$$\tan\alpha = \frac{AC}{BC} = \frac{17.9}{7} = 255714$$

Solving for a gives us

$$\alpha = \tan^{-1}(2.55714)$$

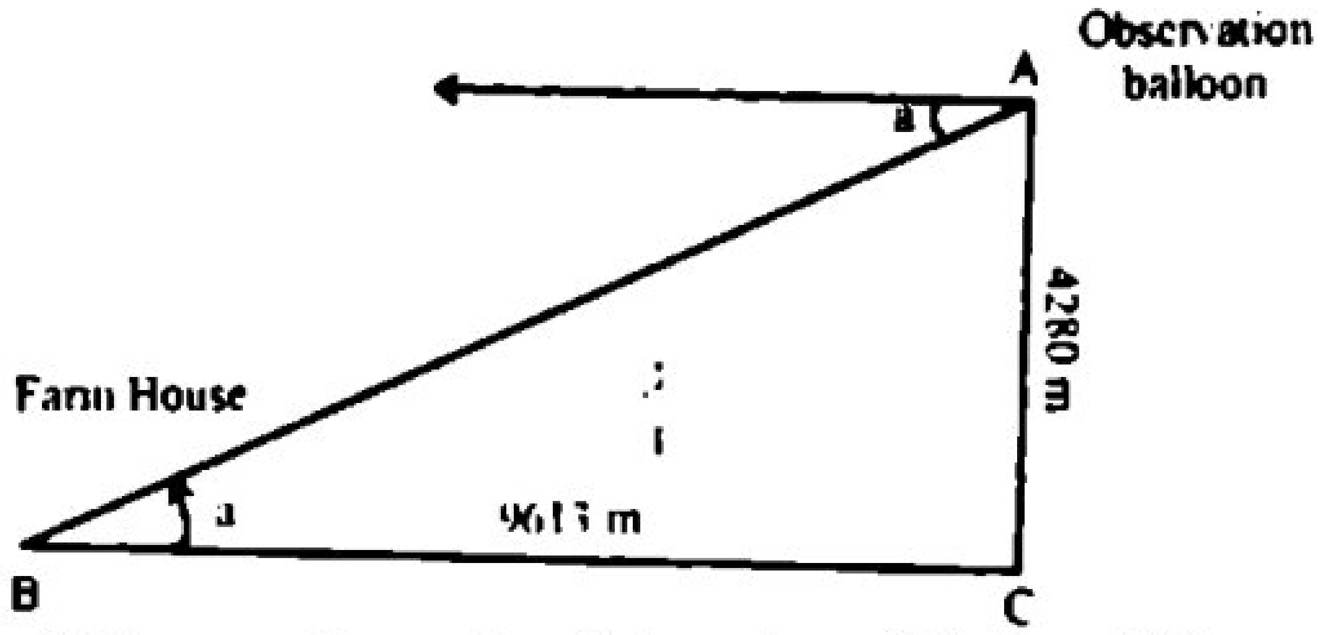
$$= (68.6666)^{\circ} = 68^{\circ}40'$$

$$\Rightarrow \quad \alpha = 68^{\circ}40'$$

Example 2

An observation balloon is 4280 meter above the ground and 9613 meter away from a farmhouse. Find the angle of depressor of the farmhouse as observed from the observation balloon.

Solution



For problems of this type the angle of elevation of A from B is considered equal to the angle of depression of B from A, as shown in the diagram.

$$\tan\alpha = \frac{AC}{BC} = \frac{4280}{9613} = 0.44523$$

$$\alpha = \tan^{-1}(0.44523) = 24^{\circ}$$

So, angle of depression is 24°.

SOLVED EXERCISE 7.5

1. Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.

Solution

From the figure, we observe that α is the angle of elevation.

