

EXERCISE 2.2

Q. Find from first principles, the derivatives of the following expressions w.r.t. their respective independent variables.

- (i) $(ax + b)^3$ (L.B 2004) (ii) $(2x + 3)^5$
 (iii) $(3t + 2)^{-2}$ (iv) $(ax + b)^{-5}$
 (v) $\frac{1}{(az - b)^7}$ (L.B 2010)

Solution:

(i) $(ax + b)^3$

Let $y = (ax + b)^3$

$$y + \delta y = [a(x + \delta x) + b]^3$$

$$\delta y = (ax + a\delta x + b)^3 - y$$

$$\delta y = (ax + b + a\delta x)^3 - (ax + b)^3 \quad \because y = (ax + b)^3$$

$$\delta y = \left[(ax + b) \left(1 + \frac{a\delta x}{ax + b} \right) \right]^3 - (ax + b)^3$$

$$\delta y = (ax + b)^3 \left(1 + \frac{a\delta x}{ax + b} \right)^3 - (ax + b)^3$$

$$\delta y = (ax + b)^3 \left[\left(1 + \frac{a\delta x}{ax + b} \right)^3 - 1 \right]$$

$$\delta y = (ax + b)^3 \left[1 + 3 \left(\frac{a\delta x}{ax + b} \right) + \frac{3(3-1)}{2!} \cdot \left(\frac{a\delta x}{ax + b} \right)^2 + \dots - 1 \right]$$

$$\delta y = (ax + b)^3 \cdot \frac{a\delta x}{ax + b} \left[3 + \frac{3(3-1)}{2!} \cdot \frac{a\delta x}{ax + b} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{(ax + b)^{3-1} \cdot a\delta x}{\delta x} \left[3 + \frac{3(3-1)}{2!} \cdot \frac{a\delta x}{ax + b} + \dots \right]$$

$$\frac{\delta y}{\delta x} = a(ax + b)^2 \left[3 + \frac{3(3-1)}{2!} \cdot \frac{a\delta x}{ax + b} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} a(ax + b)^2 \left[3 + \frac{3(3-1)}{2!} \cdot \frac{a\delta x}{ax + b} + \dots \right]$$

$$\frac{dy}{dx} = 3a(ax + b)^2$$

$$\boxed{\frac{d}{dx} (ax + b)^3 = 3a(ax + b)^2} \quad \text{Ans.}$$

(ii) $(2x + 3)^5$

$$\begin{aligned} \text{Let } y &= (2x + 3)^5 \\ y + \delta y &= [2(x + \delta x) + 3]^5 \\ \delta y &= (2x + 2\delta x + 3)^5 - y \\ \delta y &= (2x + 3 + 2\delta x)^5 - (2x + 3)^5 \quad \because y = (2x + 3)^5 \\ \delta y &= \left[(2x + 3) \left(1 + \frac{2\delta x}{2x + 3} \right) \right]^5 - (2x + 3)^5 \\ \delta y &= (2x + 3)^5 \left(1 + \frac{2\delta x}{2x + 3} \right)^5 - (2x + 3)^5 \\ \delta y &= (2x + 3)^5 \left[\left(1 + \frac{2\delta x}{2x + 3} \right)^5 - 1 \right] \\ \delta y &= (2x + 3)^5 \left[1 + 5 \left(\frac{2\delta x}{2x + 3} \right) + \frac{5(5-1)}{2!} \cdot \left(\frac{2\delta x}{2x + 3} \right)^2 + \dots - 1 \right] \\ \delta y &= (2x + 3)^5 \frac{2\delta x}{2x + 3} \left[5 + \frac{5(5-1)}{2!} \cdot \frac{2\delta x}{2x + 3} + \dots \right] \end{aligned}$$

Dividing both sides by δx .

$$\begin{aligned} \frac{\delta y}{\delta x} &= \frac{2\delta x (2x + 3)^{5-1}}{\delta x} \left[5 + \frac{5(5-1)}{2!} \cdot \frac{2\delta x}{2x + 3} + \dots \right] \\ \frac{\delta y}{\delta x} &= 2(2x + 3)^4 \left[5 + \frac{5(5-1)}{2!} \cdot \frac{2\delta x}{2x + 3} + \dots \right] \end{aligned}$$

