

EXERCISE 1.5

Q.1 Draw the graphs of the following equations.

(i) $x^2 + y^2 = 9$

(ii) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

(iii) $y = e^{2x}$

(iv) $y = 3^x$

Solution:

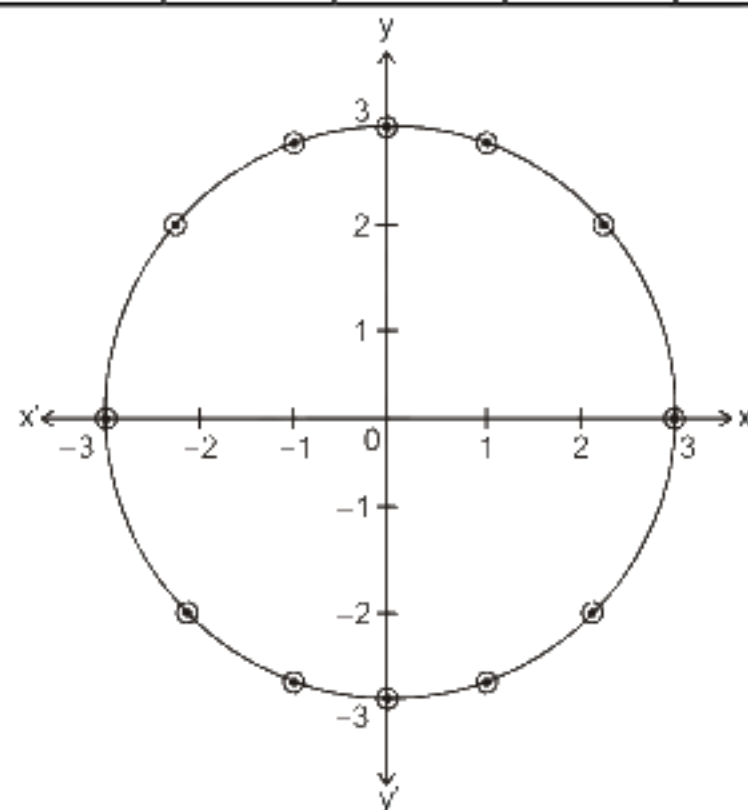
(i) $x^2 + y^2 = 9$

$$y^2 = 9 - x^2$$

$$y = \pm \sqrt{9 - x^2}$$

Its domain is $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
$y = \pm \sqrt{9 - x^2}$	0	± 2.2	± 2.8	± 3	± 2.8	± 2.2	0



(ii) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

$$\frac{y^2}{4} = 1 - \frac{x^2}{16}$$

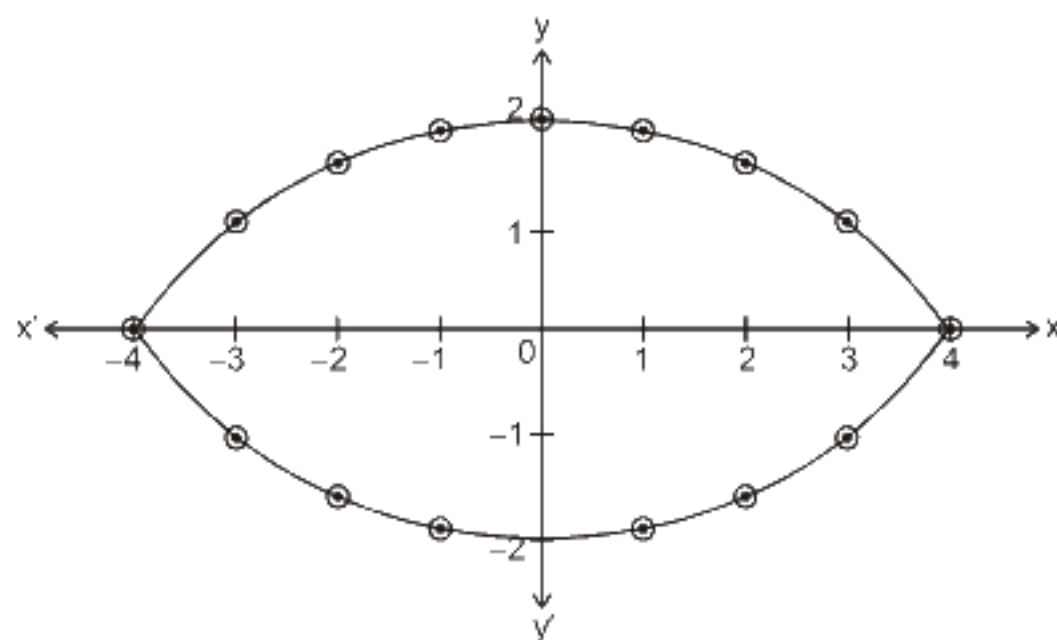
$$y^2 = 4 \left(\frac{16 - x^2}{16} \right)$$