Using the fact that

$$\tan\alpha = \frac{\overline{BC}}{\overline{AC}}$$

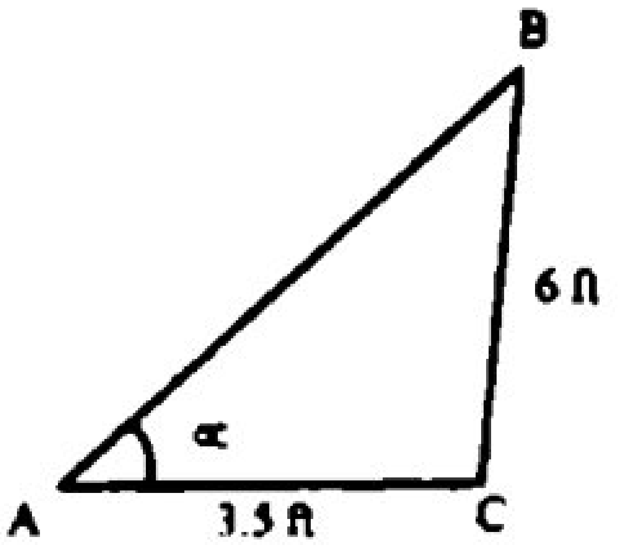
$$\tan\alpha = \frac{6}{3.5}$$

$\tan \propto = 1.7143$

$\propto = \tan^{-1}(1.7143)$

$\propto = 57.94°$

or $\propto = 59°44'37''$



2 **A tree casts a 40 meter shadow when the angle of elevation of the sun is 25°. Find the height of the tree.**

Solution

From the figure, we observe that $\propto$ is the angle of elevation.
Using the fact that

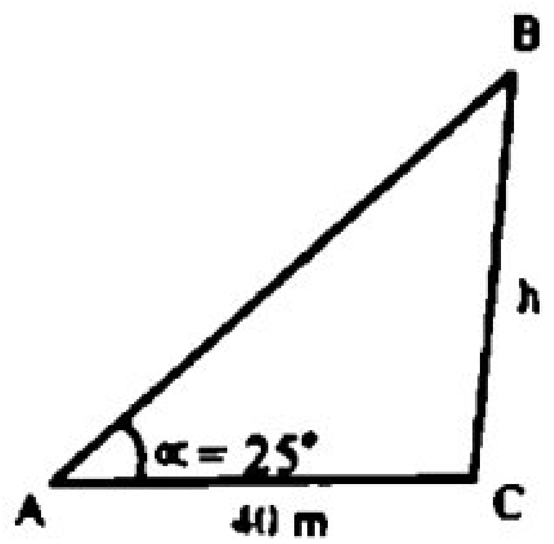
$$\tan \propto = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan 25° = \frac{h}{40}$$

$$h = 40 \tan 25°$$

$$= 40(0.44663)$$

$$= 18.652\,m$$



3. **A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.**

Solution

From the figure, we observe that $\propto$ is the angle of elevation.
Using the fact that

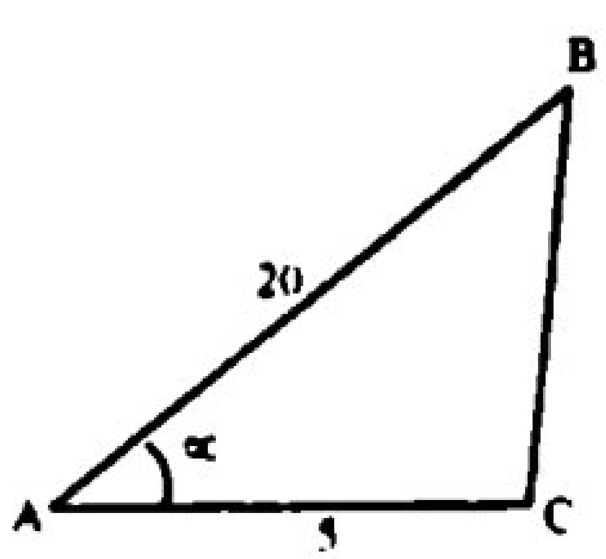
$$\cos \propto = \frac{\overline{AC}}{\overline{AB}}$$

$$\cos \propto = \frac{5}{20}$$

$$\propto = \cos^{-1}(0.25)$$

$$\propto = 75.5°$$

or $\propto = 75°30'$



4. **The base of a rectangle is 25 feet and the night of the rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.**

