r}{x} = -\frac{5}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{3}{4}$$

9. If $\tan \theta = -\frac{1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant II, find the values of $\tan \theta$, $\sec \theta$, and $\operatorname{cosec} \theta$.

As $\tan \theta = -\frac{1}{\sqrt{2}}$ and terminal said of the angle is in quadrant - IV, so.

$$y = -1 \quad \text{and } r = \sqrt{2}$$

By Phthagoras theorem, we have

$$r^2 = x^2 + y^2$$

$$x^2 = r^2 - y^2$$

$$x = \sqrt{r^2 - y^2}$$

$$x = \sqrt{(\sqrt{2})^2 - (-1)^2}$$

$$x = \sqrt{2 - 1}$$

$$x = \sqrt{1}$$

$$x = 1$$

Now

$$\tan \theta = \frac{y}{x} = \frac{-1}{1} = -1$$

$$\sec \theta = \frac{r}{x} = -\frac{\sqrt{2}}{1} = -\sqrt{2}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

10. If $\operatorname{cosec} \theta = \frac{13}{12}$ and $\sec \theta > 0$, find the remaining trigonometric functions.

As $\operatorname{Cosec} \theta = \frac{13}{12}$ and $\sec \theta > 0$ (terminal arm of angle θ is in quadrant I), so

$$y = 12 \quad \text{and} \quad r = 13$$

By Pythagoras theorem, we have

$$r^2 = x^2 + y^2$$

$$x^2 = r^2 - y^2$$

$$= \sqrt{r^2 - y^2}$$

$$= \sqrt{(\sqrt{13})^2 - (12)^2}$$

$$= \sqrt{169 - 144}$$

$$= \sqrt{25} = 5$$

The remaining trigonometric ratios are

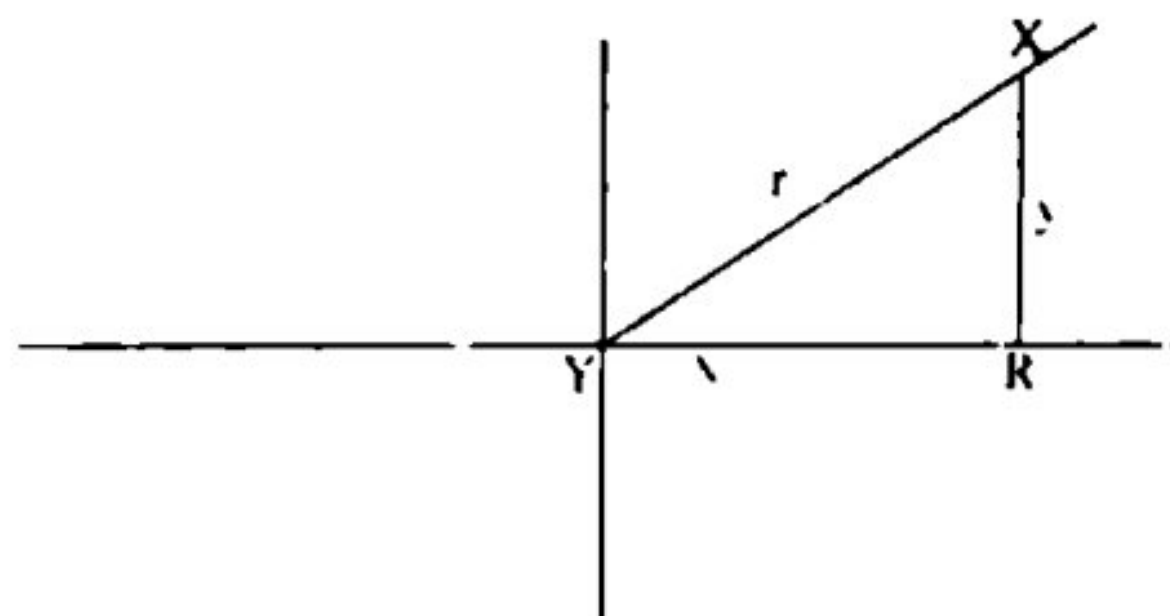
$$\sin \theta = \frac{y}{r} = \frac{12}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{5}{13}$$