$$y^2 = \frac{16 - x^2}{4}$$

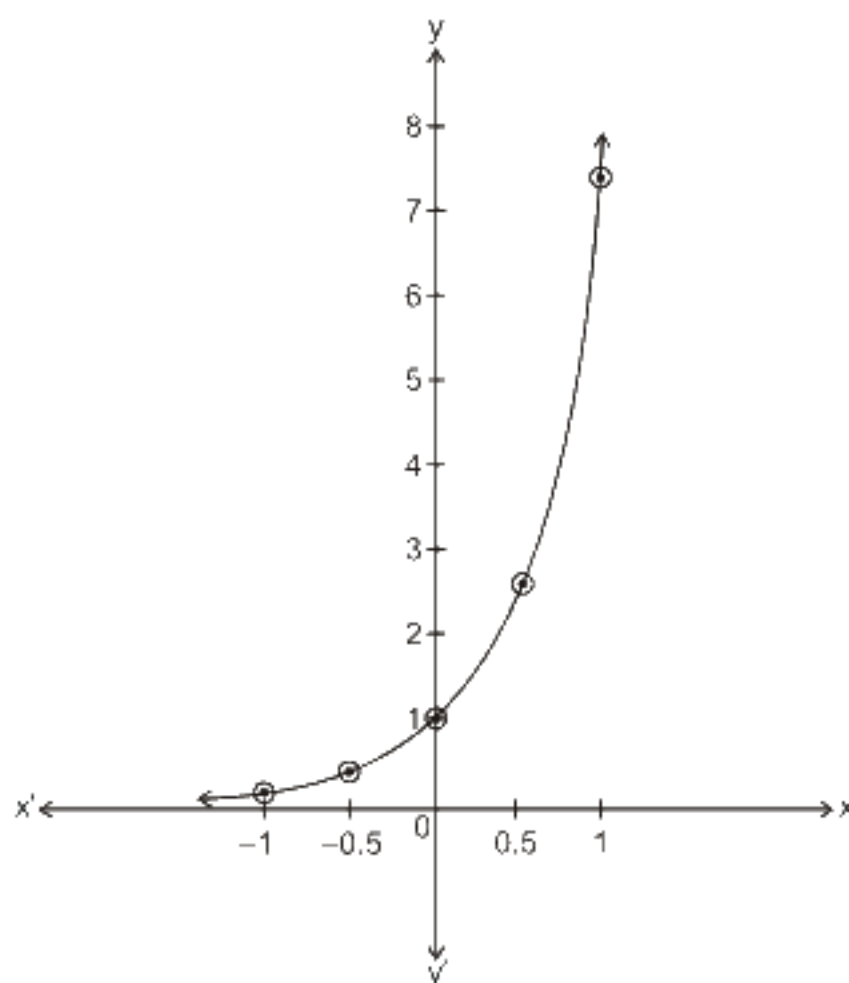
$$y = \pm \frac{\sqrt{16 - x^2}}{2}$$

Its domain is $-4 \leq x \leq 4$.

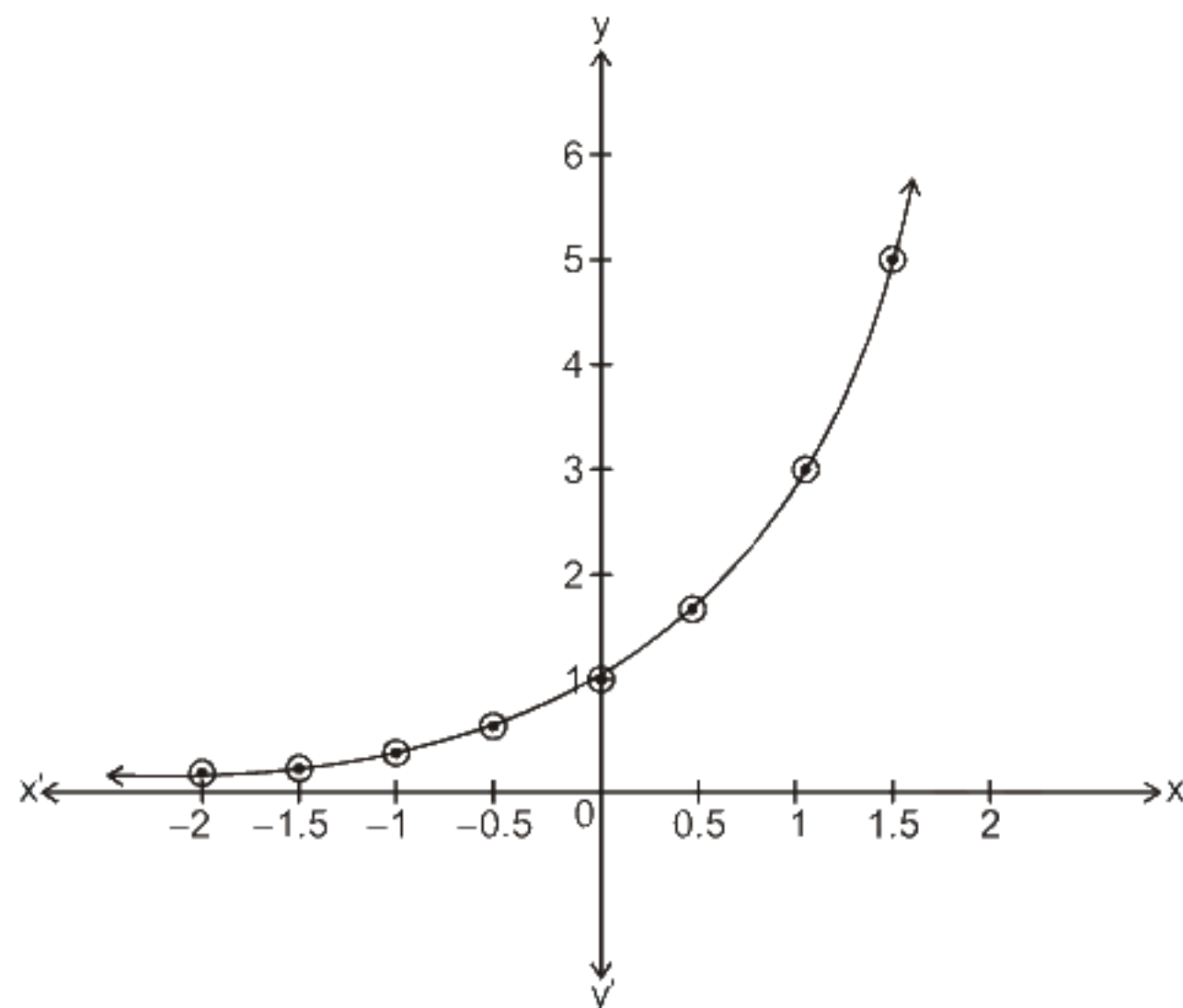
x	-4	-3	-2	-1	0	1	2	3	4
$y = \pm \frac{\sqrt{9-x^2}}{2}$	0	± 1.3	± 1.7	± 1.9	± 2	± 1.9	± 1.7	± 1.3	0