Solution

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

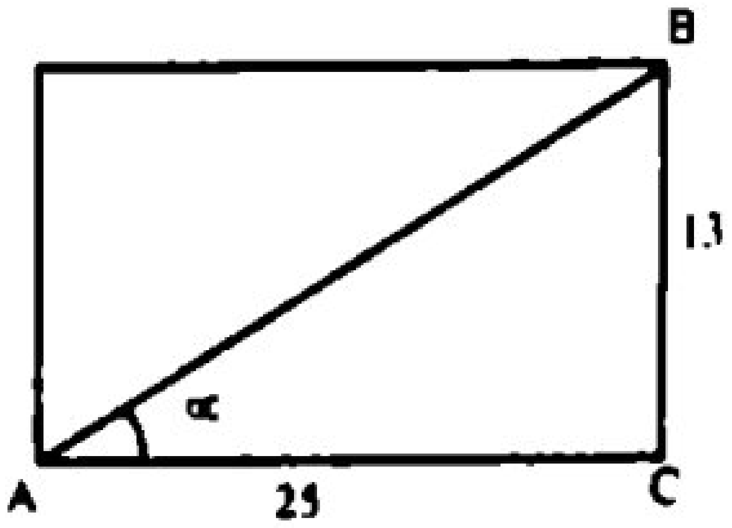
$$\tan \propto = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan \propto = \frac{13}{25}$$

$$\propto = \tan^{-1}(0.52)$$

$$\propto = 27.47^\circ$$

$$\text{or } \propto = 275^\circ 28' 28''$$



5. **A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meter.**

Solution

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

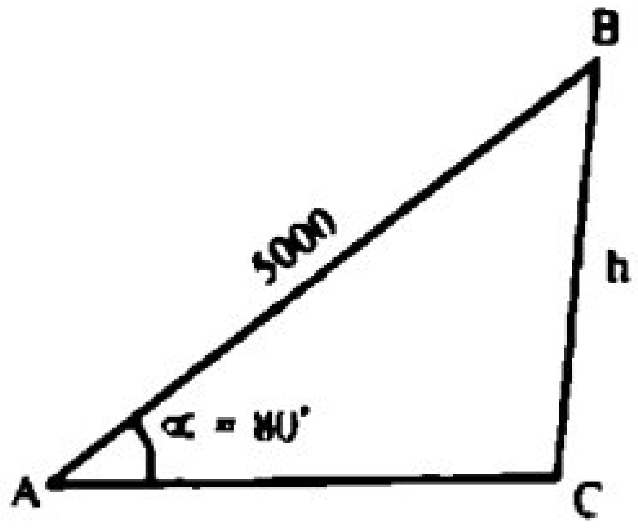
$$\sin \propto = \frac{\overline{BC}}{\overline{AB}}$$

$$\sin 80^\circ = \frac{h}{5000}$$

$$h = 5000 \sin 80^\circ$$

$$h = 5000(0.9848)$$

$$h = 4924.04 \text{ m}$$



6. **An aeroplane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descend?**

Solution

From the figure, we observe that ∝ is the angle of elevation.

Using the fact that

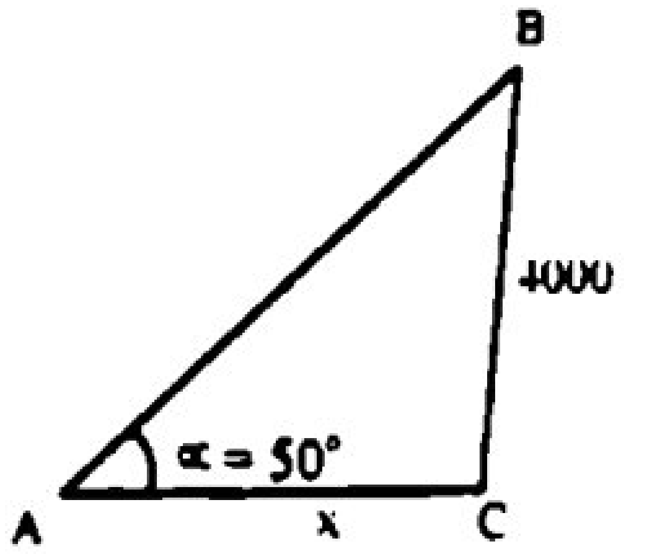
$$\tan \propto = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan 50^\circ = \frac{4000}{x}$$