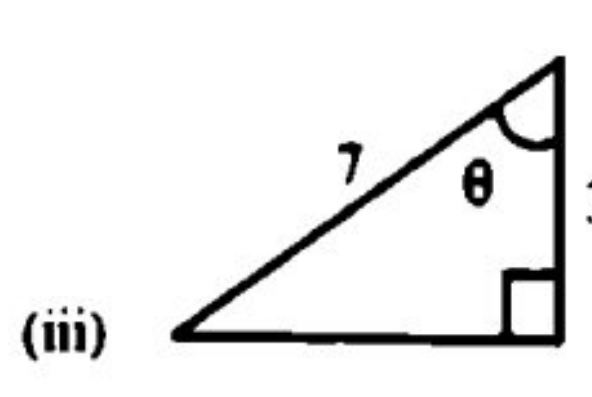
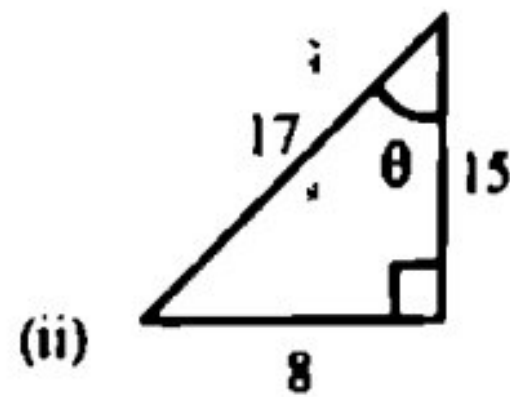
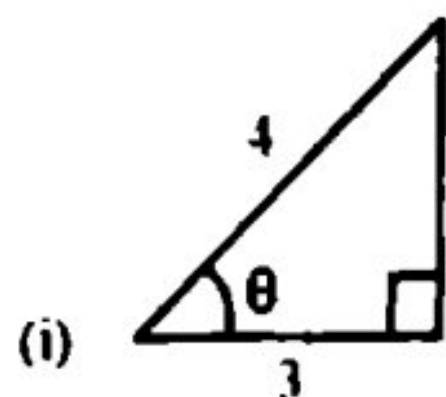
$$\tan \theta = \frac{y}{x} = \frac{12}{5}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{5}$$

$$\cot \theta = \frac{x}{y} = \frac{5}{12}$$



11. Find the values of trigonometric functions at the indicated angle θ in the right triangle.



- (i) As $x = 3$ and $r = 5$

By Pythagoras theorem, we have

$$r^2 = x^2 + y^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{(4)^2 - (3)^2}$$

$$y = \sqrt{16 - 9}$$

$$y = \sqrt{7}$$

The six trigonometric ratios are

$$\sin \theta = \frac{y}{r} = \frac{\sqrt{7}}{4}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = \frac{4}{\sqrt{7}}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{4}$$

$$\sec \theta = \frac{r}{x} = \frac{4}{3}$$

$$\tan \theta = \frac{y}{x} = \frac{\sqrt{7}}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{3}{\sqrt{7}}$$