(iii) $y = e^{2x}$

x	-1	-0.5	0	0.5	1
$y = e^{2x}$	0.1	0.4	1	2.7	7.4

(iv) $y = 3^x$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5
$y = 3^x$	0.1	0.2	0.3	0.6	1	1.7	3	5.2



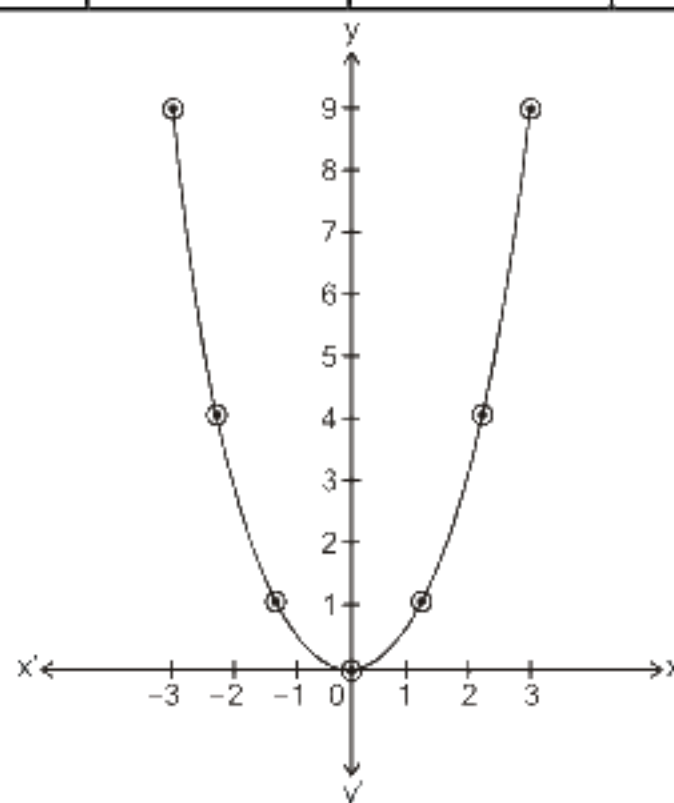
Q.2 Graph the curves that has the parametric equations given below.

- (i) $x = t$, $y = t^2$, $-3 \leq t \leq 3$ where 't' is a parameter
 (ii) $x = t - 1$, $y = 2t - 1$, $-1 < t < 5$ where 't' is a parameter
 (iii) $x = \sec\theta$, $y = \tan\theta$ where ' θ ' is a parameter

Solution:

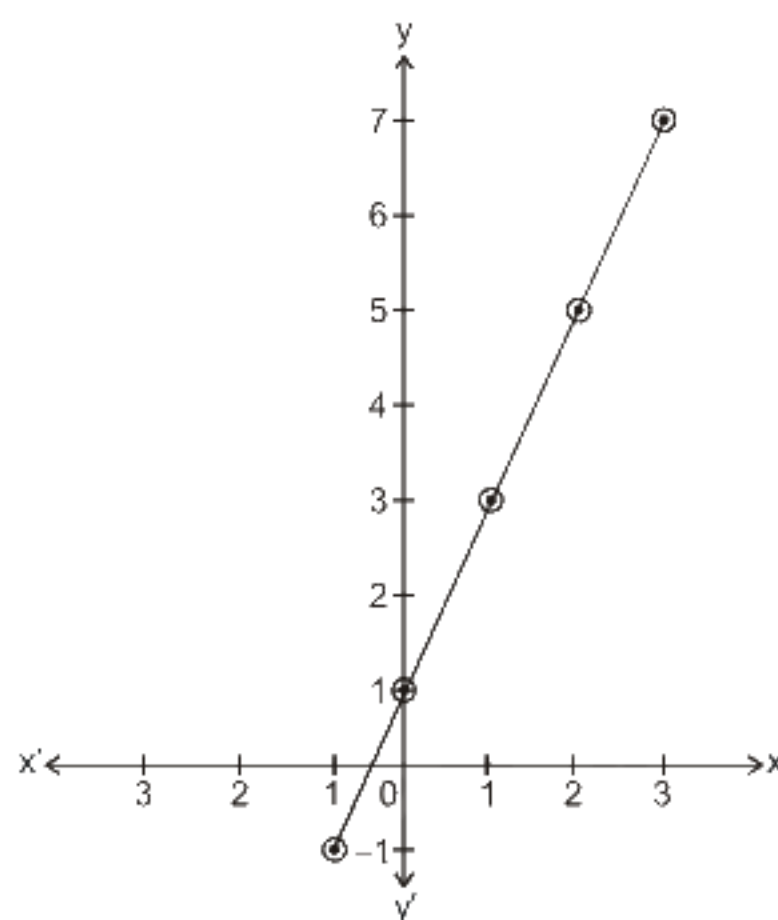
- (i) $x = t$, $y = t^2$, $-3 \leq t \leq 3$ where 't' is a parameter