$$x = \frac{4000}{\tan 50^\circ}$$

$$x = \frac{4000}{1.1918}$$

$$x = 3356.4m$$



7. A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution

From the figure, we observe that ∝ is the angle of elevation.
Using the fact that

$$\tan \propto = \frac{\overline{BC}}{AC}$$

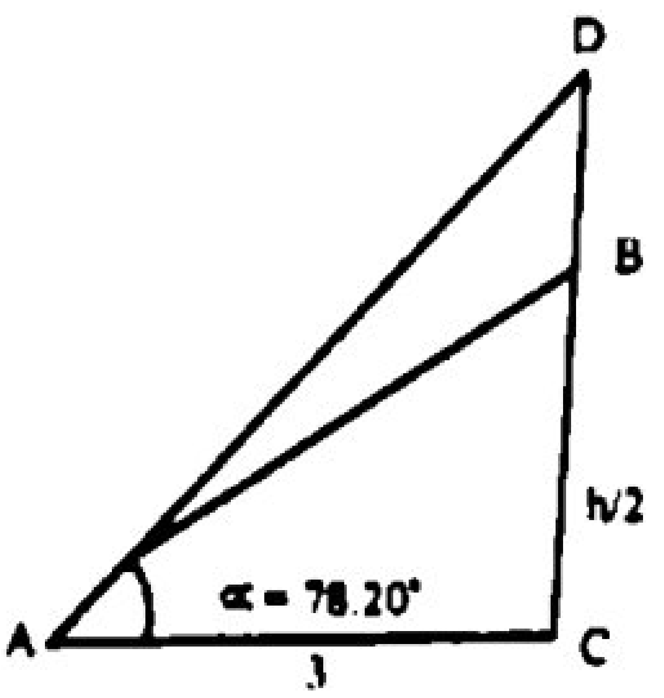
$$\tan 78.2^\circ = \frac{h/2}{3}$$

$$\frac{h}{2} = 3\tan 78.2^\circ$$

$$\frac{h}{2} = 3(4.7867)$$

$$h = 6(407867)$$

$$h = 28.72 \quad m$$



8. A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?

Solution

From the figure, we observe that ∝ is the angle of elevation.
Using the fact that

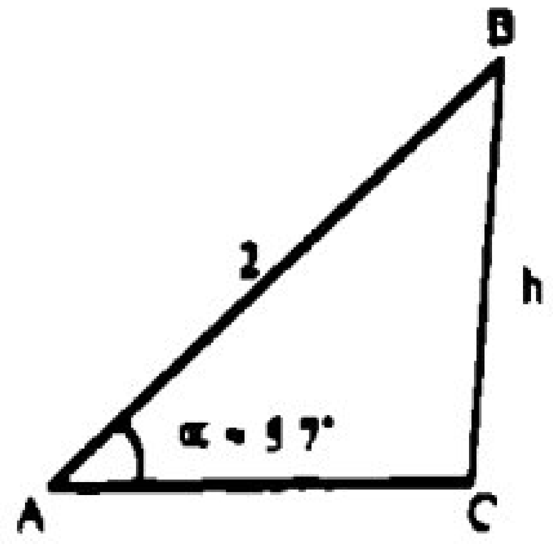
$$\cos \propto = \frac{\overline{BC}}{AB}$$

