Proof: .

Statements	Reasons
In an isosceles ΔABC with $m\overline{AB} = m\overline{AC}$. If $\angle C$ is acute,	
then $(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2m\overline{AC}.m\overline{CE}$,	By Theorem 2
$(\overline{AC})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2m\overline{AC}.m\overline{CE}$	Given mAB = mAC
$\Rightarrow$ $(\overline{BC})^2 - 2m\overline{AC}.m\overline{CE} = 0$	Cancel $(AC)^2$ on bath sides
or $(\overline{BC})^2 = 2m\overline{AC}, m\overline{CE}$	

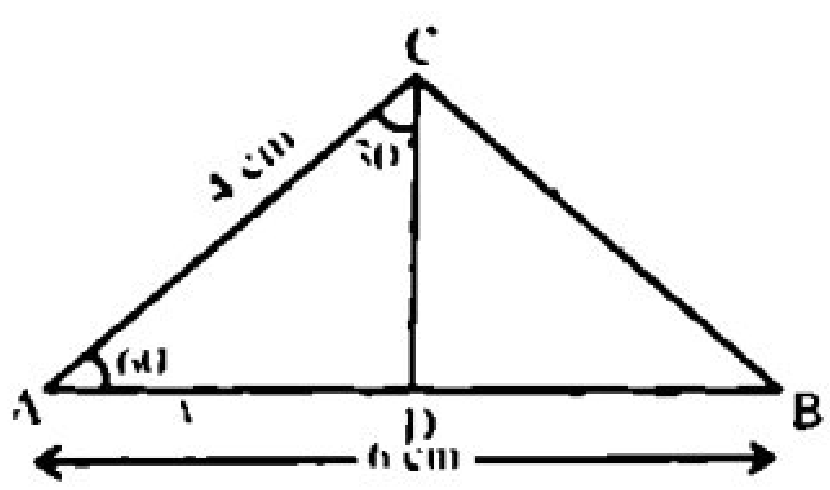
SOLVED EXERCISE 8.2

Q1. In a ΔABC calculate $m\overline{BC}$ when $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60^\circ$.

Solution:

Given: mAB = 6cm; mAC = 4cm; m CA = 60°.

Required: $m\overline{CB} = ?$



In Δ ABC, we have

$$(\overline{BC})^2 = (\overline{AB}) + (\overline{AC})^2 - 2(\overline{AB}).(\overline{AD})$$

$$= (6)^2 + (4)^2 \times 2(6)(x)$$

$$= 36 + 16 - 2(6)(2)$$

$$= 52 - 24$$

$$= 28$$

$\because \cos 60^\circ = \frac{x}{4}$

$\frac{1}{2} = \frac{x}{4}$

$2x = 4$

$\Rightarrow x = 2$

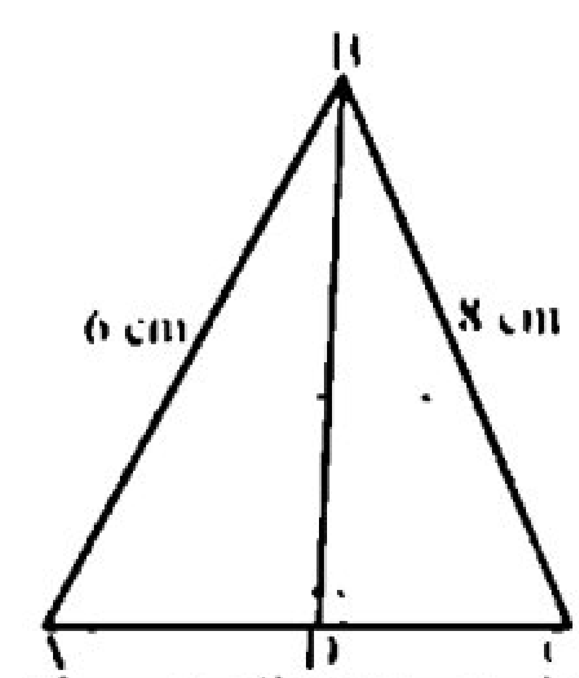
$m\overline{BC} = \sqrt{28}$

$= 2\sqrt{7}$ cm $\Rightarrow = 5.29$ cm

Q2. In ΔABC, $\overline{AB} = 6$ cm, $\overline{BC} = 8$ cm, $\overline{AC} = 9$ cm and D is the mid point of side $\overline{AC}$. Find length of the median $\overline{BD}$.