t	-3	-2	-1	0	1	2	3
$x = t$	-3	-2	-1	0	1	2	3
$y = t^2$	9	4	1	0	1	4	9



(ii) $x = t - 1$, $y = 2t - 1$, $-1 < t < 5$ where 't' is a parameter

t	0	1	2	3	4
$x = t - 1$	-1	0	1	2	3
$y = 2t - 1$	-1	1	3	5	7



(iii) $x = \sec\theta$, $y = \tan\theta$ where ' θ ' is a parameter

$$x^2 = \sec^2\theta, \quad y^2 = \tan^2\theta$$

$$x^2 - y^2 = \sec^2\theta - \tan^2\theta$$

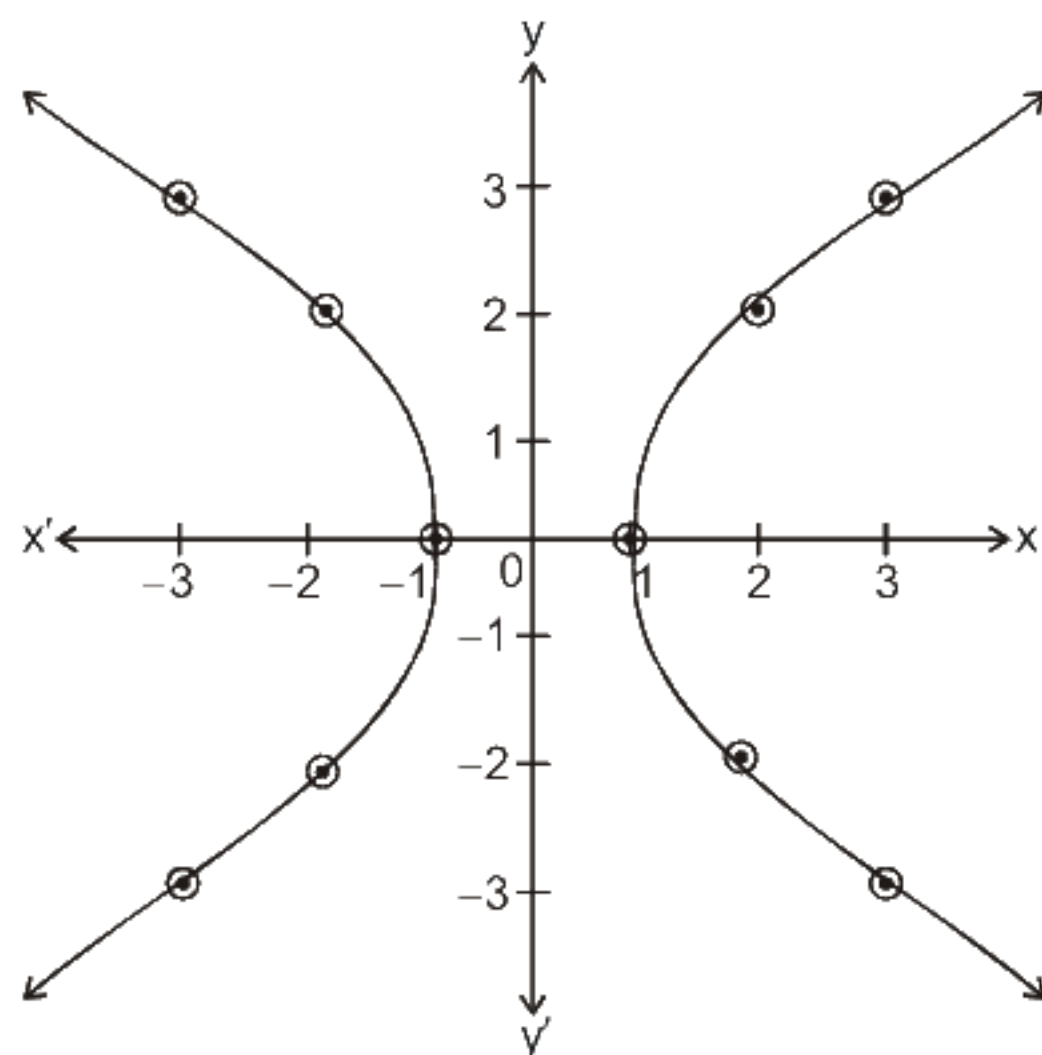
$$x^2 - y^2 = 1$$

$$(\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 = \sec^2\theta - \tan^2\theta)$$

$$y^2 = x^2 - 1$$

$$y = \pm \sqrt{x^2 - 1}$$

x	-3	-2	-1	1	2	3
$y = \sqrt{x^2 - 1}$	± 2.8	± 1.7	0	0	± 1.7	± 2.8



Q.3 Draw the graphs of the functions defined below and find whether they are continuous.

(i) $y = \begin{cases} x - 1 & \text{if } x < 3 \\ 2x + 1 & \text{if } x \geq 3 \end{cases}$

(ii) $y = \frac{x^2 - 4}{x - 2}, \quad x \neq 2$

(iii) $y = \begin{cases} x + 3 & , \quad x \neq 3 \\ 2 & , \quad x = 3 \end{cases}$