$$\cos 5.7^\circ = \frac{hc}{2}$$

$$= 2\cos 5.7^\circ$$

$$= 2(0.995)$$

$$= 1.99 \quad miles$$



9. A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of the antenna is 21.8°. Find the height of the house.

Solution

In Δ ABC

$$\tan 17^\circ = \frac{\overline{BC}}{\overline{AC}}$$

$$0.3057 = \frac{h}{AC}$$

$$AC = \frac{h}{0.3057} \qquad (1)$$

In Δ ADC

$$\tan 21.8^\circ = \frac{\overline{CD}}{\overline{AC}}$$

$$0.4 = \frac{\overline{BC} + \overline{BD}}{\overline{AC}}$$

$$0.4 = \frac{h+8}{AC}$$

$$AC = \frac{h+8}{0.4} \qquad (2)$$

Comparing eq. (1) and (2), we have

$$\frac{h}{0.3057} = \frac{h+8}{0.4}$$

$$0.4h = 0.3057(h+8)$$

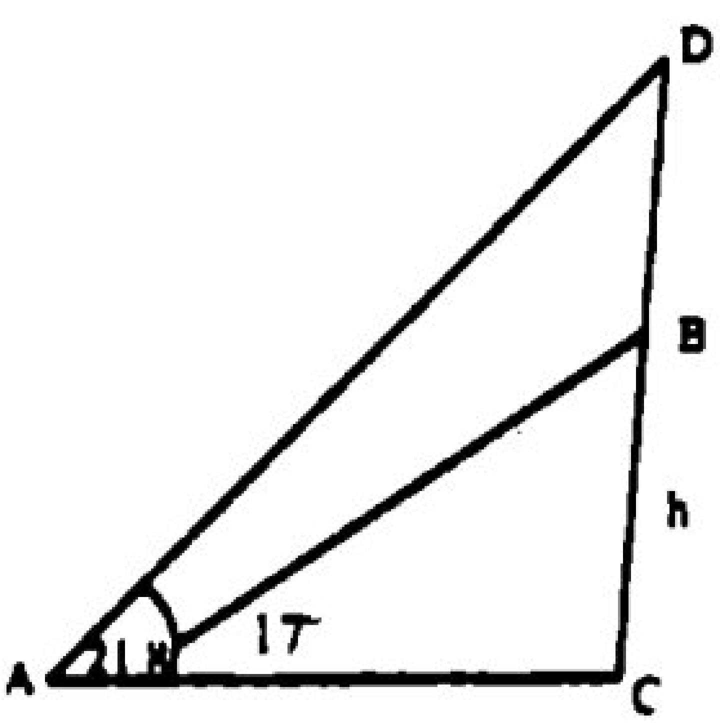
$$0.4h = 0.3057h + 2.4456$$

$$0.4h - 0.3057h = 2.4456$$

$$0.0943h = 2.4456$$

$$h = \frac{2.4456}{0.0943}$$

$$h = 25.94 \quad \text{feet}$$



10. From an observation point, the angles of depression of two boats in line with this: point are found to 30° and 45°. Find the distance between the two boats if the point of observation is 4000 feet high.

Solution

In Δ BCD

$$\tan 45^\circ = \frac{\overline{CD}}{\overline{BC}}$$

$$1 = \frac{4000}{\overline{BC}}$$

$$BC = 4000$$

In Δ ACD

$$\tan 30^\circ = \frac{\overline{CD}}{\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{\overline{AB} + \overline{BC}}$$