Taking limit $\delta x \rightarrow 0$

$$\begin{aligned} \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= \lim_{\delta x \rightarrow 0} 2(2x + 3)^4 \left[5 + \frac{5(5-1)}{2!} \cdot \frac{2\delta x}{2x + 3} + \dots \right] \\ \frac{dy}{dx} &= 2(2x + 3)^4 \cdot 5 \end{aligned}$$

$$\boxed{\frac{d}{dx} (2x + 3)^5 = 10(2x + 3)^4} \quad \text{Ans.}$$

(iii) $(3t + 2)^{-2}$

$$\begin{aligned} \text{Let } y &= (3t + 2)^{-2} \\ y + \delta y &= [3(t + \delta t) + 2]^{-2} \\ \delta y &= (3t + 3\delta t + 2)^{-2} - y \\ \delta y &= (3t + 2 + 3\delta t)^{-2} - (3t + 2)^{-2} \quad \because y = (3t + 2)^{-2} \end{aligned}$$

$$\begin{aligned}
 \delta y &= \left[(3t+2) \left(1 + \frac{3\delta t}{3t+2} \right) \right]^{-2} - (3t+2)^{-2} \\
 \delta y &= (3t+2)^{-2} \left(1 + \frac{3\delta t}{3t+2} \right)^{-2} - (3t+2)^{-2} \\
 \delta y &= (3t+2)^{-2} \left[\left(1 + \frac{3\delta t}{3t+2} \right)^{-2} - 1 \right] \\
 \delta y &= (3t+2)^{-2} \left[1 + (-2) \left(\frac{3\delta t}{3t+2} \right) + \frac{(-2)(-2-1)}{2!} \cdot \left(\frac{3\delta t}{3t+2} \right)^2 + \dots - 1 \right] \\
 \delta y &= (3t+2)^{-2} \frac{3\delta t}{3t+2} \left[-2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{3\delta t}{3t+2} + \dots \right]
 \end{aligned}$$

Dividing both sides by δt .

$$\begin{aligned}
 \frac{\delta y}{\delta t} &= \frac{3\delta t}{\delta t(3t+2)^{1+2}} \left[-2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{3\delta t}{3t+2} + \dots \right] \\
 \frac{\delta y}{\delta t} &= \frac{3}{(3t+2)^3} \left[-2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{3\delta t}{3t+2} + \dots \right]
 \end{aligned}$$

Taking limit $\delta t \rightarrow 0$

$$\begin{aligned}
 \lim_{\delta t \rightarrow 0} \frac{\delta y}{\delta t} &= \lim_{\delta t \rightarrow 0} \frac{3}{(3t+2)^3} \left[-2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{3\delta t}{3t+2} + \dots \right] \\
 \frac{dy}{dt} &= \frac{3}{(3t+2)^3} (-2)
 \end{aligned}$$

$$\boxed{\frac{d}{dt} (3t+2)^{-2} = \frac{-6}{(3t+2)^3}} \quad \text{Ans.}$$

(iv) $(ax+b)^{-5}$

Let $y = (ax+b)^{-5}$

$$y + \delta y = [a(x + \delta x) + b]^{-5}$$

$$\delta y = (ax + a\delta x + b)^{-5} - y$$

$$\delta y = (ax + b + a\delta x)^{-5} - (ax + b)^{-5} \quad \because y = (ax + b)^{-5}$$

$$\delta y = \left[(ax + b) \left(1 + \frac{a\delta x}{ax + b} \right) \right]^{-5} - (ax + b)^{-5}$$

$$\delta y = (ax + b)^{-5} \left(1 + \frac{a\delta x}{ax + b} \right)^{-5} - (ax + b)^{-5}$$

$$\delta y = (ax + b)^{-5} \left[\left(1 + \frac{a\delta x}{ax + b} \right)^{-5} - 1 \right]$$

$$\delta y = (ax+b)^{-5} \left[1 + (-5) \left(\frac{a\delta x}{ax+b} \right) + \frac{(-5)(-5-1)}{2!} \cdot \left(\frac{a\delta x}{ax+b} \right)^2 + \dots - 1 \right]$$