(iv) $y = \frac{x^2 - 16}{x - 4}, \quad x \neq 4$

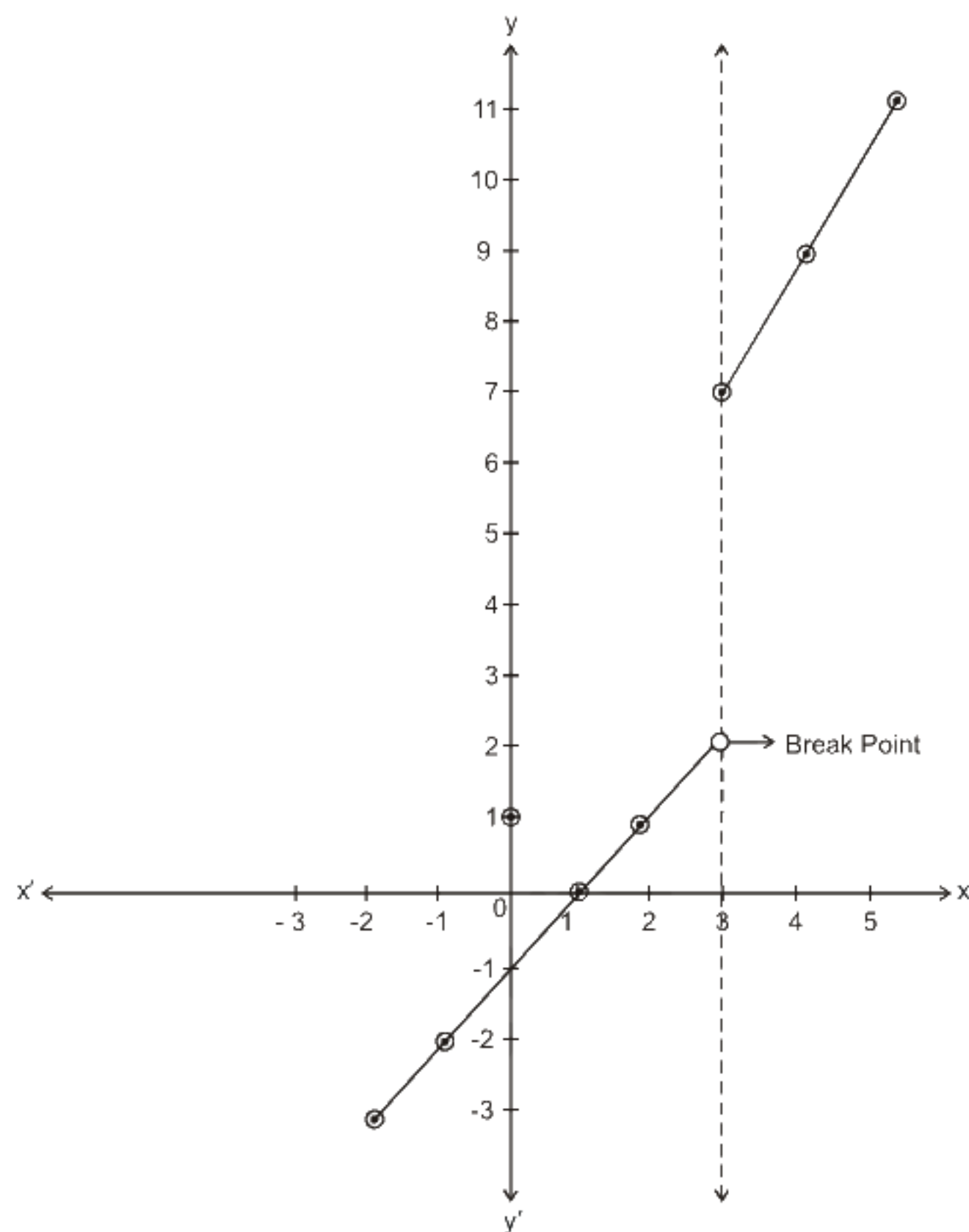
Solution:

(i) $y = \begin{cases} x - 1 & \text{if } x < 3 \\ 2x + 1 & \text{if } x \geq 3 \end{cases}$
 $y = x - 1, \quad x < 3$

x	-2	-1	0	1	2
$y = x - 1$	-3	-2	-1	0	1

$y = 2x + 1, \quad x \geq 3$

x	3	4	5
$y = 2x + 1$	7	9	11

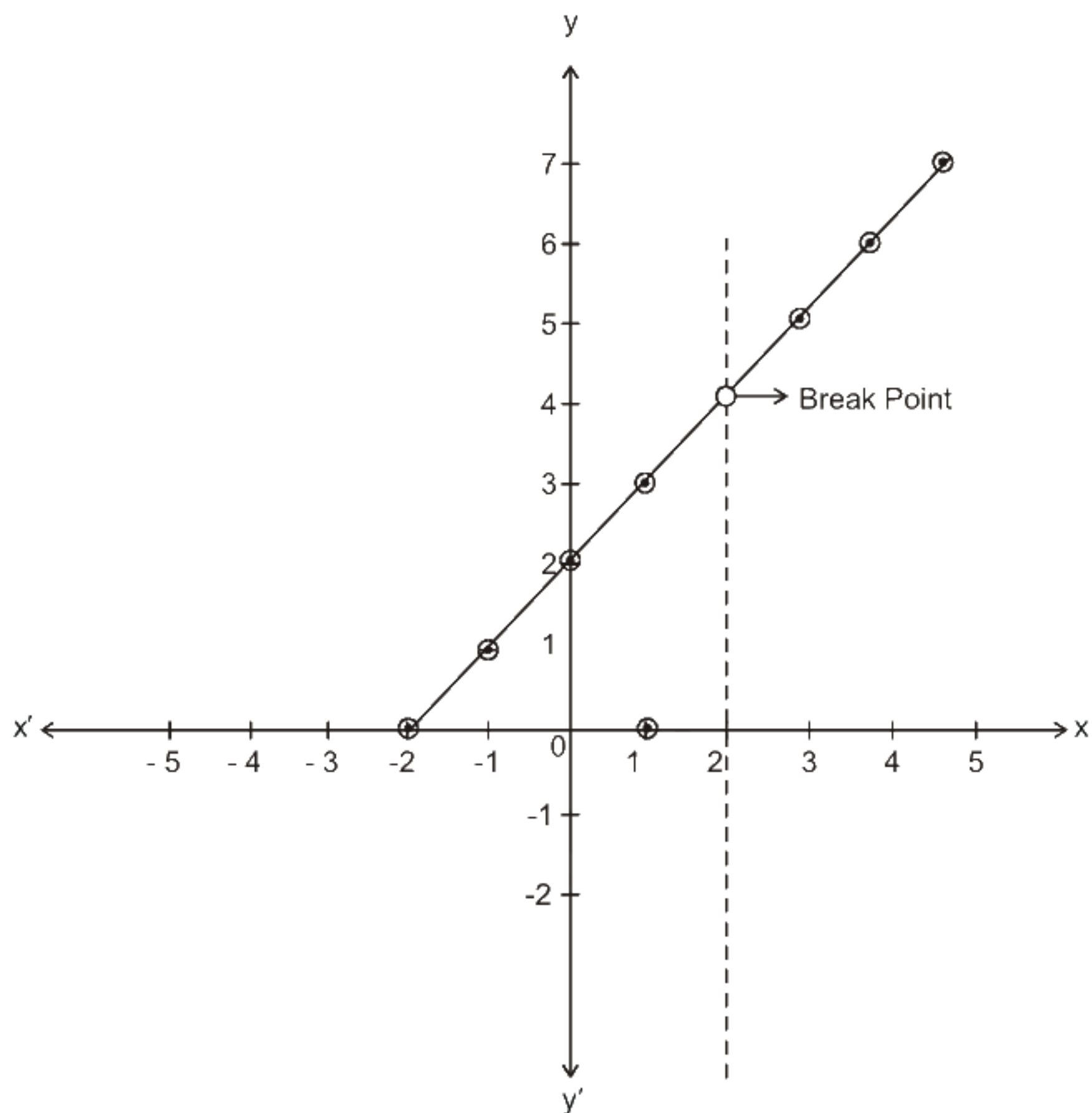


Since there is a break in a graph. So this function is not continuous.