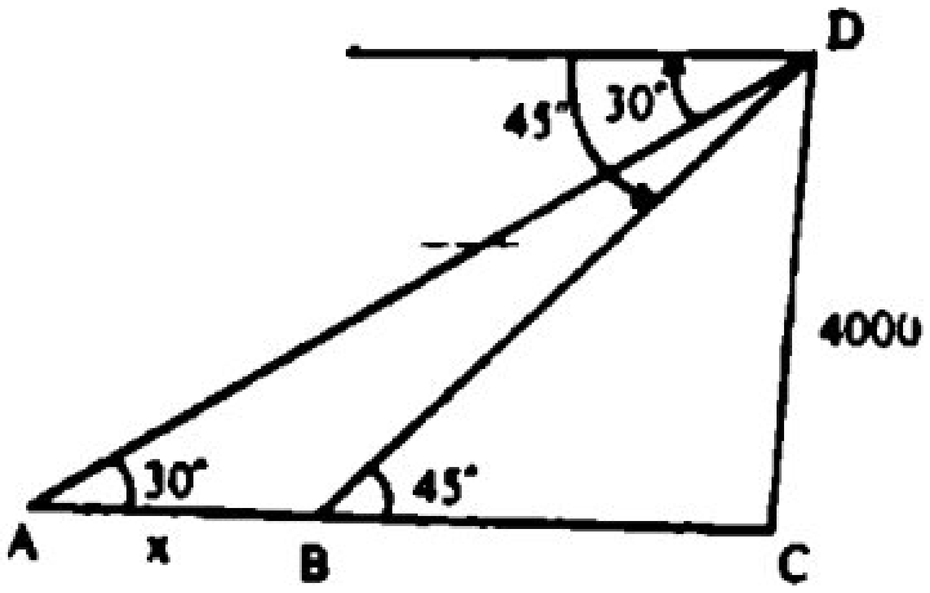
$$\frac{1}{\sqrt{3}} = \frac{4000}{\overline{AC} + 4000}$$

$$AC + 4000 = 4000\sqrt{3}$$

$$\overline{AC} = 4000\sqrt{3} - 4000$$

$$\overline{AC} = 6928.2 - 4000$$

$$\overline{AC} = 2928.2 \text{ feet}$$



11. **Two ships, which are in line with the base of a vertical cliff, are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°G, as show in the diagram.**

(a) **Calculate the distance $\overline{BC}$**

(b) **Calculate the height $\overline{CD}$, of the cliff.**

Solution

In Δ BCD

$$\tan 45^\circ = \frac{\overline{CD}}{\overline{BC}}$$

$$1 = \frac{\overline{CD}}{\overline{BC}}$$

$$\overline{BC} = \overline{CD} \qquad \_\_\_\_(1)$$

In Δ ACD

$$\tan 30^\circ = \frac{\overline{CD}}{\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{CD}}{\overline{AB} + \overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{BC}}{\overline{AB} + \overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{BC}}{120 + \overline{BC}} \qquad \because \text{From (1) } \overline{BC} = \overline{CD}$$