$$\delta y = (ax+b)^{-5} \frac{a\delta x}{ax+b} \left[-5 + \frac{(-5)(-5-1)}{2!} \cdot \frac{a\delta x}{ax+b} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{a\delta x}{\delta x(ax+b)^{1+5}} \left[-5 + \frac{(-5)(-5-1)}{2!} \cdot \frac{a\delta x}{ax+b} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{a}{(ax+b)^6} \left[-5 + \frac{(-5)(-5-1)}{2!} \cdot \frac{a\delta x}{ax+b} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{a}{(ax+b)^6} \left[-5 + \frac{(-5)(-5-1)}{2!} \cdot \frac{a\delta x}{ax+b} + \dots \right]$$

$$\frac{dy}{dx} = \frac{a}{(ax+b)^6} (-5)$$

$$\boxed{\frac{d}{dt} (ax+b)^{-5} = \frac{-5a}{(ax+b)^6}} \quad \text{Ans.}$$

(v) $\frac{1}{(az-b)^7}$

Let $y = \frac{1}{(az-b)^7}$

$$y = (az-b)^{-7}$$

$$y + \delta y = [a(z + \delta z) - b]^{-7}$$

$$\delta y = (az + a\delta z - b)^{-7} - y$$

$$\delta y = (az - b + a\delta z)^{-7} - (az - b)^{-7} \quad \because y = (az - b)^{-7}$$

$$\delta y = \left[(az - b) \left(1 + \frac{a\delta z}{az - b} \right) \right]^{-7} - (az - b)^{-7}$$

$$\delta y = (az - b)^{-7} \left(1 + \frac{a\delta z}{az - b} \right)^{-7} - (az - b)^{-7}$$

$$\delta y = (az - b)^{-7} \left[\left(1 + \frac{a\delta z}{az - b} \right)^{-7} - 1 \right]$$

$$\delta y = (az - b)^{-7} \left[1 + (-7) \left(\frac{a\delta z}{az - b} \right) + \frac{(-7)(-7-1)}{2!} \cdot \left(\frac{a\delta z}{az - b} \right)^2 + \dots - 1 \right]$$

$$\delta y = (az - b)^{-7} \frac{a\delta z}{az - b} \left[-7 + \frac{(-7)(-7-1)}{2!} \cdot \frac{a\delta z}{az - b} + \dots \right]$$

Dividing both sides by δz .

$$\frac{\delta y}{\delta z} = \frac{a\delta z}{\delta z(az-b)^{1+7}} \left[-7 + \frac{(-7)(-7-1)}{2!} \cdot \frac{a\delta z}{az-b} + \dots \right]$$

$$\frac{\delta y}{\delta z} = \frac{a}{(az-b)^8} \left[-7 + \frac{(-7)(-7-1)}{2!} \cdot \frac{a\delta z}{az-b} + \dots \right]$$

Taking limit $\delta z \rightarrow 0$

$$\lim_{\delta z \rightarrow 0} \frac{\delta y}{\delta z} = \lim_{\delta z \rightarrow 0} \frac{a}{(az-b)^8} \left[-7 + \frac{(-7)(-7-1)}{2!} \cdot \frac{a\delta z}{az-b} + \dots \right]$$

$$\frac{dy}{dz} = \frac{a}{(az-b)^8} (-7)$$

$$\boxed{\frac{d}{dz} \left[\frac{1}{(az-b)^7} \right] = \frac{-7a}{(az-b)^8}} \quad \text{Ans.}$$

EXERCISE 2.3

Q.1 Differentiate w.r.t. 'x'

$$x^4 + 2x^3 + x^2$$

Solution:

$$\text{Let } y = x^4 + 2x^3 + x^2$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} (x^4) + 2 \frac{d}{dx} (x^3) + \frac{d}{dx} (x^2)$$

$$\frac{dy}{dx} = 4x^{4-1} \cdot \frac{d}{dx} (x) + 2 \cdot 3x^{3-1} \cdot \frac{d}{dx} (x) + 2x^{2-1} \cdot \frac{d}{dx} (x)$$

$$\frac{dy}{dx} = 4x^3 \cdot 1 + 6x^2 \cdot 1 + 2x \cdot 1$$

$$\boxed{\frac{dy}{dx} = 4x^3 + 6x^2 + 2x} \quad \text{Ans.}$$

Q.2 $x^{-3} + 2x^{-3/2} + 2$

Solution:

$$\text{Let } y = x^{-3} + 2x^{-3/2} + 2$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} (x^{-3}) + 2 \frac{d}{dx} (x^{-3/2}) + \frac{d}{dx} (2)$$