$$\begin{aligned} \text{(ii)} \quad y &= \frac{x^2 - 4}{x - 2}, \quad x \neq 2 \\ &= \frac{(x + 2)(x - 2)}{x - 2}, \quad x \neq 2 \end{aligned}$$

$$y = x + 2, \quad x \neq 2$$

x	-3	-2	-1	0	1	3	4	5
y	-1	0	1	2	3	5	6	7



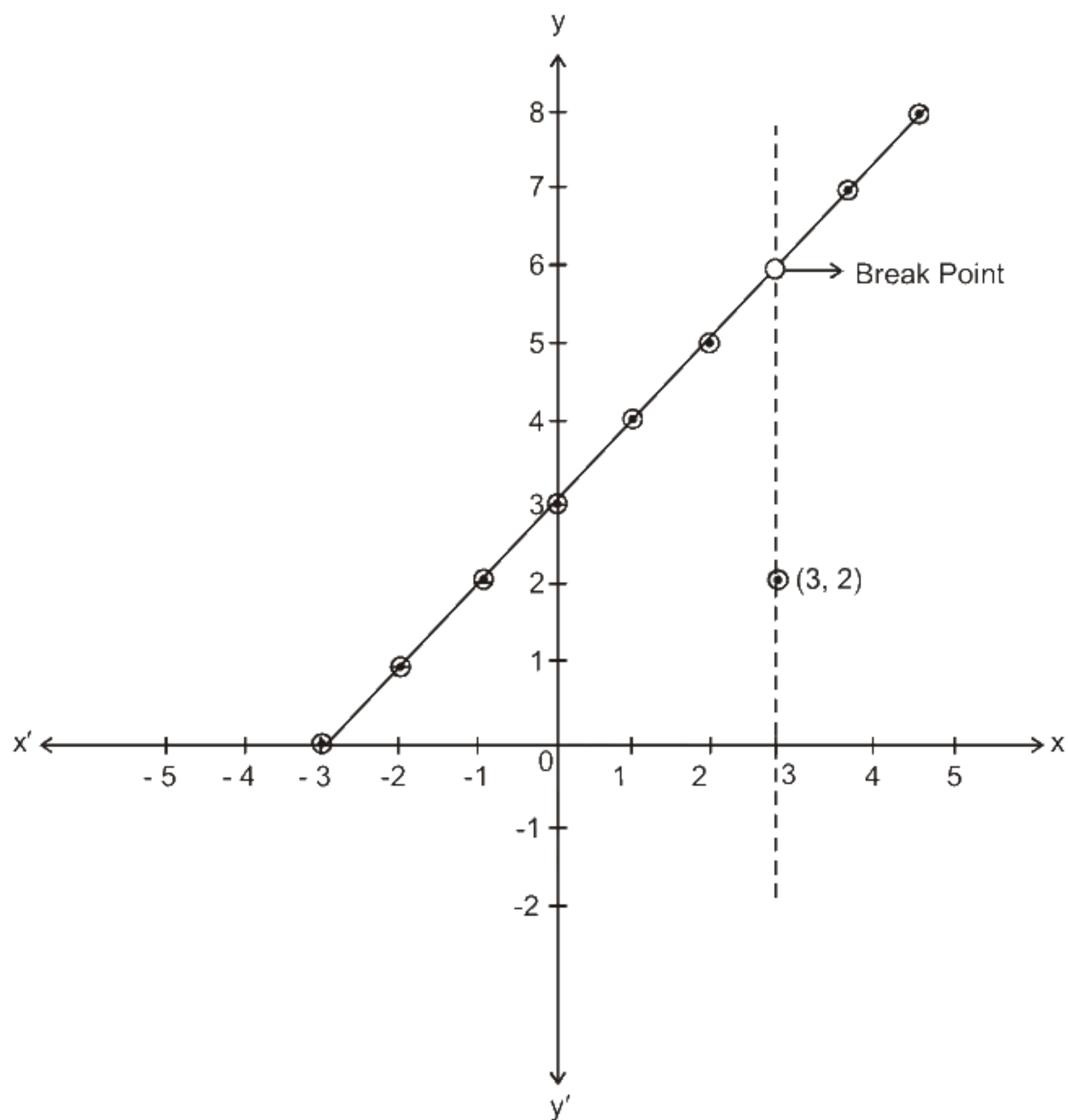
Since there is a break in a graph so this function is not continuous.

$$(iii) \quad y = \begin{cases} x + 3 & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$$