$$\sqrt{3}\,\overline{BC} = 120 + \overline{BC}$$

$$\sqrt{3}\,\overline{BC} - \overline{BC} = 120$$

$$\left(\sqrt{3} - 1\right)\overline{BC} = 120$$

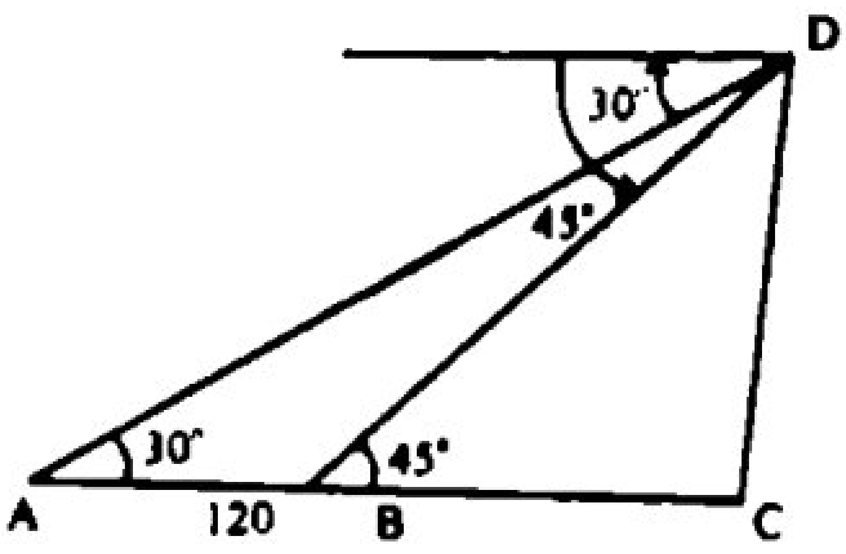
$$0.732\,\overline{BC} = 120$$

$$\overline{BC} = \frac{120}{0.732}$$

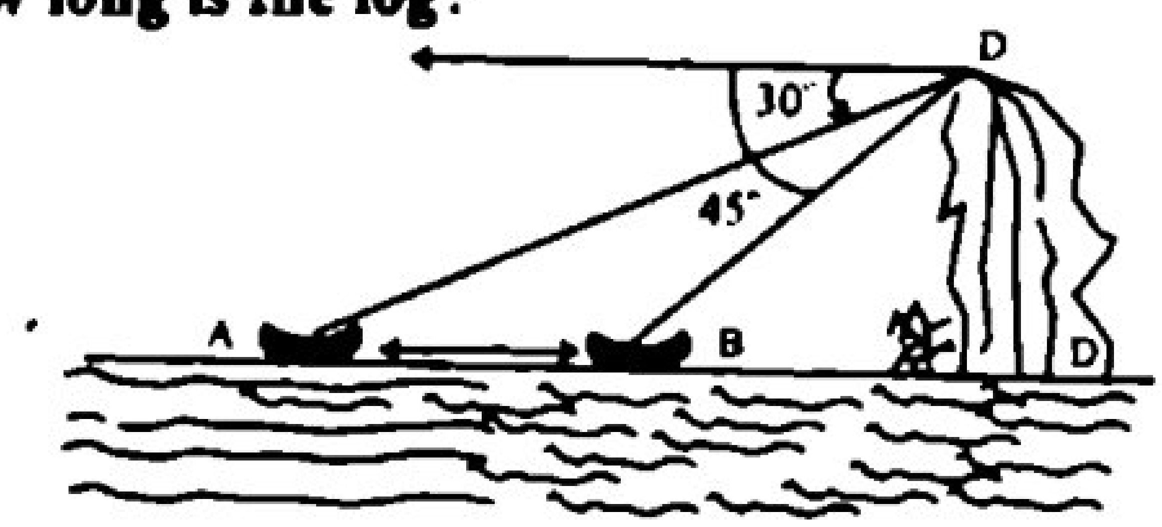
$$\overline{BC} = 164\text{ m}$$

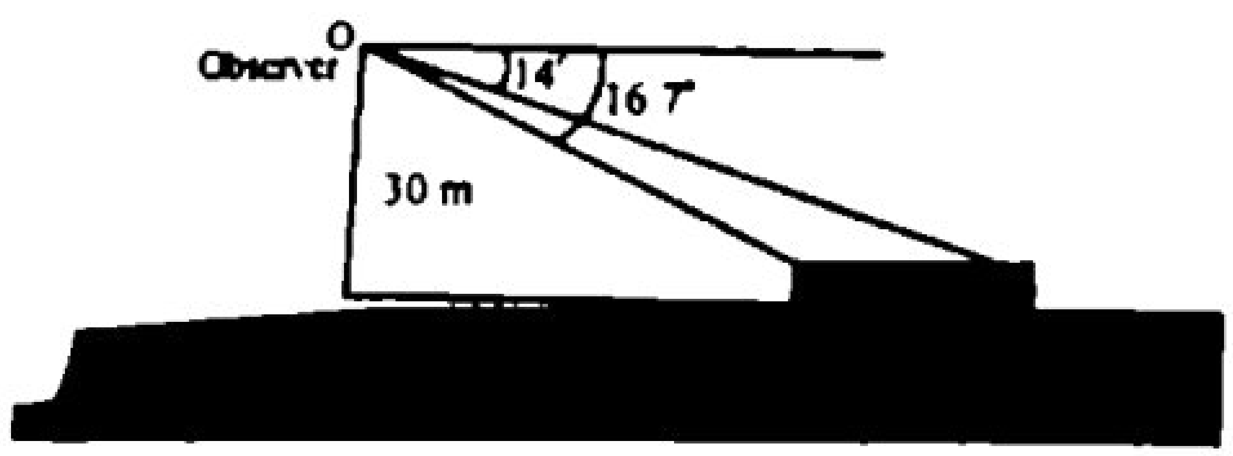
Put $\overline{BC} = 164$ in eq.(1), we get