(ii) As $r = 17$, $x = 15$, $y = 8$

The six trigonometric ratios are

$$\sin \theta = \frac{y}{r} = \frac{8}{17}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = \frac{17}{8}$$

$$\cos \theta = \frac{x}{r} = \frac{15}{17}$$

$$\sec \theta = \frac{r}{x} = \frac{17}{15}$$

$$\tan \theta = \frac{y}{x} = \frac{8}{15}$$

$$\cot \theta = \frac{x}{y} = \frac{15}{8}$$

(iii) As $x = 3$ and $r = 7$

By Pythagoras theorem, we have

$$r^2 = x^2 + y^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{(7)^2 - (3)^2}$$

$$y = \sqrt{49 - 9}$$

$$y = \sqrt{40}$$

$$y = 2\sqrt{10}$$

The six trigonometric ratios are

$$\sin \theta = \frac{y}{r} = \frac{2\sqrt{10}}{7}$$

$$\operatorname{Cosec} \theta = \frac{r}{y} = \frac{7}{2\sqrt{10}}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{7}$$

$$\sec \theta = \frac{r}{x} = \frac{7}{3}$$

$$\tan \theta = \frac{y}{x} = \frac{2\sqrt{10}}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{3}{2\sqrt{10}}$$

12. Find the values of the trigonometric functions. Do not use trigonometric tables or calculator.

(i) $\tan 30^\circ$

Solution

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

(ii) $\tan 330^\circ$

Solution

We know that

$$2k\pi + \theta, \theta \text{ where } k \in \mathbb{Z}$$

$$\begin{aligned} \tan 330^\circ &= \tan(360^\circ - 30^\circ) \\ &= \tan(2(1)\pi - 30^\circ) \\ &= \tan(-30^\circ) \\ &= -\tan 30^\circ \\ &= -\frac{1}{\sqrt{3}} \end{aligned}$$

(iii) $\sec 330^\circ$

Solution

We know that

$$2k\pi + \theta = \theta, \text{ where } k \in \mathbb{Z}$$

$$\begin{aligned} \sec 330^\circ &= \sec(360^\circ - 30^\circ) \\ &= \sec(2(1)\pi - 30^\circ) \\ &= \sec(-30^\circ) \\ &= \sec 30^\circ \\ &= \frac{1}{\cos 30^\circ} \\ &= \frac{2}{\sqrt{3}} \end{aligned}$$

(iv) $\cot \frac{\pi}{4}$

Solution

We know that

$2k\pi + \theta$, θ where $k \in \mathbb{Z}$

$$\begin{aligned}\cot \frac{\pi}{4} &= \frac{1}{\tan \frac{\pi}{4}} \\ &= \frac{1}{1} = 1\end{aligned}$$

(v) $\cos \frac{2\pi}{3}$

Solution

We know that

$2k\pi + \theta$, θ where $k \in \mathbb{Z}$

$$\begin{aligned}\cos\left(\frac{2\pi}{3}\right) &= \cos\left(2\left(\frac{1}{2}\right)\pi - \frac{\pi}{3}\right) \\ &= \cos\left(-\frac{\pi}{3}\right) \\ &= \cos \frac{\pi}{3} \\ &= -\frac{1}{2} \because (\text{In quad. II } \cos \theta < 0)\end{aligned}$$

(vi) $\operatorname{cosec} \frac{7\pi}{6}$

Solution

We know that

$2k\pi + \theta = \theta$

$$\begin{aligned}\operatorname{cosec}\left(\frac{2\pi}{3}\right) &= \operatorname{cosec}\left(2\left(\frac{1}{2}\right)\pi - \frac{\pi}{3}\right) \\ &= \operatorname{cosec}\left(-\frac{\pi}{3}\right) \\ &= \frac{1}{\sin(-\pi/3)} \\ &= -\frac{1}{\sin(\pi/3)} \\ &= \frac{1}{\sin(\pi/3)} \because (\text{In quad. II } \sin > 0)\end{aligned}$$