$$y = x + 3 \quad \text{if } x \neq 3$$

x	-3	-2	-1	0	1	3	4	5
y	0	1	2	3	4	5	7	8

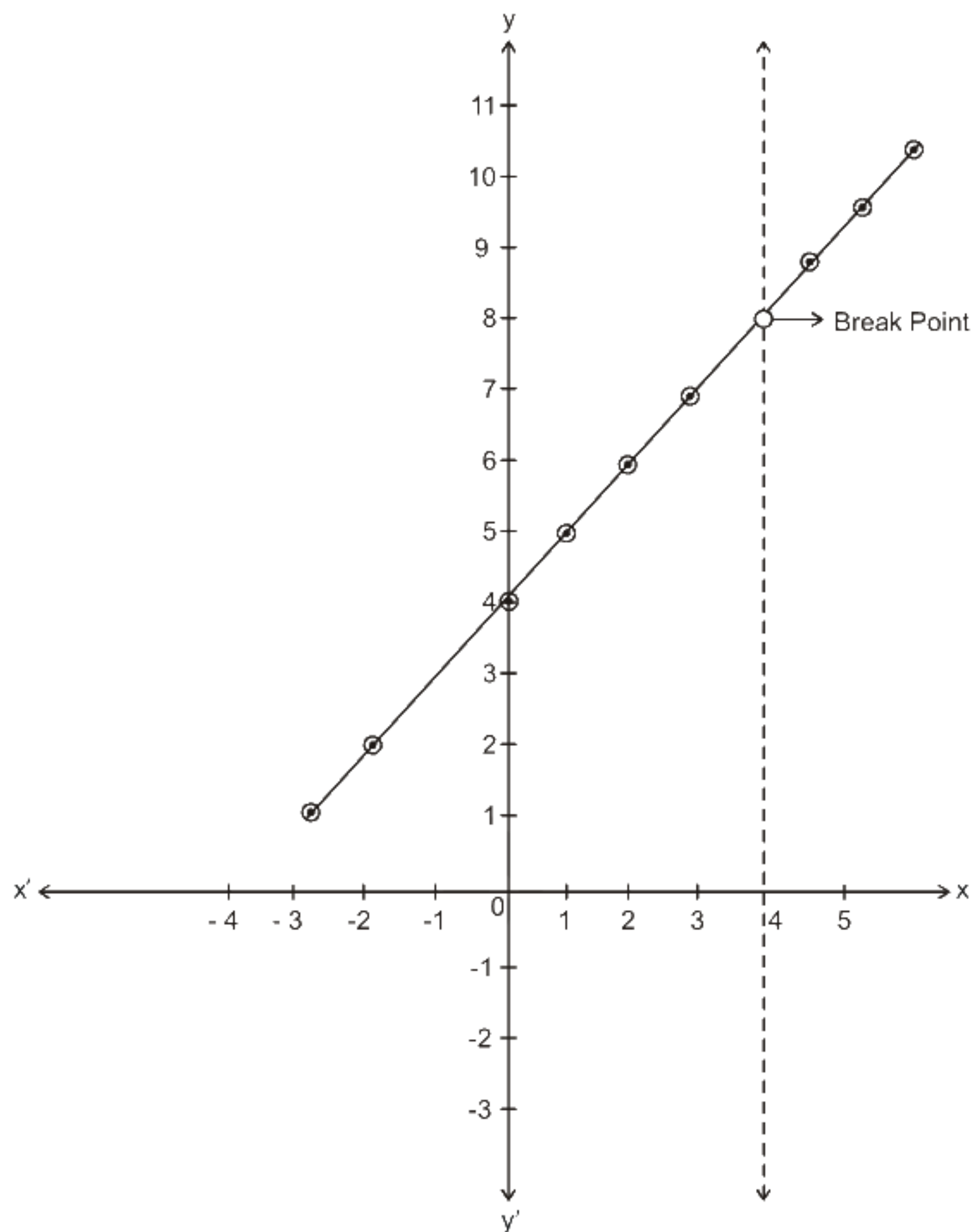
$$y = 2 \quad \text{if } x = 3$$



Since there is a break in a graph. So this function is not continuous at $x = 3$.

$$\begin{aligned}
 \text{(iv)} \quad y &= \frac{x^2 - 16}{x - 4}, \quad x \neq 4 \\
 &= \frac{(x + 4)(x - 4)}{x - 4}, \quad x \neq 4
 \end{aligned}$$

x	-3	-2	-1	0	1	2	3	5	6
y	1	2	3	4	5	6	7	9	10



Since there is a break in a graph. So this function is not continuous at $x = 4$.

Q.4 Find the graphical solution of the following equations.

(i) $x = \sin 2x$

(ii) $\frac{x}{2} = \cos x$

(iii) $2x = \tan x$

Solution:

(i) Let $y = x = \sin 2x$

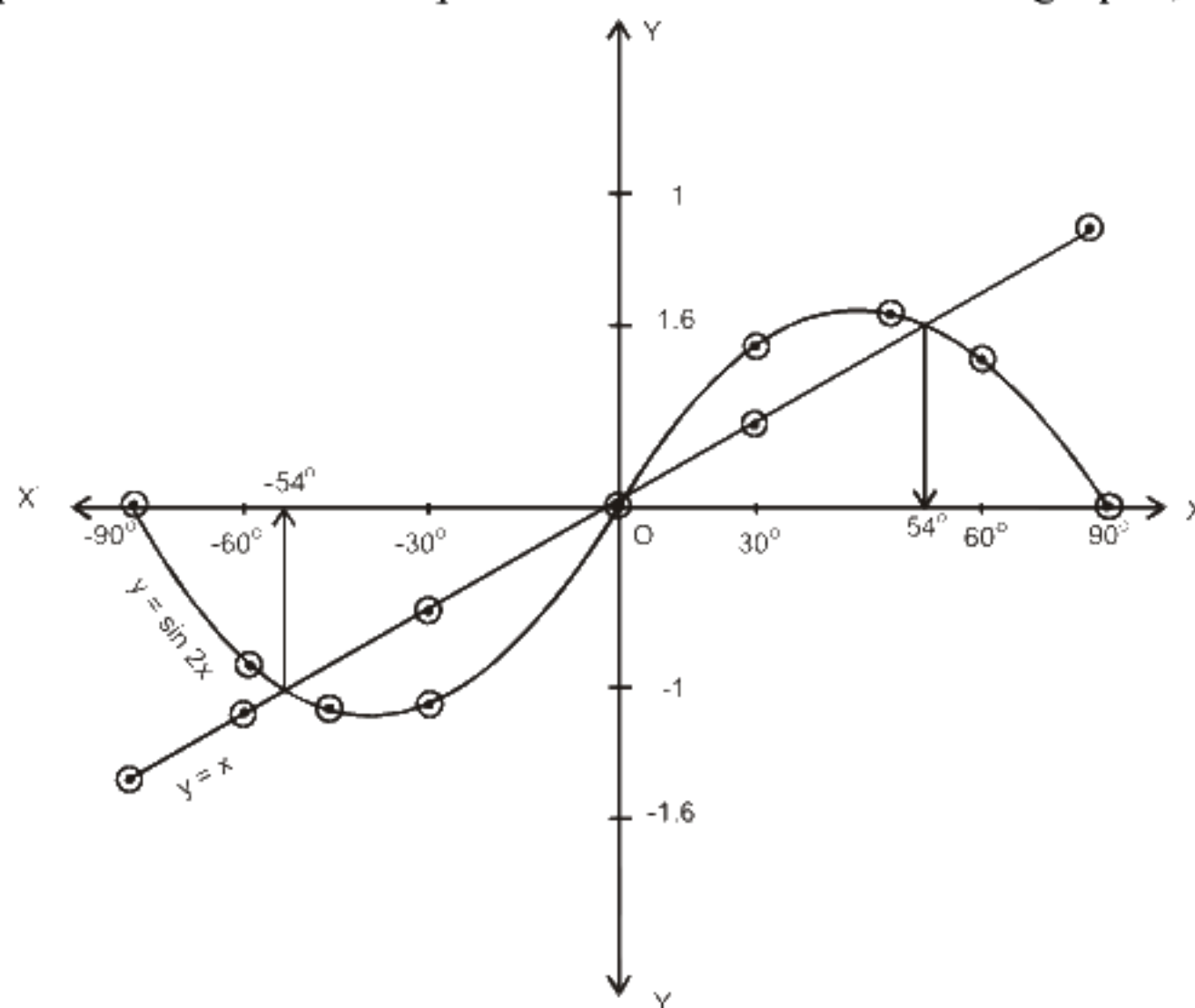
Therefore $y = x$ and $y = \sin 2x$

x	-90°	-60°	-30°	0°	30°	60°	90°
$y = x$	$-\pi/2 = -1.6$	$-\pi/3 = -1.05$	$-\pi/6 = -0.52$	0	$\pi/6 = 0.52$	$\pi/3 = 1.05$	$\pi/2 = 1.6$

$$y = \sin 2x$$

x	-90°	-60°	-30°	0°	30°	60°	90°
$y = \sin 2x$	0	-0.87	-0.87	0	0.87	0.87	0

The graphical solution is the points of intersection of two graphs, i.e. $x = 0^\circ, 54^\circ$



(ii) Let $y = \frac{x}{2} = \cos x$

Therefore $y = \frac{x}{2}$ and $y = \cos x$

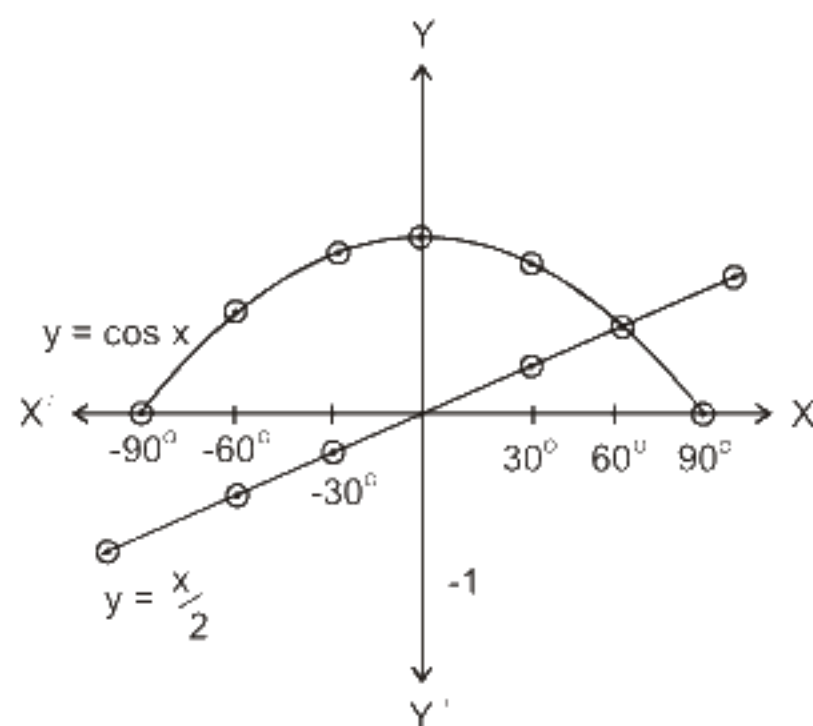
$$y = \frac{x}{2}$$

x	-90°	-60°	-30°	0°	30°	60°	90°
$y = \frac{x}{2}$	$-\pi/4$ = -0.79	$-\pi/6$ = -0.52	$-\pi/12$ = -0.26	0	$\pi/6$ = 0.26	$\pi/6$ = 0.52	$\pi/4$ = 0.79

$$y = \cos x$$

x	-90°	-60°	-30°	0°	30°	60°	90°
$y = \cos x$	0	0.5	0.87	1	0.87	0.5	0

The graphical solution is the point on x-axis, which is just below the point of intersection of two graphs. Hence $x = 60^\circ$.



- (iii) Let $y = 2x = \tan x$
 Therefore $y = 2x$ and $y = \tan x$
 $y = 2x$

x	-90°	-60°	-30°	0°	30°	60°	90°
$y = 2x$	$-\pi - 3.14$	$-2\pi/3 - 2.09$	$-\pi/3 - 1.05$	0	$\pi/3 - 1.05$	$2\pi/3 - 2.09$	$\pi - 3.14$

$$y = \tan x$$

x	-90°	-60°	-30°	0°	30°	60°	90°
$y = \tan x$	∞	-1.73	-0.58	0	0.58	1.73	∞

The graphical solution is the point of intersection of two graphs, i.e. $x = 0^\circ$.