$$\overline{CD} = 164\text{ m}$$



12. Suppose that we are standing on a bridge 30 feet above a river watching a log (piece of wood) floating toward we. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°, how long is the log?





Solution

In Δ BOC

$$\tan 16.7^\circ = \frac{\overline{OC}}{\overline{BC}}$$

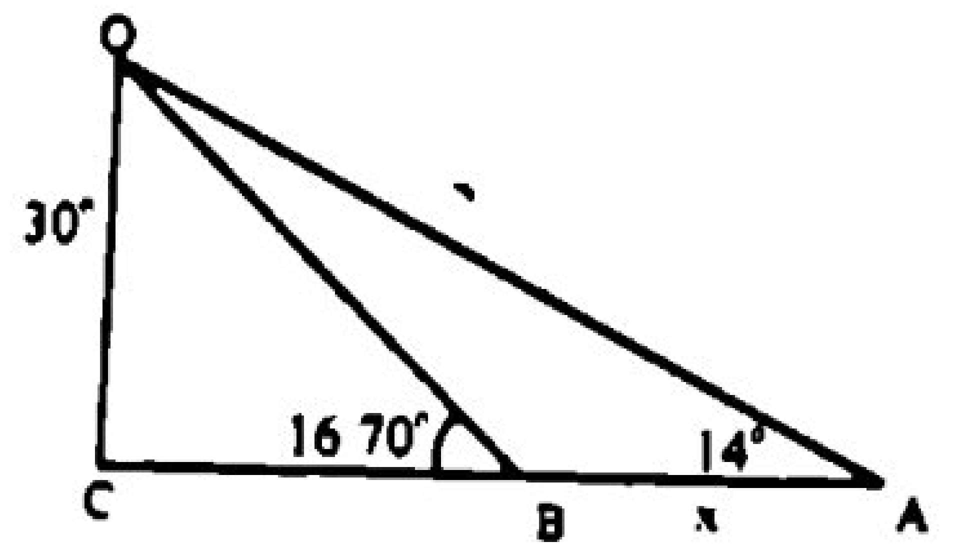
$$0.3 = \frac{30}{\overline{BC}}$$

$$\overline{BC} = \frac{30}{0.3}$$

$$\overline{BC} = 100m$$

In Δ AOC

$$\tan 14° = \frac{\overline{OC}}{\overline{AC}}$$



$$0.2493 = \frac{30}{\overline{AB} + \overline{BC}}$$

$$0.2493 = \frac{30}{x + 100}$$

$$0.2493(x + 100) = 30$$

$$0.2493x + 24.93 = 30$$

$$0.2493x = 30 - 24.93$$

$$0.2493x = 5.07$$

$$x = \frac{5.07}{0.2493}$$

$$x = 20.33 \text{ m}$$

SOLVED MISCELLANEOUS EXERCISE - 7

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The union of two non-collinear rays, which have common end point is called

(a) an angle (b) a degree (c) a minute (d) a radian

(ii) The system of measurement in which the angle is measured in radians is called

(a) CGS system (b) sexagesimal system

(c) MKS system (d) circular system

(iii) $20° =$

(a) 360′ (b) 630′ (c) 1200′ (d) 3600′

(iv) $\frac{3\pi}{4}$ radians =

(a) 115° (b) 135° (c) 150° (d) 150°

(v) If $\tan\theta = \sqrt{3}$, then θ is equal to

(a) 90° (b) 45° (c) 60° (d) 30°

(vi) $\sec 2\theta =$