Solution:

According to the figure, we have



$m\overline{AD} = \overline{DC}$

and $m\overline{AC} = m\overline{AD} + m\overline{DC}$

$m\overline{AC} = m\overline{AD} + m\overline{AD}$

$9 = 2m\overline{AD}$

Or $2m\overline{AD} = 9$

$m\overline{AD} = \frac{9}{2} = 4.5$ cm

We know that

$$(\overline{AC})^2 + (\overline{BC})^2 = 2\left[(\overline{AD})^2 + (BD)^2\right]$$

$$(6)^2 + (8)^2 = 2\left[(4.5)^2 + (\overline{BD})^2\right]$$

$$36 + 64 = 2(4.5)2 + 2(\overline{BD})^2$$

$$100 = 40.5 + 2\ (\overline{BD})^2$$

Or $$2(BD)^2 = 100 - 40.5$$

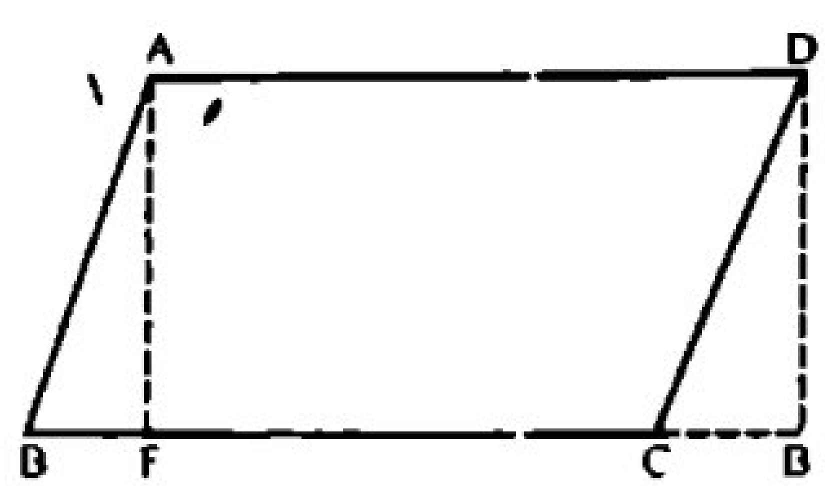
$$2\,\overline{BD}^2 = 59.5$$

$$\Rightarrow \quad \overline{BD}^2 = 29.75$$

$$\Rightarrow \quad \overline{BD} = \sqrt{29.75} \doteq 5.45 \text{ cm}$$

Q3. In a quadrilateral $\overline{AB}$ CD prove that $(\overline{AC})^2 + (\overline{AD})^2 = 2\left[(\overline{AB})^2 + (\overline{BC})^2\right]$

Solution:



$$(\overline{BD})^2 = (\overline{CD})^2 + (\overline{BC})^2 + 2\ (\overline{BC})(\overline{CE}) \qquad (1)$$

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2\ (\overline{BC})\ (\overline{BF}) \qquad (2)$$

Adding (1) and (2), we get

$$(\overline{AC})^2 + (\overline{BD})^2 = (\overline{CD})^2 + (\overline{BC})^2 + 2(\overline{BC})\ \overline{CE} + (\overline{AB})^2 + (\overline{BC})^2 - 2(\overline{BC})\ (\overline{BF})$$

$$= (\overline{AB})^2 + (\overline{CD})^2 + 2(\overline{BC})^2 + 2(\overline{BC})\ \ (\overline{CE})^2 - 2(\overline{BC})\ (\overline{BF})$$

In parallelogram opposite sides are congruent, so

$\overline{AB} = \overline{DC}$, $\overline{AD} = \overline{BC}$, and $\overline{BF} = \overline{CE}$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2(\overline{AB})^2 + (\overline{AB})^2\ 2(\overline{BC})^2 + 2\ (\overline{CE}) - 2\ (\overline{BC})\ \overline{CE}$$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2(\overline{AB})^2 + 2(\overline{BC})^2$$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2\left[(\overline{AB})^2 + (\overline{BC})^2\right]$$

