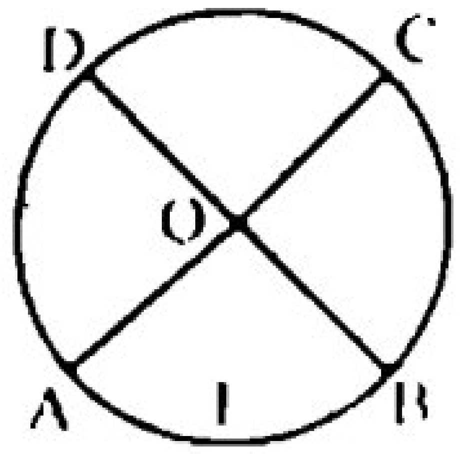
SOLVED EXERCISE 9.1

1. Prove that, only the diameters of a circle are the intersecting chords which bisect each other.



Given: A circle having diameters $\overline{AC}$ and $\overline{BD}$ which passes through centre O.

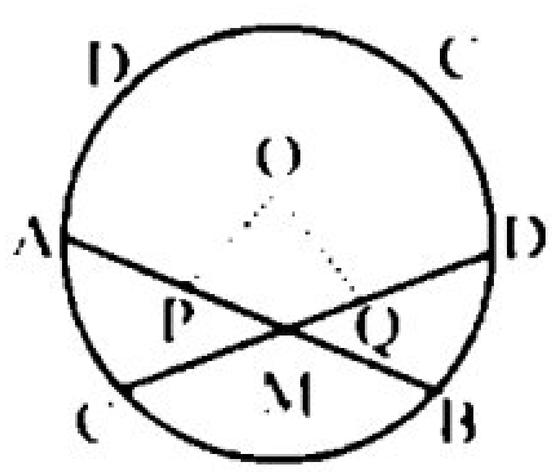
To Prove: Diameters $\overline{AC}$ and $\overline{BD}$ bisect each other.

Proof:

Statements		Reasons
$\overline{OA} \cong \overline{OC}$	(i)	Common
Similarly $m\overline{OC} \cong \overline{OD}$	(ii)	
$m\overline{OA} = m\overline{OD}$	(iii)	
From (i), (ii) and (iii), we have		radii of the same circle
$m\overline{OA} = m\overline{OB} = m\overline{OC} = \overline{OD}$		

Hence AC and BD are intersecting chords which bisect each other.

2. Two chords of a circle do not pass through the centre. Prove that they cannot bisect each other,



Given:

A circle with centre O having two chords $\overline{AB}$ and $\overline{CD}$

To Prove:

M is not the mid-point of chords $\overline{AB}$ and $\overline{CD}$

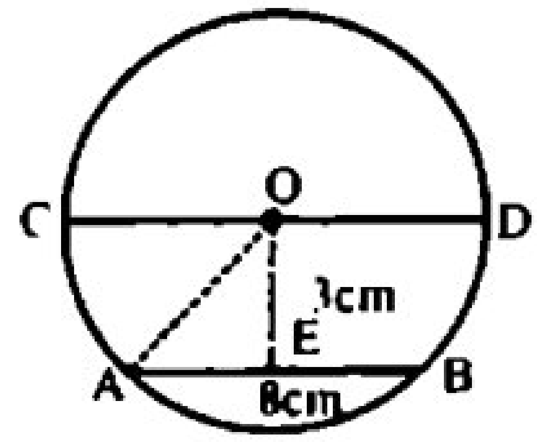
Construction:

Join O to P and Q such that $\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$

Proof:

Statements	Reasons
O is the centre of the circle with $\overline{OP} \perp \overline{AB}$	Construction
Thus $\overline{OP} \perp \overline{AB}$	
Now point M lies between P and B. Therefore M is not the midpoint of AB.	
Hence $\overline{AB}$ and $\overline{CD}$ cannot bisect each other.	

3. If the length of the chord AB = 8cm. Its distance from the centre is 3 cm, then measure the diameter of such circle.



Given: mAB = 8cm, mOE = 3cm

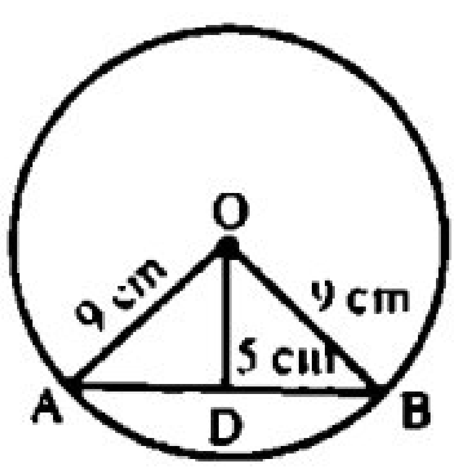
Required: to find the length of diameter i.e., mCD = ?