$$= \frac{1}{\sqrt{3/2}}$$

$$= \frac{2}{\sqrt{3}}$$

(vii) $\cos(-450^\circ)$

Solution

We know that

$$2k\pi + \theta = \theta$$

$$\cos(-450^\circ) = \cos(2(-1)\pi - 90^\circ)$$

$$= \cos(-90^\circ)$$

$$= \cos 90^\circ = 0$$

(viii) $\tan(-9\pi)$

Solution

We know that

$$2k\pi + \theta = \theta$$

$$\tan(-9\pi) = \tan(2(-5)\pi + \pi)$$

$$= \tan(\pi)$$

$$= 0$$

(ix) $\cos\left(-\frac{5\pi}{6}\right)$

Solution

We know that

$$2k\pi + \theta = \theta$$

$$\cos\left(-\frac{5\pi}{6}\right) = \cos\left(2\left(-\frac{1}{2}\right)\pi + \frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$$\therefore (\text{In quad. II } \cos < 0)$$

(x) $\sin \frac{7\pi}{6}$

Solution

We know that

$$2k\pi + \theta = \theta$$

$$\begin{aligned}\sin\left(\frac{7\pi}{6}\right) &= \sin\left(2\left(\frac{1}{2}\right)\pi + \frac{\pi}{6}\right) \\ &= \sin\left(\frac{\pi}{6}\right) \\ &= \frac{1}{2} \quad \because (\text{In quad. III } \sin < 0).\end{aligned}$$

(xi) $\cot\left(\frac{7\pi}{6}\right)$

Solution

We know that

$$2k\pi + \theta = \theta$$

$$\begin{aligned}\sin\left(\frac{7\pi}{6}\right) &= \sin\left(2\left(\frac{1}{2}\right)\pi + \frac{\pi}{6}\right) \\ &= \sin\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\tan(\pi/6)} \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

(In quad. II $\cos < 0$)

(xii) $\cos 225^\circ$

Solution

We know that

$$2k\pi + \theta = \theta$$

$$\begin{aligned}\cos(225^\circ) &= \cos\left(2\left(\frac{1}{2}\right)\pi + 45^\circ\right) \\ &= \cos 45^\circ \\ &= -\frac{1}{\sqrt{2}}\end{aligned}$$

\therefore (In quad. II $\cos < 0$)

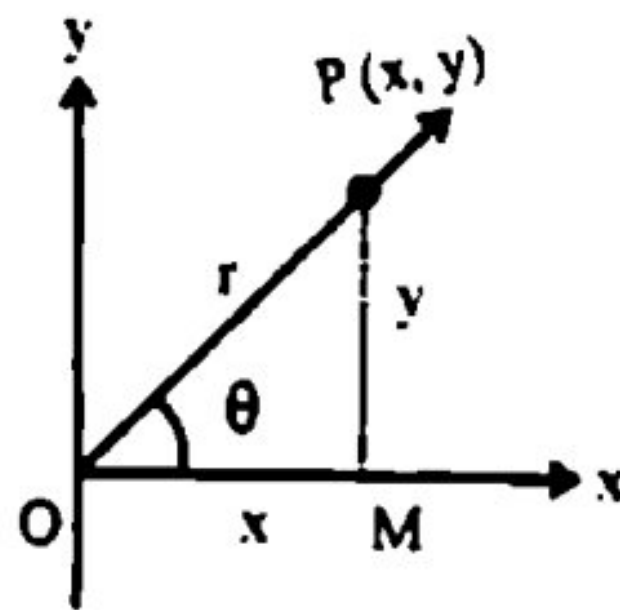
Trigonometric Identities:

Consider an angle $\angle MOP = \theta$ radian in standard position. Let point P (x, y) be on the terminal side of the angle. By Pythagorean theorem, we have from right triangle OMP.