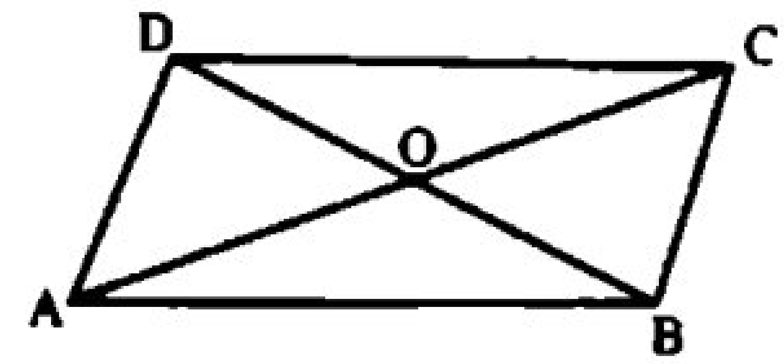
Hence Proved.

Q4. Prove that the sum of the squares of the sides of a parallelogram is equal to sum of the squares of its diagonals.

Solution:

Given:

ABCD is a parallelogram with $\overline{AC}$ and $\overline{BD}$ are its diagonals.



To Prove

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{DA}^2 = \overline{AC}^2 + \overline{BD}^2$$

In ΔACD

$$\overline{DC}^2 + \overline{AD}^2 = 2\overline{OD}^2 + \overline{OA}^2 \quad \text{\_\_\_\_\_\_ (i)}$$

And In ΔABC

$$\overline{AB}^2 + \overline{BC}^2 = 2\overline{OB}^2 + \overline{OA}^2 \quad \text{\_\_\_\_\_\_ (ii)}$$

Adding (i) & (ii)

$$\overline{DC}^2 + \overline{AD}^2 + \overline{AB}^2 + \overline{BC}^2 = 4\overline{OA}^2 + 2\overline{OD}^2 + 2\overline{OB}^2$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = 4\overline{OA}^2 + 2\overline{OD}^2 + 2\overline{OD}^2 \qquad \left[\because \overline{OB} = \overline{OD}\right]$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = 4\overline{OA}^2 + 4\overline{OD}^2$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = \left(2\overline{OA}\right)^2 + \left(2\overline{OD}\right)^2$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = \overline{AC}^2 + \overline{BD}^2$$

Hence proved

SOLVED MISCELLANEOUS EXERCISE 8

Q1. In a ΔABC, $m\angle A = 60°$, prove that $(\overline{BC})^2 = (\overline{AB})^2 + \overline{AC}^2 - m\,\overline{AB}\,.\,m\,\overline{AC}$.

Solution:

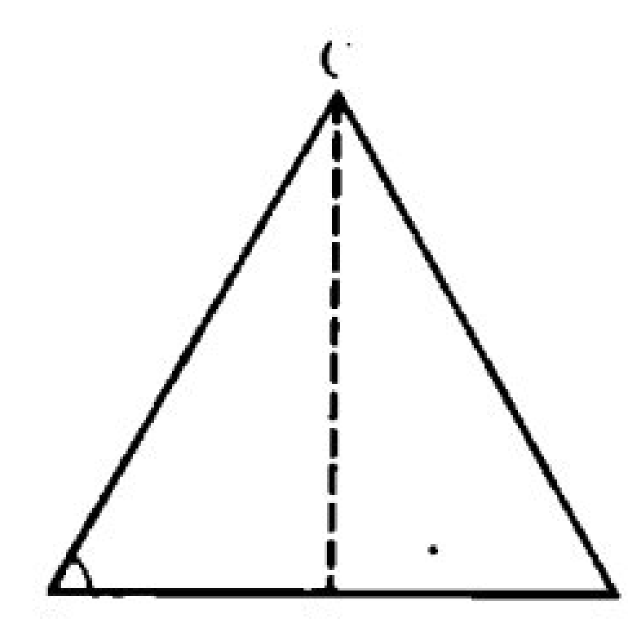
In a ΔABC, $m\angle A = 60°$,

Given:

In a ΔABC, $m\angle A = 60°$

Required:

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \overline{AB}.\overline{AC}$$



Construction:

Draw $\overline{CD} \perp \overline{AB}$, so that the Projection of $\overline{AC}$ on AB.

Proof:

In right angle ΔACD

$\angle A = 60°$ and $\angle ACD = 30°$ (being complement of $\overline{CA}$)