Construction: Join O to A and E.

Proof:

Statements	Reasons
In Δ AEO	$m\overline{AO} = m\overline{OC} = m\overline{OD} = 5cm$
$(AO)^2 = \overline{AE}^2 + \overline{EO}^2$	
$= \left[\frac{1}{2}(\overline{AB})\right]^2 + (3)^2$	$\Rightarrow \overline{CD} = \overline{CO} + m\overline{OD}$
	$= 5cm + 5cm$
$= \left[\frac{1}{2} \times 8\right]^2 + 9$	$= 10cm$
$= (4)^2 + 9$	Hence
$= 16 + 9 = 25cm$	Diameter = 10cm
$\Rightarrow \overline{AO} = \sqrt{25} = 45cm$	

4. Calculate the length of a chord which stands at a distance 5cm from the centre of a circle whose radius is 9cm.



Given:

$m\overline{OA} = m\overline{OB} = 8cm.$

$m\overline{OD} = 5cm$

Required:

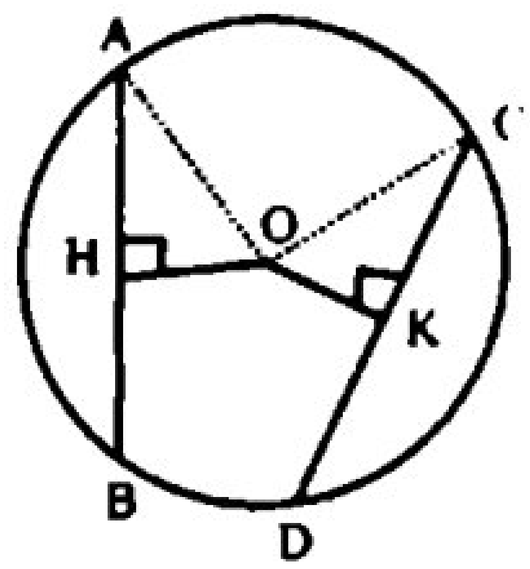
$m\overline{AB} = ?$

Proof:

Statements	Reasons
In Δ OAD.	
$m\overline{OA}^2 = m\overline{OD}^2 + m\overline{AD}^2$	
$m\overline{OA}^2 - m\overline{OD}^2 = m\overline{AD}^2$	$\left[\because AD = \frac{1}{2}\overline{AB}\right]$
$9^2 - 5^2 = \left[\frac{1}{2}m\left(\overline{AB}\right)\right]^2$	
$\left[\frac{1}{2}m\left(\overline{AB}\right)\right]^2 = 81 - 25$	
$\frac{1}{4}m\left(\overline{AB}\right)^2 = 56$	
$\Rightarrow m\overline{AB}^2 = 56 \times 4 = 224$	
$AB = \sqrt{224}$ 14.97cm	

THEOREM 4

9.1 (iv) If two chords of a circle are congruent then they will be equidistant from the centre.



Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{UK} \perp \overline{CD}$.

To prove:

$m\ \overline{OH} = m\ \overline{OK}$

Construction:

Join O with A and O with C So that we have ∠ rt Δ' OAH and OCX.

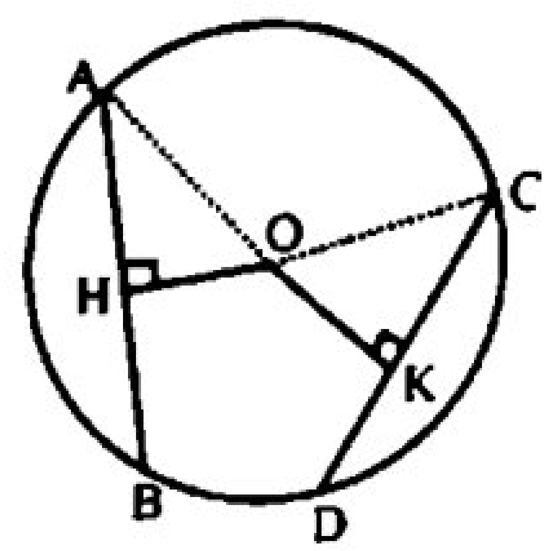
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	OH ⊥ AB (By Theorem 3)
i.e., $m\ \overline{AH} = \frac{1}{2}\ m\overline{AB}$ (i)	
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ (By Theorem 3)
i.e., $m\overline{CK} = \frac{1}{2}\ m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii)& (iii)
Now in ∠rt Δ' OAH ↔ OCK	Given $\overline{OH} \perp \overline{AB}$ and OK ⊥ $\overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle

$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\Delta OAH \cong \Delta OCK$	H. S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	

THEOREM 5

9.1 (v) Two chords of a circle which are equidistant from the centre, are congruent.