$$OM^2 + MP^2 = OP^2$$

$$x^2 + y^2 = r^2 \quad \dots\dots (i)$$

Dividing both sides by r^2 we get



$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

$$\Rightarrow \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$\Rightarrow (\cos^2 \theta) + (\sin^2 \theta) = 1$$

$$\therefore \boxed{\cos^2 \theta + \sin^2 \theta = 1} \quad (1)$$

Dividing (i) by x^2 , we have

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{x^2}$$

$$\Rightarrow 1 + \left(\frac{y}{x}\right)^2 + \left(\frac{r}{x}\right)^2 \quad \because \tan \theta = \frac{y}{x} \text{ and } \sec \theta = \frac{r}{x}$$

$$\Rightarrow 1 + (\tan^2 \theta) + (\sec \theta)^2 = 1$$

$$\therefore \boxed{1 + \tan^2 \theta = \sec^2 \theta} \text{ or } \sec^2 \theta - \tan^2 \theta = 1 \quad (2)$$

Again dividing both sides of (i) by y^2 , we get

$$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$$

$$\left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2 \quad \because \cot \theta = \frac{x}{y} \text{ and } \operatorname{cosec} \theta = \frac{r}{y}$$

$$\Rightarrow (\cot \theta)^2 + 1 = (\operatorname{cosec} \theta)^2$$

$$\therefore \boxed{1 + \cot^2 \theta = \operatorname{cosec}^2 \theta} \text{ or } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \quad (3)$$

The identities (1), (2) and (3) are also known as Pythagorean Identities.

The fundamental identities are used to simplify expressions involving trigonometric functions

Example 1:

Verify that $\cot \theta \sec \theta = \operatorname{cosec} \theta$

Solution

Expressing left hand side in terms of sine and cosine, we have

$$\begin{aligned}
 \text{L.H.S} &= \cot \theta \sec \theta = \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \\
 &= \frac{1}{\sin \theta} = \operatorname{cosec} \theta \\
 &= \text{R.H.S}
 \end{aligned}$$

Example 2

Verify that $\tan \theta + \tan^2 \theta \tan^2 \theta \sec^2 \theta$

Solution

$$\begin{aligned}
 \text{L.H.S} &= \tan^4 \theta + \tan^2 \theta = \tan^2 \theta (\tan^2 \theta + 1) & \because \tan^2 \theta + 1 = \sec^2 \theta \\
 &= \tan^2 \theta \sec^2 \theta \\
 &= \text{R.H.S}
 \end{aligned}$$

Example 3

Show that $\frac{\cot^2 \alpha}{\operatorname{cosec} \alpha - 1} = \operatorname{cosec} \alpha + 1$

Solution

$$\frac{\cot^2 \alpha}{\operatorname{cosec} \alpha - 1} \quad \left(\begin{array}{l} \because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \\ \cot^2 \theta = \operatorname{cosec}^2 \theta - 1 \end{array} \right)$$

Solution

$$= \frac{(\operatorname{cosec}^2 \alpha - 1)}{\operatorname{cosec} \alpha - 1} = \frac{(\operatorname{cosec} \alpha - 1)(\operatorname{cosec} \alpha + 1)}{(\operatorname{cosec} \alpha - 1)} = \operatorname{cosec} \alpha + 1 = \text{R.H.S}$$

Example 4

Express the trigonometric functions in terms of $\tan \theta$.

Solution

By using reciprocal identity, we can express $\cot \theta$ in terms of $\tan \theta$.

$$\text{i.e., } \cot \theta = \frac{1}{\tan \theta}$$

By solving the identity $1 + \tan^2 \theta \sec^2 \theta$

We have expressed $\sec \theta$ in terms of $\tan \theta$.

$$\sec \theta = \pm \sqrt{\tan^2 \theta + 1} \because$$

$$\cos \theta = \frac{1}{\sec \theta} \Rightarrow \cos \theta = \frac{1}{\pm \sqrt{\tan^2 \theta + 1}}$$

Because $\sin \theta = \tan \theta \cos \theta$, we have

$$\sin \theta = \tan \theta \left(\frac{1}{\pm \sqrt{\tan^2 \theta + 1}} \right) = \frac{\tan \theta}{\pm \sqrt{\tan^2 \theta + 1}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\pm \sqrt{\tan^2 \theta + 1}}{\tan \theta}$$