Given:

$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with centre at O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{UH} = m\overline{OK}$

To prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O. So that we can form ∠rt Δ' OAH and OCK.

Proof:

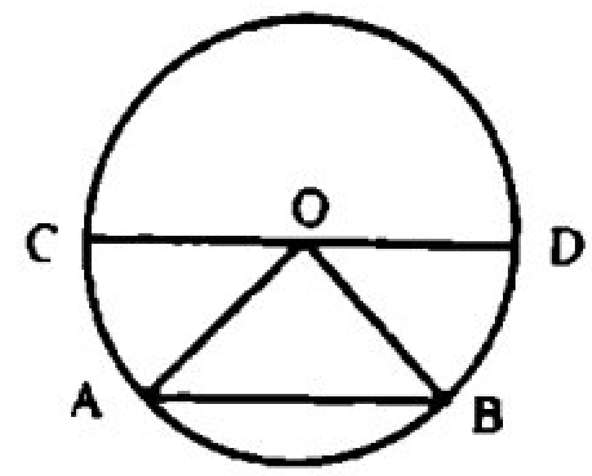
Statements	Reasons
In ∠rt Δ' OAH ↔ OCK.	
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle.
$m\overline{OH} = m\overline{OK}$	Given
$\Delta O\overline{AH} \cong \Delta OCK$	H.S Postulate
So $m\overline{AH} = mCK$ (i)	
But $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	OH ⊥ chord AB (Given)
Similarly $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (iii)	OK ⊥ chord CD (Given)
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\frac{1}{2} m\overline{AB} = m\overline{CD}$	Using (ii)& (iii)
or $m\overline{AB} = m\overline{CD}$	

Example:

Prove that the largest chord in a circle is the diameter.

Given:

$\overline{AB}$ is a chord and $\overline{CD}$ is the diameter of a circle with centre point O.



To prove:

If $\overline{AB}$ and $\overline{CD}$ are distinct, then $m\overline{CD} = m\overline{AB}$.

Construction:

Join O with A and 0 with B then form a ΔOAB.

Proof:

Sum of two sides of a triangle is greater than its third side.

In $\Delta OAS \Rightarrow m\ \overline{OA} + m\ \overline{OB} > m\ \overline{AB}$ ... (i)

But $\overline{OA}$ and $\overline{OB}$ are the radii of the same circle with centre O.

So that $m\ \overline{OA} + m\ \overline{OB} = m\ \overline{CD}$ ... (ii)

$\Rightarrow$ Diameter $\overline{CD}$ > chord $\overline{AB}$ using (i) & (ii).

Hence, diameter CD is greater than any other chord drawn in the circle.

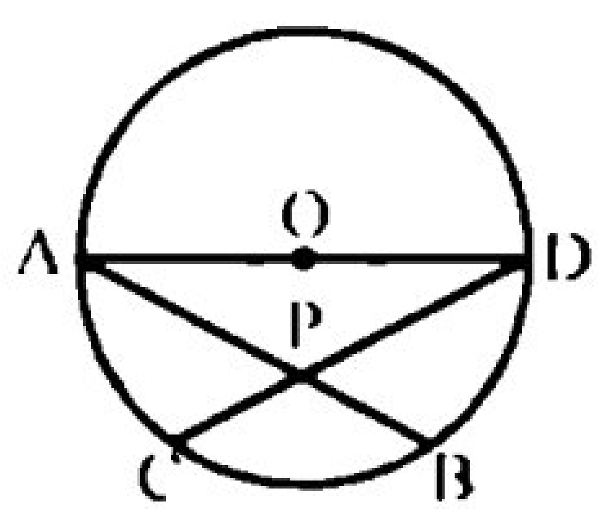
SOLVED EXERCISE 9.2

1. Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.

Given:

In a circle with radius O, we have

$m\overline{AB} = m\overline{CD}$



To Prove:

$\overline{AB} = \overline{CD}$

Construction:

Join O to A and D

Proof:

Because $\overline{AB}$ and $\overline{CD}$ intersect each other, so $m\ \overline{AB} = \overline{AP} + \overline{BP}$

and $m\overline{CD} = m\overline{CP} + m\ \overline{PD}$

$\overline{AP} = m\overline{CP}$ and $m\ \overline{PB} = m\ \overline{PP}$

So $m\ \overline{AB} = m\overline{CP}$

Hence proved